<u>Overall</u>: The mathematics tasks focus on developing CONCEPTUAL UNDERSTANDING and encouraging ALL students to make sense of the mathematics and to persevere in solving mathematical problems. As you observe, check to see if <u>STUDENTS</u> exhibited the following behaviors in solving mathematics problems and if <u>TEACHERS</u> facilitated these behaviors by providing cognitively demanding tasks and encouraging sense making for ALL students.

Mathematical	Teacher:	Student:
Practice Standard	Actions/Responsibilities	Actions/Responsibilities
1. MAKES SENSE OF	Teacher:	Students:
PROBLEMS AND PERSEVERES IN SOLVING THEM	 Provides an open-ended problem with no solution pathway evident and/or non-routine problems with multiple solutions. Provides time and facilitates discussion in problem. 	 Are actively engaged in solving problems & thinking is visible (i.e., DOING MATHEMATICS vs. FOLLOWING STEPS OR PROCEDURES). Are applying givens, constraints, relationships, and goals
	solutions.	(NOT the teacher).
	 Facilitates discourse in the classroom so that students UNDERSTAND the approaches of others. Provides opportunities for students to explain themselves, the meaning of a problem, etc. Provides opportunities for students to connect concepts 	 Are discussing with one another, making conjectures, planning a solution pathway, not jumping into a solution attempt or guessing at the direction to take. Relate current "situation" to concept or skill previously learned and check answers using different methods.
	 to "their" world. Provides students TIME to think and become "patient" problem solvers. Facilitates and encourages students to check their answers using different methods (not calculators). Provides problems that focus on relationships and are "generalizable". 	 Continually ask self, does this make sense?

Mathematical	Teacher:	Student:
Practice Standard	Actions/Responsibilities	Actions/Responsibilities
2. REASONING ABSTRACTLY AND QUANTITATIVELY	 Teacher: Provides a range of representations of math problem situations and encourages various solutions. Provides opportunities for students to make sense of quantities and their relationships in problem situations. Provides problems that require flexible use of properties of operations and objects. Emphasizes quantitative reasoning which entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them and/or rules; and knowing and flexibly using different properties of operations and objects. 	 Students: Use varied representations and approaches when solving problems. Make sense of quantities and their relationships in problem situations. Are <i>decontextualizing</i> (abstract a given situation and represent it symbolically and manipulate the representing symbols), and <i>contextualizing</i> (pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Use quantitative reasoning that entails creating a coherent representation of the problem at hand, considering the units involved, and attending to the meaning of quantities, NOT just how to compute them.
3. CONSTRUCTING VIABLE ARGUMENTS AND CRITIQUING THE ARGUMENTS OF OTHERS	 Teacher: Uses tasks that allow students to analyze situations by breaking them into cases and then justify, defend/refute and communicate examples and counterexamples, etc. Provides ALL students opportunities to understand and use stated assumptions, definitions, and previously established results in constructing arguments. Provides ample time for students to make conjectures and build a logical progression of statements to explore the truth of their conjectures. Provides opportunities for students to construct arguments and critique arguments of peers. Facilitates and guides students in recognizing and using counterexamples. Encourages and facilitates students justifying their conclusions, communicating, and responding to the arguments of others. Asks useful questions to clarify and/or improve students' arguments. 	 Students: Make conjectures and explore the truth of their conjectures. Recognize and use counterexamples. Justify and defend ALL conclusions and communicates them to others. Recognize and explain flaws in arguments. (After listening or reading arguments of others, they respond by deciding whether or not they make sense. They ask useful questions to improve arguments.) <u>Elementary</u> Students: construct arguments using concrete referents such as objects, drawings, diagrams, actions. Later, students learn to determine the domains to which an argument applies.

Mathematical	Teacher:	Student:
Practice Standard	Actions/Responsibilities	Actions/Responsibilities
4. MODEL WITH MATHEMATICS	 Teacher: Provides problem situations that apply to everyday life. Provides rich tasks that focus on conceptual understanding, relationships, etc. 	 Students: Apply the mathematics they know to everyday life, society, and the workplace. Write equations to describe situations. Are comfortable in making assumptions and approximations to simplify complicated situations. Analyze relationships to draw conclusions. Improve their model if it has not served its purpose.
5. ATTENDS TO PRECISION	 Teacher: Facilitates, encourages and <u>expects</u> precision in communication including correct usage of mathematical vocabulary. Provides opportunities for students to explain and/or write their reasoning to others. 	 Students: Use and clarify mathematical definitions in discussions and in their own reasoning (orally and in writing). Use, understand and state the meanings of symbols. Express numerical answers with a degree of precision.
6. APPROPRIATE TOOLS USED	 Teacher: Provides a variety of tools and technology for students to explore to deepen their understanding of math concepts. Provides problem solving tasks that require students to consider a variety of tools for solving. (Tools might include pencil/paper, concrete models, ruler, protractor, calculator, spreadsheet, computer algebra system, statistical package, or dynamic geometry software, etc.) 	 Students: Consider available tools when solving a mathematical problem. Are familiar with a variety of mathematics tools and use them when appropriate to explore and deepen their understanding of concepts.

Mathematical	Teacher:	Students:
Practice Standard	Actions/Responsibilities	Actions/Responsibilities
7. LOOK FOR AND MAKE USE OF STRUCTURE	 Teacher: Provides opportunities and time for students to explore patterns and relationships to solve problems. Provides rich tasks and facilitates pattern seeking and understanding of relationships in numbers rather than following a set of steps and/or procedures. 	 Students: Look closely to discern patterns or structure. Associate patterns with properties of operations and their relationships. Step back for an overview and can shift perspective. See complicated things, such as algebraic expressions, as single objects or as composed of several objects. (Younger children decompose and compose numbers.)
8. LOOK FOR AND EXPRESS REGULARITY IN REPEATED REASONING	 Teacher: Provides problem situations that allow students to explore regularity and repeated reasoning. Provides rich tasks that encourage students to use repeated reasoning to form generalizations and provides opportunities for students to communicate these generalizations. 	 Students: Notice if calculations are repeated and look for both general methods and shortcuts. Pay attention to regularity and use to solve problems. Use regularity and use this to lead to a general formula and generalizations. Maintain oversight of the process of solving a problem while attending to details and continually evaluates the reasonableness of immediate results.