

Lesson Plan: 8.EE.B.5 - Graphing Proportional Relationships

(This lesson should be adapted, including instructional time, to meet the needs of your students.)

Background Information	
Content/Grade Level	Expressions and Equations/ Grade 8
Unit	Understand the connections between proportional relationships, lines, and linear equations.
Essential Questions/Enduring Understandings Addressed in the Lesson	<p>How can you use data in multiple formats to determine similarities and differences among them?</p> <p>Real-world data can be expressed in multiple contexts; there is an essential need to interpret the data regardless of the format.</p>
Standards Addressed in This Lesson	<p>8.EE.B.5: Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.</p> <p>It is critical that the Standards for Mathematical Practice be incorporated in ALL lesson activities throughout each unit as appropriate. It is not the expectation that all eight Mathematical Practices will be evident in every lesson. The Standards for Mathematical Practice make an excellent framework on which to plan instruction. Look for the infusion of the Mathematical Practices throughout this unit.</p>
Lesson Topic	Relating and comparing multiple representations of proportional relationships
Relevance/Connections	<p>8.EEB.6: Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin, and the equation $y = mx + b$ for a line intercepting the vertical axis at b.</p> <p>8.F.A.1: Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output (function notation is not required in Grade 8).</p> <p>8.F.A.2: Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).</p> <p>8.F.B.4: Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values,</p>

Background Information	
	<p>including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.</p> <p>8.F.B.5: Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.</p> <p>8.SP.A.2: Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.</p> <p>8.SP.A.3: Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.</p>
Student Outcomes	<ul style="list-style-type: none"> • Ability to relate and compare graphic, symbolic, and numerical representations of proportional relationships • Ability to determine constant rate of change/slope of a line graphically
Prior Knowledge Needed to Support This Learning	<p>6.EE.C.9: Represent and analyze quantitative relationships between dependent and independent variables.</p> <p>7.EE.B.3 and 7.EE.B.4: Solve real-life and mathematical problems using numerical and algebraic expressions and equations.</p> <p>7.RP.A.1-3: Analyze proportional relationships and use them to solve real-world and mathematical problems.</p>
Method for determining student readiness for the lesson	<p>Use questions from attached activity <i>Determining Student Readiness for Lesson</i> to assess student understanding of:</p> <ul style="list-style-type: none"> • Ability to read and interpret tables • Ability to read and interpret graphs on a coordinate plane • Ability to use variables to represent quantities in a real-world problem and create a graph <p>Note: To inform instruction of this lesson, the teacher may wish to measure student readiness the day before this lesson is to be taught.</p>
Materials	Attached

Learning Experience		
<i>Component</i>	<i>Details</i>	<i>Which Standards for Mathematical Practice does this activity address? How is each Practice used to help students develop proficiency?</i>
<i>Warm Up/Motivation for Activity 1</i>	<p>Provide students with the following scenario:</p> <p>A conveyer belt in a factory moves at a constant rate of speed of $39\frac{1}{2}$ feet per minute. After how many minutes will the conveyer belt have moved $177\frac{3}{4}$ feet?</p> <p>Correct Response: $4\frac{1}{2}$ minutes</p> <p>Teacher Note: Give students choices of how they can solve the given problem. Students may:</p> <ul style="list-style-type: none"> • Create an equation and solve. • Create a function table. • Use continuous addition or subtraction. • Create a graph. • Perform operations without an equation. 	
<p><i>Activity 1</i></p> <p>UDL Components</p> <ul style="list-style-type: none"> • Multiple Means of Representation • Multiple Means for Action and Expression • Multiple Means for Engagement <p>Key Questions Formative Assessment Summary</p>	<p>UDL Components:</p> <ul style="list-style-type: none"> • <u>Principle I: Representation</u> is present in the activity. Prior knowledge is activated through the Warm-up and Motivation. Options for language, mathematical expressions and symbols are supported by dealing with data in a table, a graph, and an equation. • <u>Principle II: Expression</u> is present in the activity. The grouping and re-grouping of students for different tasks provides options for physical activity. Similarly, students interact with and engage other students about their ideas as they review each other's work posted on the wall. 	<ul style="list-style-type: none"> • Make sense of problems and persevere in solving them by requiring students to display text message data using tables, graphs, and equations; and compare data represented in these different formats. (SMP # 1) • Reason abstractly and quantitatively by requiring students to perceive relationships between number of text messages and time; and to decontextualize these quantitative relationships. (SMP #2)

Learning Experience

- Principle III: Engagement is present in the activity. Sources of information are in an authentic, student-centered context. The task allows for active participation, exploration and experimentation, and also invites personal response, evaluation and reflection.

Directions:

Present the students with the following scenario:

Sound travels through dry air at the rate of 343.2 meters per second. Create a representation that shows how far sound travels during any given length of time.

Split the class into groups to develop one representation per group for the scenario:

- Assign some groups to create a table of values.
- Assign some groups to create a graph.
- Assign some groups to create an equation.

Regroup the students into new groups of 3: one from graphing, one from tables, and one from equations:

- Each student should report to the other two on what they did in the first group and share their data.
- The students should then look for similarities and differences among all three methods. Each group should complete the attachment *Activity 1: Similarity/Difference Table*.
- Each group should post its completed table on a wall.
- Provide student groups with time to get up and review each posted table of similarities/differences.
- Then, facilitate a full class discussion to allow students time to talk about their findings, remaining questions, etc. from the walk/review.
- If students do not bring up “unit rate” be certain to lead them into a brief discussion of 7.RP.A.2b. (constant of proportionality).

- Construct viable arguments and critique the reasoning of others by requiring students to justify their conclusions with mathematical ideas in their group discussions. (SMP #3)
- Attend to precision by expecting students to calculate their equations accurately; use clear mathematical language when expressing their ideas about how they represented the text message in a table, graph, or equation; understand the meaning of variables x and y and the constant in the context of the problem. (SMP #6)

Learning Experience		
<i>Closure</i>	Use the attached activity sheet, <i>Activity 1: Pulling It Together</i> to provide students with connections to standards in other grade 8 domains.	
<i>Warm-up/Motivation for Activity 2</i>	<p>Show a short video of someone filling up a pool, (e.g., http://www.istockphoto.com/stock-video-16947873-filling-up-a-pool-with-hose.php)</p> <p>Present the students with the following scenario: <i>A swimming pool needs to be filled by using a hose. What information do we have to identify in order to determine the amount of time needed to fill the pool?</i></p> <p>Possible responses:</p> <ul style="list-style-type: none"> • Size of pool • Rate of water from hose – connect with “unit rate,” “constant rate,” and “constant of proportionality” • Size of hose • Lead the students to the conclusion that the sizes of the pool and the hose are fixed values and the real determining factor is the rate of the water flow from the hose. • Connect this idea to “slope” 	
<p><i>Activity 2</i></p> <p>UDL Components</p> <ul style="list-style-type: none"> • Multiple Means of Representation • Multiple Means for Action and Expression • Multiple Means for Engagement <p>Key Questions</p> <p>Formative Assessment</p> <p>Summary</p>	<p>UDL Components:</p> <ul style="list-style-type: none"> • <u>Principle I: Representation</u> is present in the activity. Prior knowledge is activated through the Warm-up and Motivation. Key elements in the task, such “fixed values” and “rate of change,” are emphasized so students are better able to identify them when they set up the problem. • <u>Principle II: Expression</u> is present in the activity. The tasks are prompted and depend on scaffolding. Students work cooperatively in a group on several examples before completing and independent task. . • <u>Principle III: Engagement</u> is present in the activity. Sources of information are in an authentic, student-centered context. 	<ul style="list-style-type: none"> • Make sense of problems and persevere in solving them by requiring students to analyze the given information, and decide upon a strategy for determining the comparative unit rates for the pools. (SMP #1) • Reason abstractly and quantitatively by requiring students to attend to the meaning of quantities represented in the graphs and to their relationships, as well as requiring students to apply general mathematical

Learning Experience

	<p>The task allows for active participation, exploration and experimentation, and also invites personal response, evaluation and reflection</p> <p>Directions: Put the students into groups of 2 or 3:</p> <ul style="list-style-type: none"> • Provide students with Activity 2: Filling the Swimming Pool activity sheets. Each group needs one copy of the problem and graphs, but every student needs a copy of the tasks. • Each group should complete tasks 1- 3 cooperatively. • Each group then should share their results with the class. • After class discussion, students should independently complete task 4. <p>NOTE: If the technology is available, this activity would lend itself well to being dropped into a computerized, interactive template that allows students to move a cursor along the graph of each line to determine unit rate, slope, and generally to compare how the water levels change as each swimming pool is filled with water.</p>	<p>rules, such as unit rate and slope, to specific situations in life. (SMP #2)</p> <ul style="list-style-type: none"> • Construct viable arguments and critique the reasoning of others by requiring students to explain and justify the mathematics they used to determine the water flow rates, and to defend their conclusions with mathematical ideas on their individual activity sheets. (SMP #3) • Attend to precision by requiring students to compute the water flow rates accurately and efficiently, and to communicate their ideas precisely using meaningful, exact mathematics vocabulary. (SMP #6)
<p>Closure</p>	<p>Distribute one copy of Activity 2: Exit Pass to each student. Ask them to place a checkmark next to the graphs which show a function that also represents a proportional relationship.</p>	
<p>Warm-up/Motivation for Activity 3</p>	<p>Show the students one or more of the following videos describing the process of picking crab meat from a crab. Provide written transcripts of videos for students who need them.</p> <p>http://www.youtube.com/watch?v=Cr5LDwWMma8 http://www.youtube.com/watch?v=FVBegtAvgLA</p> <p>Have the students read the following article to get background information with regard to crab picking competitions and the picking rate of champion crab pickers. http://baltimore.cbslocal.com/2011/09/04/eastern-shore-shows-its-crab-picking-might-at-crab-derby/</p>	

Learning Experience

Debrief afterwards and find the rates per minute of the two pickers.
(Ruth = 0.26 lbs / min; Greg = 0.09 lbs / min)

Activity 3

UDL Components

- Multiple Means of Representation
- Multiple Means for Action and Expression
- Multiple Means for Engagement

Key Questions

Formative Assessment Summary

UDL Components:

- Principle I: Representation is present in the activity. Options for perception are provided in the Warm-up and Motivation. Students are accommodated visually through the video, as well as with transcripts for the video. These techniques also pre-teach critical prerequisite concepts in preparations for the main task of the activity.
- Principle II: Expression is present in the activity. Students are provided with graphing calculators and pre-formatted graph paper. Also, students complete guided questions and discuss their answers with a small group for purposes of self-monitoring.
- Principle III: Engagement is present in the activity. It is designed so that outcomes are authentic, communicate to real audiences, and are purposeful. Students are placed in groups so they discuss the processes they used to solve to answer the activity questions, and to obtain feedback from peers.

Directions:

- Tell students to use what they know about proportional relationships to represent the scenario described in **Activity 3: Picking Blue Crabs** (attached) in as many different ways as they can think of. This provides the teacher with a means of informally assessing what each student has learned up to this point in the lesson. **NOTE:** Teachers need to have graph paper available for students who wish to represent the scenario on a coordinate plane.
- Once students have each completed all of their representations independently, put the students into groups of four and have them discuss possible advantages and/or disadvantages of each type of representation for this scenario.
- Still sitting in their groups, ask the students to individually

Which Standards for Mathematical Practice does this activity address? How are the Practices used to help students develop proficiency?

- Make sense of problems by asking students to use what they know about proportional reasoning and the problem to describe in as many ways as they can think of the relationship between quantities of crabs and time.
(SMP #1)
- Reason abstractly and quantitatively by requiring students to define the Make sense of problems by asking students to make meaning of the problem use all they know about proportional.
(SMP #2)
- Construct viable arguments and critique the reasoning of others by placing students into groups to discuss the advantages or disadvantages of their representations.
(SMP #3)
- In the group and whole-class discussions, students must look for and express regularity in repeated reasoning to develop and express their understanding of the broader application of patterns, as well as their ability to identify the structure in the similar situations they find throughout the

Learning Experience		
	<p>complete the three questions for Activity 3: Picking Blue Crabs, and then confirm their answers with their group members.</p> <ul style="list-style-type: none"> Summarize Activity 3 with a whole-class debriefing. 	activities in this lesson. (SMP # 8)
Closure	On Ask students “What is the importance of the point (0, 0) on a graph in regard to proportional relationships?” Discussion should include how all proportional relationships graph back to the origin.	

Supporting Information	
<p>Interventions/Enrichments</p> <ul style="list-style-type: none"> Students with Disabilities/Struggling Learners ELL Gifted and Talented 	<p>For students who need reinforcement or an additional experience with direct variation and proportional reasoning, the historic First Moon Walk on July 20, 1969 provides an interesting scenario for students. They can further develop their conceptual understanding of proportional relationships by reasoning about their weight (or Neil Armstrong’s weight) on Earth versus on the Moon. Show a news clip/video to provide students visual information on the actual event. An example: http://www.bing.com/videos/search?q=first+moon+walk&view=detail&mid=7790908CF7F3356037947790908CF7F335603794&first=0&qpv=first+moon+walk</p> <p>If a motivation is needed, the following video shows Michael Jackson dancing his own version of the Moon Walk: http://www.youtube.com/watch?v=QZaw_J0D5p8&feature=fvwrel</p> <p>Although all students can benefit from the visual experiences that tie the videos in this lesson to the activities, ELL students especially need the visual support of videos. For additional, meaningful topics, ELL students can be encouraged to relate an authentic example/scenario from their native culture with something from their experiences living in the United States/Maryland.</p> <p>For students who are ready for a challenge, ask them to find an authentic application of direct variation with negative values (i.e., below sea level, sub-freezing temperature), which when graphed on the coordinate plane are in Quadrant IV. Research/discuss whether or not they think of an authentic scenario that places the linear relationship in Quadrant II or Quadrant III when graphed?</p>
Technology	<ul style="list-style-type: none"> A classroom projection device will be needed to display certain handouts; handouts may need adjustment depending on projection used. A means of playing and hearing the videos will be necessary. Access to graphing calculators is important.

Supporting Information

Resources	<p>Examples of Possible Resources for this Lesson:</p> <ul style="list-style-type: none">• http://www.istockphoto.com/stock-video-16947873-filling-up-a-pool-with-hose.php• http://www.youtube.com/watch?v=Cr5LDwWMma8• http://www.youtube.com/watch?v=FVBegtAvgLA• http://baltimore.cbslocal.com/2011/09/04/eastern-shore-shows-its-crab-picking-might-at-crab-derby/• http://www.bing.com/videos/search?q=first+moon+walk&view=detail&mid=7790908CF7F3356037947790908CF7F335603794&first=0&qpv=first+moon+walk• http://www.youtube.com/watch?v=QZaw_J0D5p8&feature=fvwrel

Determining Student Readiness for 8.EE.B.5

1. John has \$75 in his savings account. Each week he deposits \$7.

Which table represents the amount of money John has in his savings account after 5 weeks?

A

Week	Amount
1	75
2	80
3	85
4	90
5	95

B

Week	Amount
1	75
2	82
3	87
4	92
5	99

C

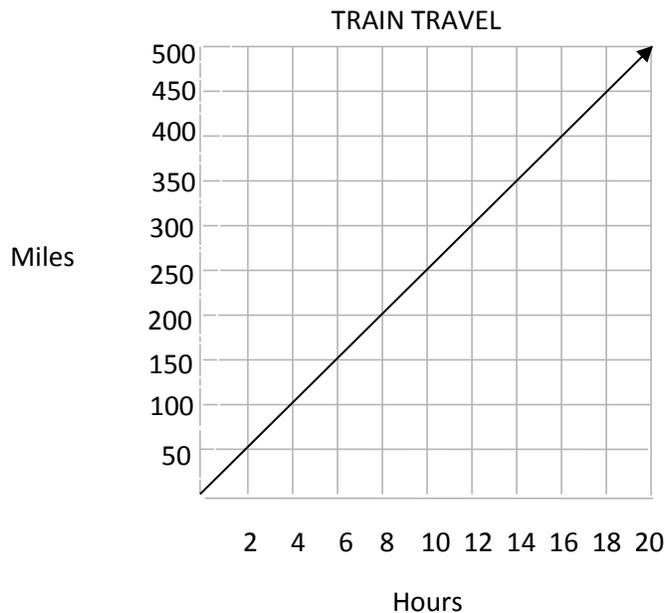
Week	Amount
1	75
2	82
3	89
4	96
5	103

D

Week	Amount
1	75
2	68
3	61
4	54
5	47

2. This graph represents the distance a train traveled over a 20 hour time period.

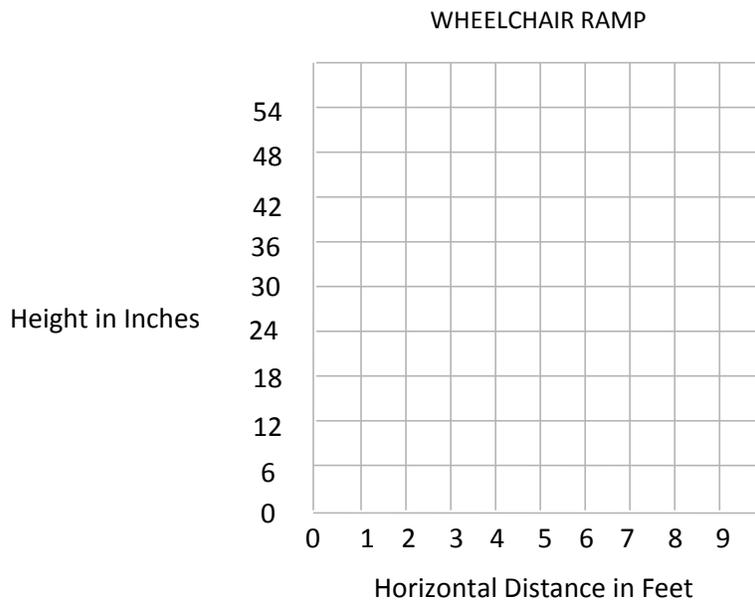
Approximately how far has the train traveled after 1 hour? After 5 hours?



3. The steepness of a wheelchair ramp measures 6 inches in height for every 12 inches of horizontal distance.
- Complete the table of values for the first 8 feet of a ramp.

Horizontal Distance (ft)	Height (in.)
1	
2	
3	18
4	
5	
6	
7	
8	

- Write an equation that can be used to represent the height (y) of the ramp in relation to the horizontal distance (x).
- Graph the equation that represents the steepness of the wheelchair ramp.



Answer Key

Determining Student Readiness for 8.EE.B.5

4. John has \$75 in his savings account. Each week he deposits \$7.

Which table represents the amount of money John has in his savings account after 5 weeks? Why do you think so?

A

Week	Amount
1	75
2	80
3	85
4	90
5	95

B

Week	Amount
1	75
2	82
3	87
4	92
5	99

C

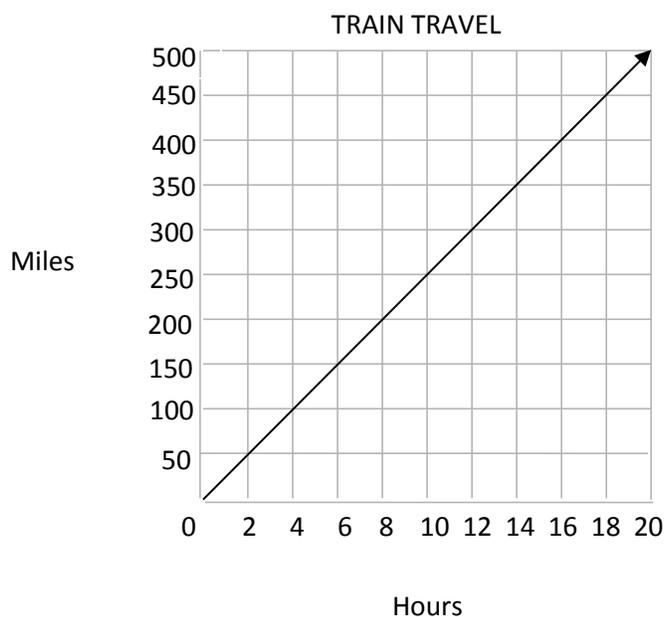
Week	Amount
1	75
2	82
3	89
4	96
5	103

D

Week	Amount
1	75
2	68
3	61
4	54
5	47

5. This graph represents the distance a train traveled over a 20 hour time period.

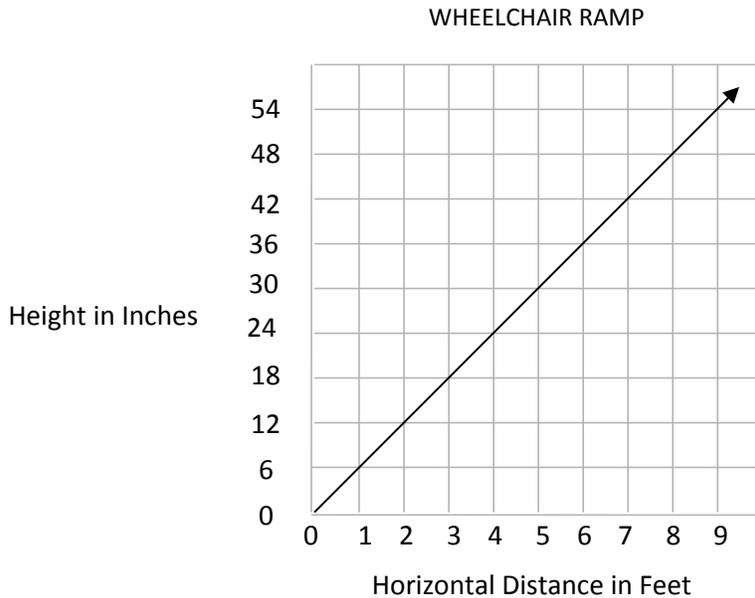
Approximately how far has the train traveled after 1 hour? After 5 hours? **Approximately 25 miles; 125 miles**



6. The steepness of a wheelchair ramp measures 6 inches in height for every 12 inches of horizontal distance.
- Complete the table of values for the first 8 feet of a ramp.

Horizontal Distance (ft)	Height (in.)
1	6
2	12
3	18
4	24
5	30
6	36
7	42
8	48

- Write an equation that can be used to represent the height (y) of the ramp in relation to the horizontal distance (x). $y = 6x$
- Graph the equation that represents the steepness of the wheelchair ramp. See graph below.



Activity 1: Similarity/Difference Table

<u>Similarities</u>	<u>Differences</u>
<u>Conclusions</u>	

- 1) What do you notice about the three representations? How are they the same? How are they different? Make notes in the table above.
- 2) How does the value of x change from one hour to the next? How do the values of y change in relationship to the values for x ? Fill in the sentence: **As the x -values change by _____, the y -values change by _____.**
- 3) Look at your answer to #2. Compare the values you wrote for the change in x and the change in y to the value of y when $x = 1$. Describe what you notice about each change in x and y *compared to* the value of y when $x = 1$?
- 4) Look at the graph your group member created with a graph. Describe what you notice about the change in x compared to the value of y , as the line rises from left to right?
- 5) Write a sentence or two about your group's most important observations and conclusions in the bottom space of the table.

Function

The relationship between the length of the circumference (C) of a circle and the length of its diameter (d) can be represented by the linear function: $C = \pi d$.

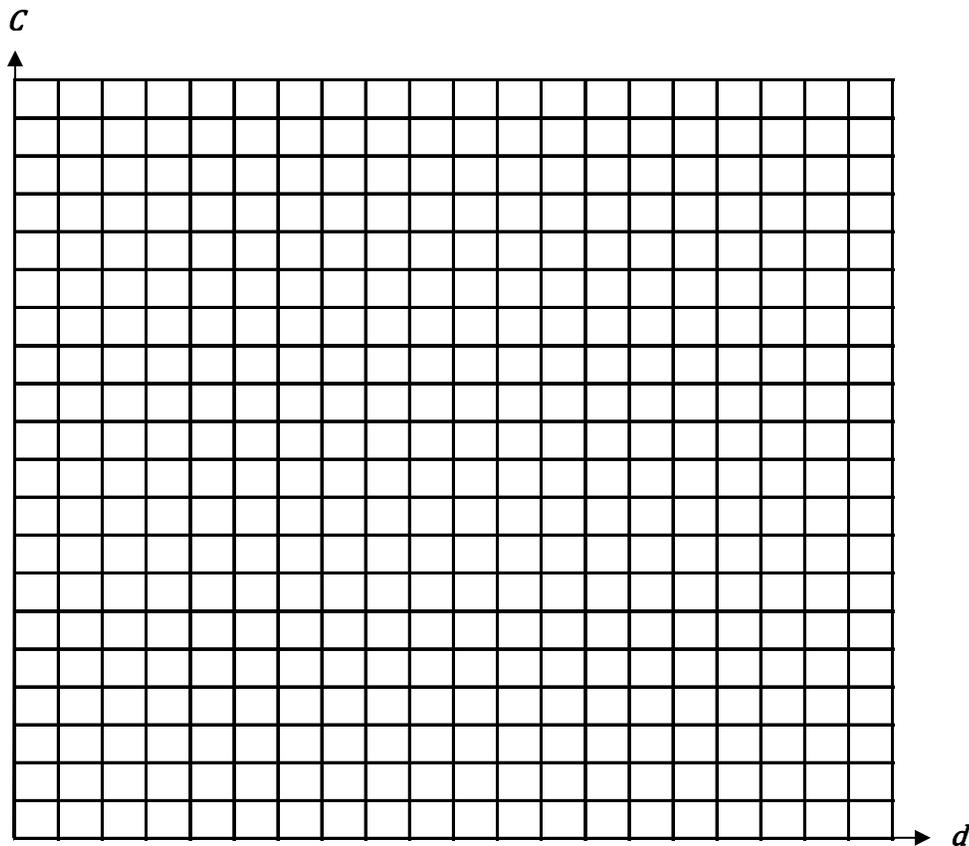
If we use 3.14 for π , describe the ratio between the circumference and the diameter of a circle.

Function Table

d (Input) centimeters	C (Output) centimeters
2.5	
	18.84
10.2	
27.6	
	103.62

Use 3.14 for π to determine the missing values in the function table.

- (1) Determine appropriate scales for the x-axis and y-axis. (2) Plot the values from the function table on the graph.

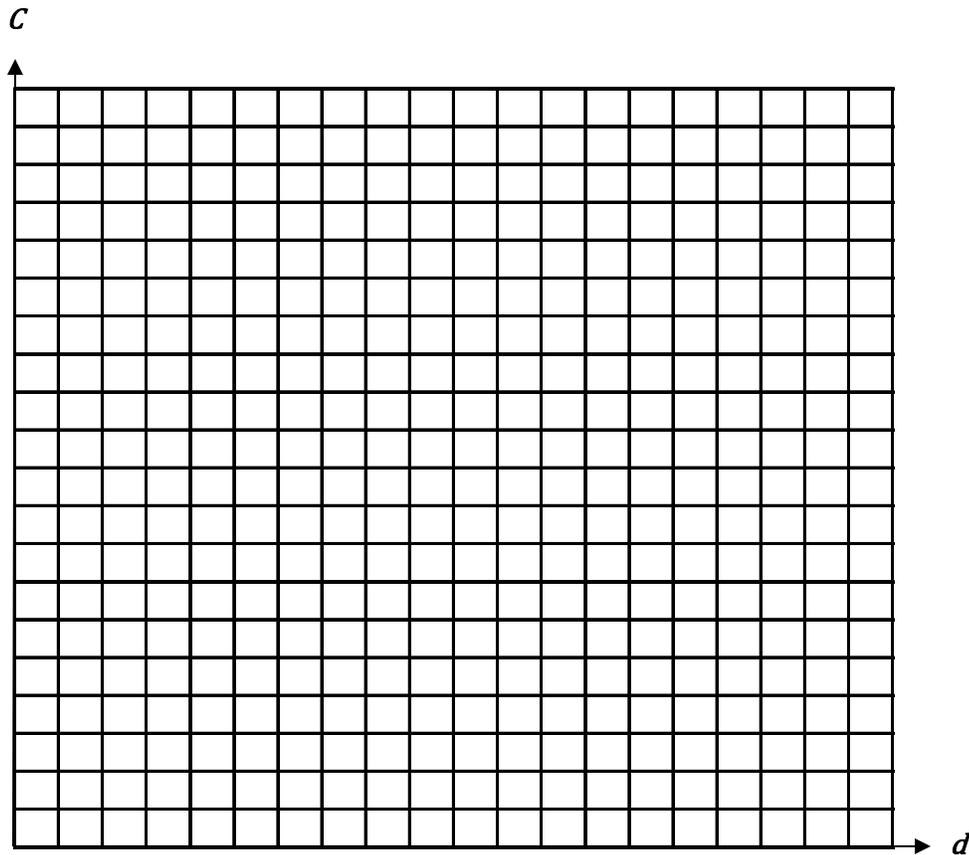


Think About It:

Is the linear function $C = \pi d$ a proportional relationship? Justify your reasoning.

Relationships between Quantities:

Sketch a graph of the function $C = \pi d$ to show the relationship that has been described above.



Draw three similar right triangles, using the graphed line of the equation $C = \pi d$ as the hypotenuse for each triangle.

- What are the values of $\frac{\Delta C}{\Delta d}$ for each triangle?
- Describe any possible relationship between these values and the slope of the line that represents the function.

Function

The relationship between the length of the circumference (C) of a circle and the length of its diameter (d) can be represented by the linear function: $C = \pi d$.

If we use 3.14 for π , describe the ratio between the circumference and the diameter of a circle.

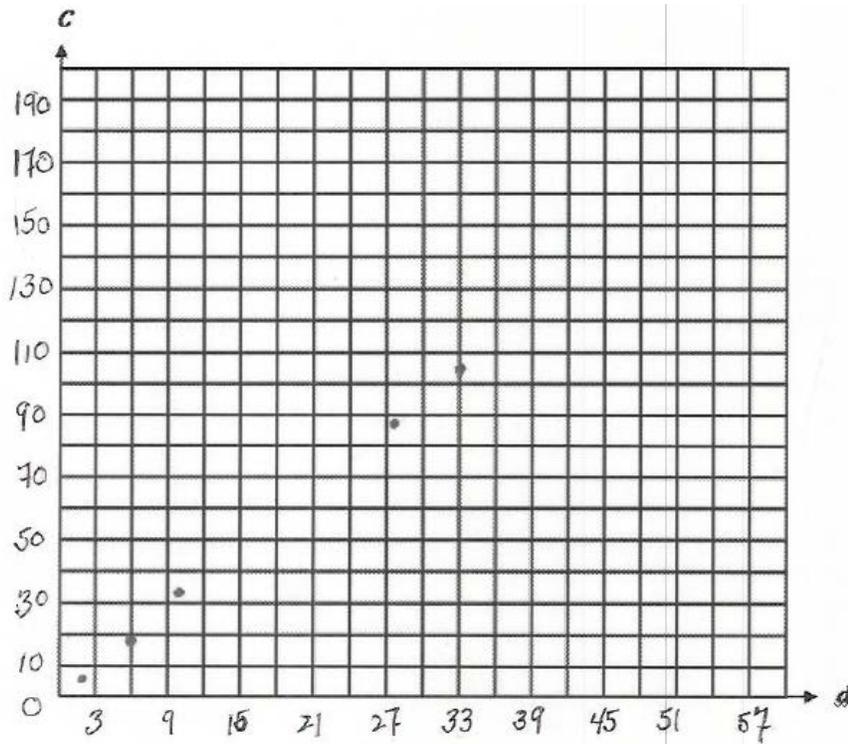
$C = 3.14d$

Function Table

d (Input) centimeters	C (Output) centimeters
2.5	7.85
6	18.84
10.2	32.03
27.6	88.66
33	103.62

Use 3.14 for π to determine the missing values in the function table.

(2) Determine appropriate scales for the x-axis and y-axis. (2) Plot the values from the function table on the graph.

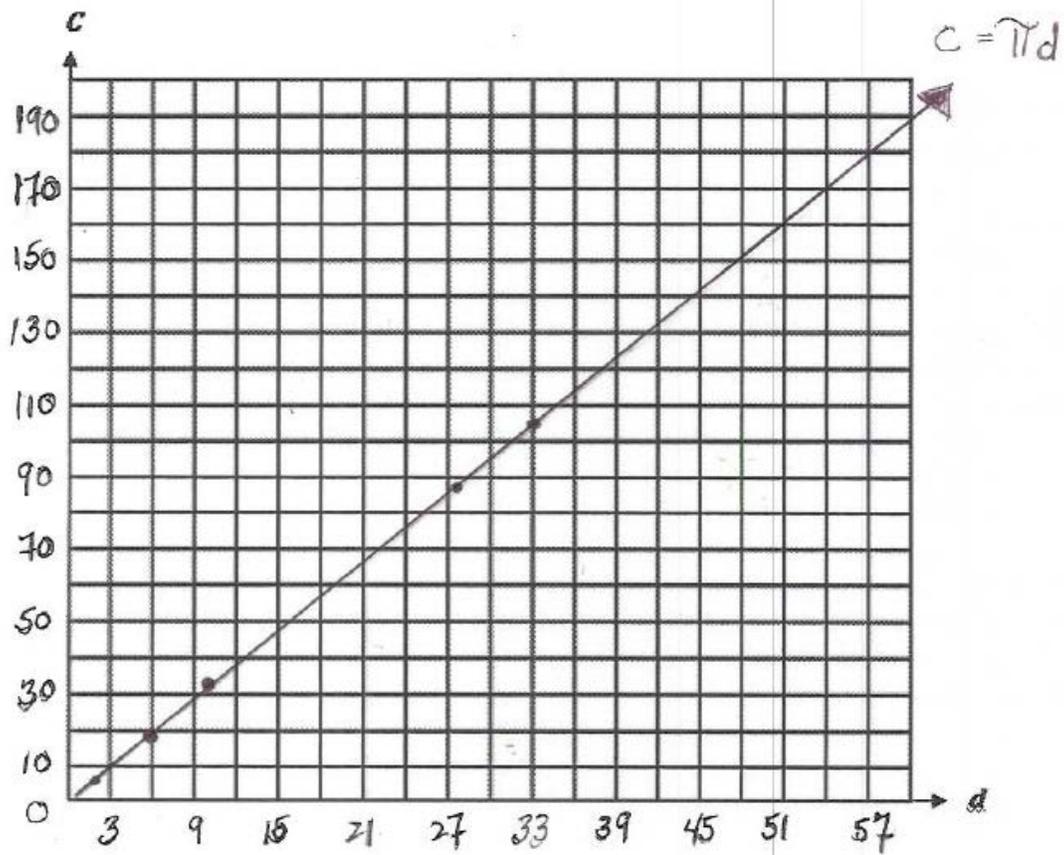


Think About It:

Is the linear function $C = \pi d$ a proportional relationship? Justify your reasoning.

Relationships between Quantities:

Sketch a graph of the function $C = \pi d$ to show the relationship that has been described above.



Draw three similar right triangles, using the graphed line of the equation $C = \pi d$ as the hypotenuse for each triangle.

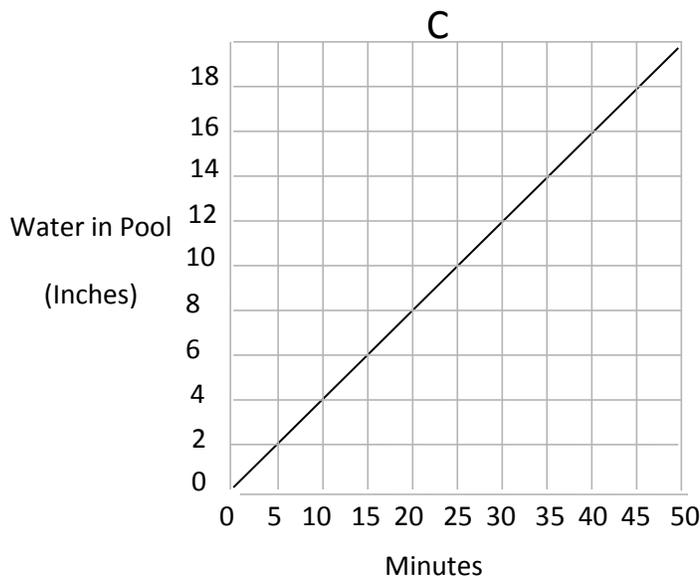
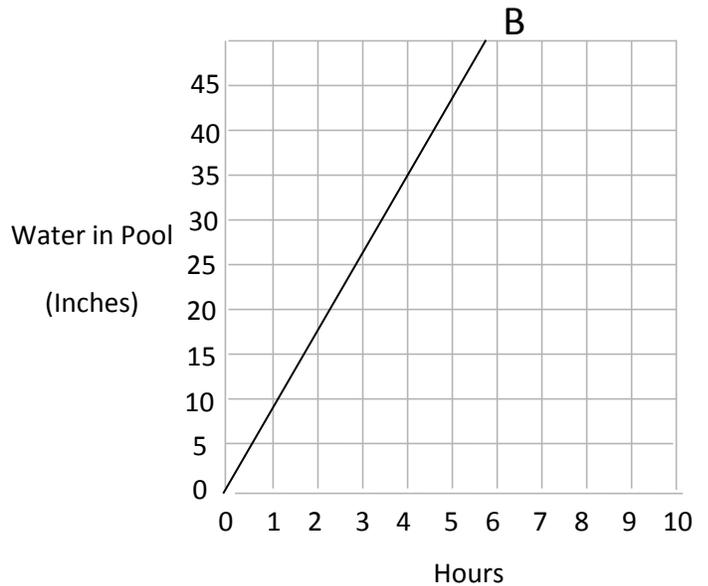
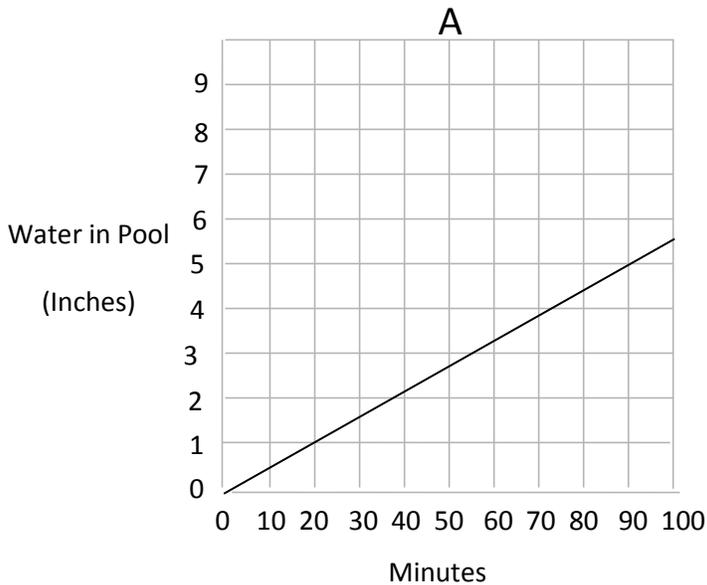
- What are the values of $\frac{\Delta C}{\Delta d}$ for each triangle? **When simplified, they are all close to 3, give or take.**
- Describe any possible relationship between these values and the slope of the line that represents the function.

Activity 2: Filling the Swimming Pool

Susan works for the Department of Water Works in her town. She has received complaints about water flow rate in different neighborhoods of her town.

To investigate the complaints, Susan found three identically shaped swimming pools in various neighborhoods that she used to help her determine the flow rates. She used the same equipment and measuring tools at each location in order to measure the flow rates. Susan began to fill each pool with water, and then she graphed the results, as shown.

- Graph A represents the first pool.
- Graph B represents the second pool.
- Graph C represents the third pool.



Activity 2: Filling the Swimming Pool

ANSWER KEY

1) Which pool has the fastest flow rate? Pool B

a. Pool B: $\frac{10}{60} = 0.16$ inches per minute (approximately), or $\frac{1}{6} = 0.16$ inches per minute

b. Pool B: $y = \frac{1}{6}x$

2) The slowest flow rate? Pool A

a. Pool A: $\frac{1}{20} = 0.05$ inches per minute

b. Pool A: $y = \frac{1}{20}x$

3) Describe the process you used to determine your answers. Use what you know about proportional relationships to justify why your answers are correct.

$\frac{\Delta y}{\Delta x} = \frac{\text{change in inches}}{\text{change in minutes}}$ to determine *slope*, *unit rate* or *constant of proportionality*

Pool A: $\frac{1}{20} = 0.05$ inches per minute

Pool B: $\frac{10}{60} = 0.16$ inches per minute (approximately), or $\frac{1}{6} = 0.16$ inches per minute

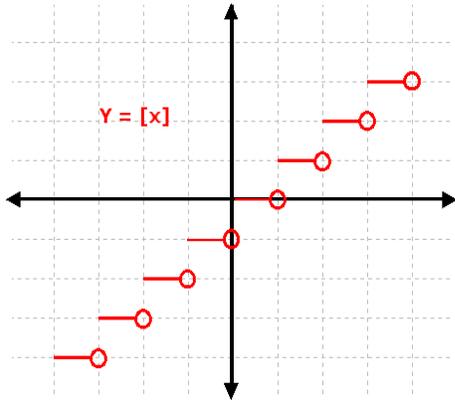
Pool C: $\frac{8}{20} = 0.4$ inches per minute, or $\frac{2}{5} = 0.4$ inches per minute

Pool C: $y = \frac{2}{5}x$

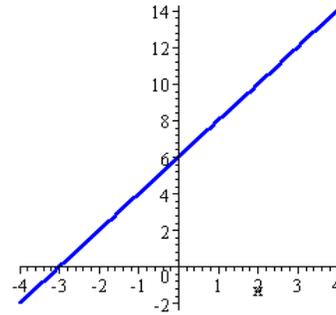
4) At the next meeting of the Department of Water Works, Susan must share her findings. To help Susan justify her findings, write a brief summary on the back of this paper about the direct relationship between the number of inches of water added to the pool and the number of minutes the hose was running. Include all calculations that you used to determine the answers to Tasks 1 through 3. Be sure to accurately use the terms, *unit rate*, *constant of proportionality*, *slope*, and *function*.

Activity 2: Exit Pass

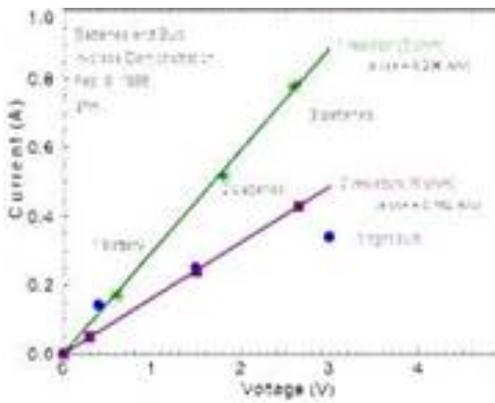
A.



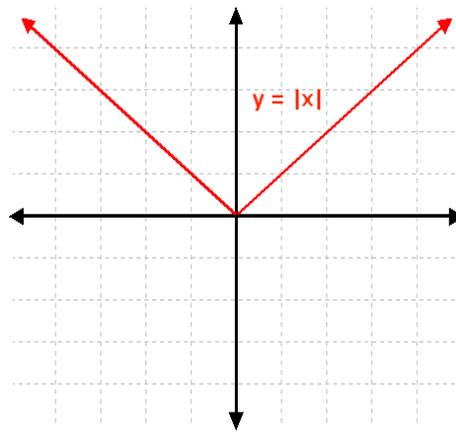
B.



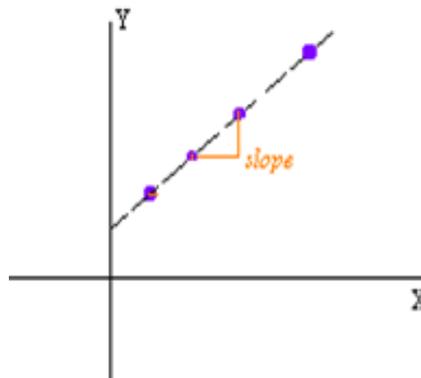
C.



D.

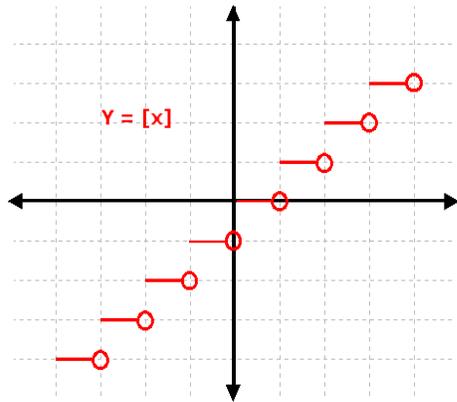


E.

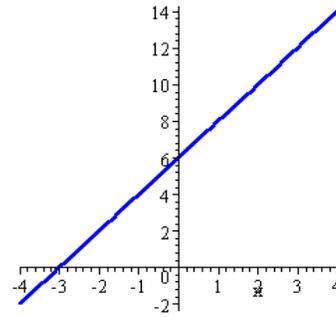


Activity 2: Exit Pass - ANSWER KEY

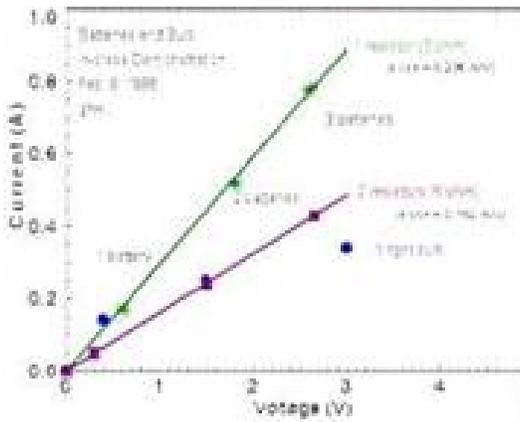
A.



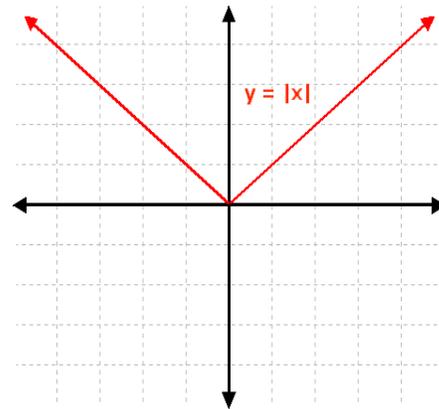
B.



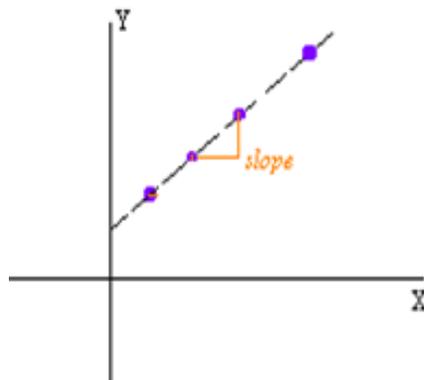
C. ✓



D.



E.



Activity 3: Picking Blue Crabs (8.EE.B.5)

At an eighth grade picnic, one of the events will be a crab picking contest. Teachers at the picnic will select several students to participate in the contest. A local seafood provider has offered to donate the crabs. You have researched crab picking rates and have determined that an 8th grade student can pick crab meat at a rate of one and one half pounds of meat in thirty minutes, on average.

Use what you know about proportional relationships to display the amount of crab meat an 8th grade student could pick for any length of contest up to sixty minutes. Represent this scenario in **as many ways as you know how**.

Discussion Questions

1. Which two points on the graph were most critical in your construction of the graph? Explain why.

2. Determine how many pounds of crab meat would be picked at the following intervals:

5 minutes _____

50 minutes _____

20 minutes _____

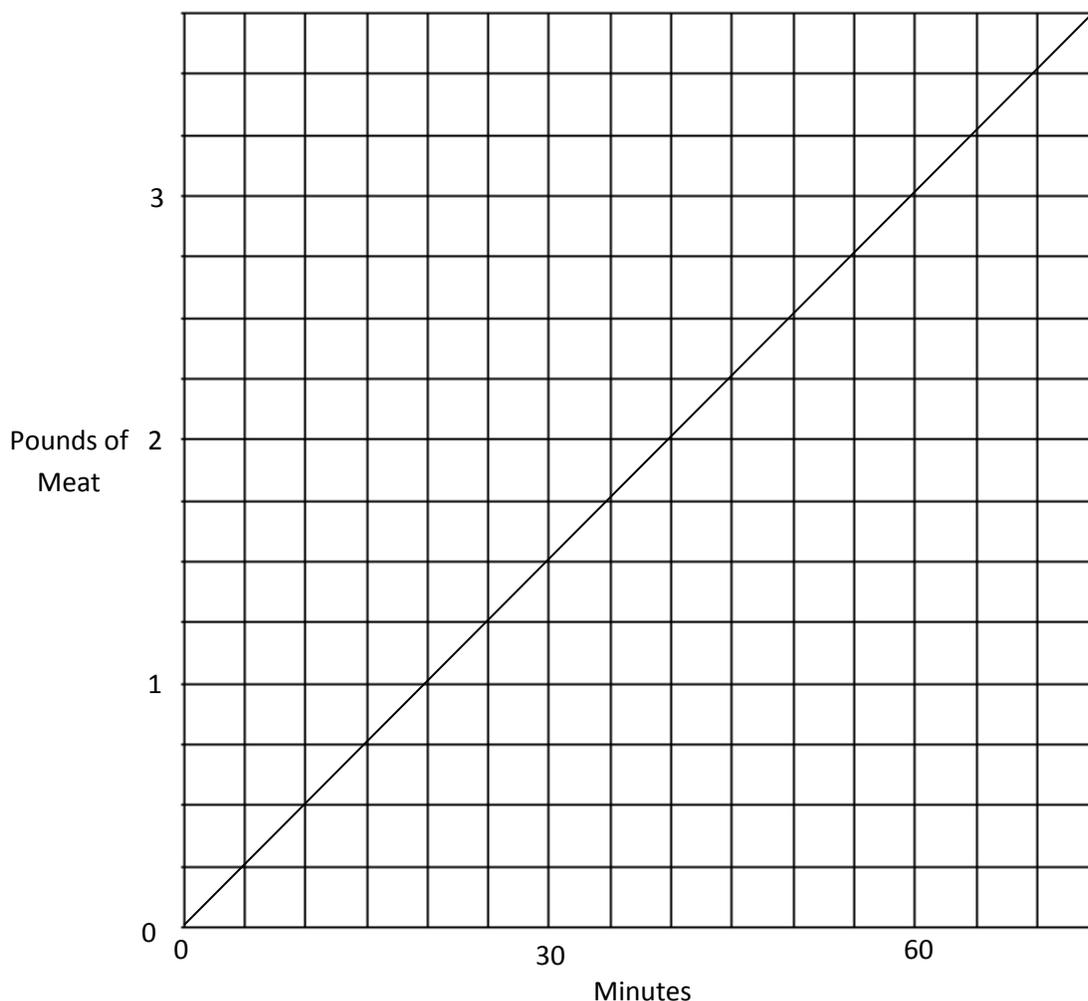
60 minutes _____

3. Write a proportion to justify your answer for fifty minutes.

Picking Blue Crabs: 8.EE.B.5

ANSWER KEY

Amount Crab Meat Picked



1. Which two points on the graph were most critical in your construction of the graph?
I needed to use the given information as a point. One and one half pounds in thirty minutes is (1.5, 30). Since I needed another point to make a line, I needed to realize that there were zero pounds of meat at zero minutes. This lead me to plot the point (0,0).
2. Determine how much meat would be picked at the following intervals:
5 minutes .25 pound 50 minutes 2.5 pounds
20 minutes 1 pound 60 minutes 3 pounds
3. Write a proportion to justify your answer for fifty minutes. $\frac{1.5}{30} = \frac{x50}{2.5 \text{ pounds}}$

Lesson Plan: 8.F.A.1 Investigating Functions

(This lesson should be adapted, including instructional time, to meet the needs of your students.)

Background Information	
Content/Grade Level	Functions/Grade 8
Unit	Define, evaluate, and compare functions
Essential Questions/Enduring Understandings Addressed in the Lesson	How can students demonstrate understanding of the fact that a function is a rule that assigns exactly one output for every input?
Standards Addressed in This Lesson	8.F.A.1: Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output (function notation is not required in Grade 8).
Lesson Topic	Investigating the definition of function
Relevance/Connections	<p>8.F.A.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).</p> <p>8.F.A.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.</p> <p>8.F.B.4 Construct a function to model a linear relationship between two quantities.</p> <p>8.F.B.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.</p>
Student Outcomes	<ul style="list-style-type: none"> • Students will be able to view a table/graph and determine whether or not it is a function. • Students will be able to view a table and find the rule. • Students will be able to use the vertical line test to determine validity
Prior Knowledge Needed to Support This Learning	<ul style="list-style-type: none"> • Independent/dependent variable • Recognize patterns • Complete a function table given a rule

	<ul style="list-style-type: none"> Evaluating expressions
Method for determining student readiness for the lesson	Informal observations during warm-up
Materials	<ul style="list-style-type: none"> Activity #1 Worksheet Activity #2 Worksheet Activity #2 Formative Assessment Two: Graph

Learning Experience														
<i>Component</i>	<i>Details</i>	<i>Which Standards for Mathematical Practice does this component address?</i>												
<i>Warm Up/Drill</i>	<ul style="list-style-type: none"> Give students an incomplete function table and provide the rule. Have students complete the function table. For example: Roberto works part time to earn some extra money. For every hour he works, he earns \$5.00. The table below shows the relationship between hours worked and amount of money earned. <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th style="text-align: center;">Hours worked x</th> <th style="text-align: center;">Amount earned $y = 5x$</th> </tr> </thead> <tbody> <tr> <td style="text-align: center;">1</td> <td style="text-align: center;">\$5</td> </tr> <tr> <td style="text-align: center;"></td> <td style="text-align: center;">\$15</td> </tr> <tr> <td style="text-align: center;">5</td> <td style="text-align: center;"></td> </tr> <tr> <td style="text-align: center;">7</td> <td style="text-align: center;">\$35</td> </tr> <tr> <td style="text-align: center;"></td> <td style="text-align: center;">\$42.50</td> </tr> </tbody> </table> <p>Suggested conversations:</p> <ul style="list-style-type: none"> Have students complete the table <ul style="list-style-type: none"> “What does the 7 represent in the table?” 	Hours worked x	Amount earned $y = 5x$	1	\$5		\$15	5		7	\$35		\$42.50	
Hours worked x	Amount earned $y = 5x$													
1	\$5													
	\$15													
5														
7	\$35													
	\$42.50													

	<ul style="list-style-type: none"> ○ “What does the \$15 represent in the table?” ○ “If Roberto wants to earn \$129 for a new phone, how would you determine how many hours he needs to work?” ● Discussion of dependent versus independent variables in the context of this problem <ul style="list-style-type: none"> ○ “What are the dependent and independent variables in this problem?” ○ “How does the amount earned relate to the number of hours worked?” 	
<p>Motivation</p>	<ul style="list-style-type: none"> ● Intro: “Today we will be learning about functions” ● Ask students if they have ever seen the machine that makes Krispy Kreme donuts. Show the video of Krispy Kreme donuts being made (http://www.youtube.com/watch?v=t0bph6Cbh9Y). (Real World Connection). <p>NOTE: If you can’t log on to You Tube in your school, try this:</p> <div style="text-align: center;">  <p>how krispy kremes are made (1).mp4</p> </div> <ul style="list-style-type: none"> ● Connect the video ~“dough going into a machine and coming out a delicious donut.” Something happens in the machine to produce the donut. Talk to a partner and discuss other real life examples where you start with something, do something to it, and it changes its form. This is where the factory concept originated. (Other examples include Play-Doh ~put in machine ~out comes a shape; chocolate ~ put into machine ~ out comes Reese’s cups; etc. Compare this to a function machine. Start with a number, do something to it, and the output is the result. Discuss independent and dependent variables. ● Provide time for the students to explore one of the following website(s) individually, in small groups, or as a class: (You may also choose a similar website that shows a virtual function table. See appendix for additional function machine websites) <p>http://www.mathplayground.com/functionmachine.html (this website offers varying levels, requires student to enter the function as an equation, and gives students the option to enter their own input or computer generated input—the numbers can go as high as 150. See additional information in the appendix. If the website does not open with the link,</p>	

	<p>open up an internet browser and paste the link into the address bar.)</p> <p>http://www.learningtoday.com/corporate/files/games/Algebra_Functions_L3_V1_T4a.swf (this website is more at a beginning level to functions and asks for a rule rather than the equation as the above website does. See additional information in the appendix.)</p> <ul style="list-style-type: none"> • After providing independent exploration time, ask the students to write about what they think the rule is for a few examples they explored on the chosen website. 	
<p>Activity 1</p> <p>UDL Components</p> <ul style="list-style-type: none"> • Multiple Means of Representation • Multiple Means for Action and Expression • Multiple Means for Engagement <p>Key Questions</p> <p>Formative Assessment</p> <p>Summary</p>	<p>UDL Components:</p> <ul style="list-style-type: none"> • <u>Principle I: Representation</u> is present in the activity. Students have choices for completing the <i>Function Sort Chart</i> on paper or on an interactive whiteboard. The formative assessment also provides choices of equivalent yet different tasks. • <u>Principle II: Expression</u> is present in the activity. Resources in the Motivation and in the Appendix include links to computer-based instructional materials that provide alternatives in the requirements for rate, timing, amplitude, and range of motor action necessary to interact with instructional materials and technologies. • <u>Principle III: Engagement</u> is present in the activity. Throughout this activity, students are given choices regarding level of challenge, type of materials used. <p>Directions: <i>Identifying a function using a table of values.</i></p> <p>Complete Activity One: Function Sort Chart (Note: this activity could be completed using an interactive whiteboard)</p> <p>Suggested conversations:</p> <ul style="list-style-type: none"> • Sort the first two functions for the students – yes it is a function and no it is not a function. Have the students work with a partner or a group to discuss the similarities and differences between the two tables to help students discover the properties of functions. • Share one function table at a time. Ask students to predict if the example is or is not a function and their reasoning. • Challenge students to notice the similarities and differences between those that are 	<p>Students will construct viable arguments and critique the reasoning of others as they justify conclusions while sorting their cards. (SMP#3)</p> <p>Students will make sense of problems and persevere in solving them as they plan a solution pathway instead of jumping to a solution as they compare the function cards. (SMP#1)</p> <p>Students will attend to precision as they notice the similarities and differences between the function</p>

	<p>functions compared to those that are not functions.</p> <ul style="list-style-type: none"> • Use the vocabulary domain and range when referring to the table of values for x (input) and y (output). • Have students justify the answer they chose to one or more of the examples. <p>Culminating Question/Summary: What defines a function? Answer: A single input value in the domain results in a unique (only one) output value.</p> <p>Formative Assessment:</p> <p>Option 1: Give students several table of value examples. Students need to determine if the table is a function or not and justify.</p> <p>Possible Examples:</p> <table border="1" data-bbox="619 592 852 841"> <thead> <tr> <th>input</th> <th>output</th> </tr> </thead> <tbody> <tr> <td>3</td> <td>5</td> </tr> <tr> <td>3</td> <td>10</td> </tr> <tr> <td>3</td> <td>12</td> </tr> <tr> <td>3</td> <td>14</td> </tr> <tr> <td>2</td> <td>15</td> </tr> </tbody> </table> <table border="1" data-bbox="892 592 1125 841"> <thead> <tr> <th>input</th> <th>output</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>5</td> </tr> <tr> <td>5</td> <td>13</td> </tr> <tr> <td>9</td> <td>21</td> </tr> <tr> <td>10</td> <td>23</td> </tr> <tr> <td>12</td> <td>27</td> </tr> </tbody> </table> <p>Option 2: Students create an example of a table that is a function and one that is not and justify.</p>	input	output	3	5	3	10	3	12	3	14	2	15	input	output	1	5	5	13	9	21	10	23	12	27	<p>cards. (SMP#6)</p>
input	output																									
3	5																									
3	10																									
3	12																									
3	14																									
2	15																									
input	output																									
1	5																									
5	13																									
9	21																									
10	23																									
12	27																									
<p>Activity 2</p> <p>UDL Components</p> <ul style="list-style-type: none"> • Multiple Means of Representation • Multiple Means for Action and Expression • Multiple Means for Engagement 	<p>UDL Components:</p> <ul style="list-style-type: none"> • <u>Principle I: Representation</u> is present in the activity. Students have choices for completing the <i>Function Sort Chart</i> on paper or on an interactive whiteboard. The formative assessment also provides choices of equivalent yet different tasks. • <u>Principle II: Expression</u> is present in the activity. Resources in the Motivation and in the Appendix include links to computer-based instructional materials that provide alternatives in the requirements for rate, timing, amplitude, and range of motor action necessary to interact with instructional materials and technologies. • <u>Principle III: Engagement</u> is present in the activity. Throughout this activity, students are given choices regarding level of challenge, type of materials used. 	<p>Students will reason abstractly and quantitatively as they look at each graph. (SMP#2)</p> <p>Students will make sense of the graphing problems and persevere in deciding if it is a function or</p>																								

Key Questions
Formative Assessment
Summary

Directions:

What is a function, using a graph?

Complete Activity Two: Function Sort Chart. (Note: this activity could be completed using an interactive whiteboard)

Suggested conversations:

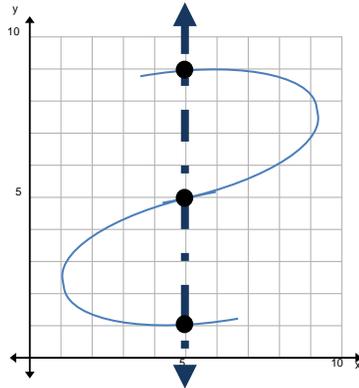
- Sort the first two graphs with their corresponding table for the students – yes it is a function and no it is not a function. Have the students work with a partner or a group to discuss the similarities and differences between the two graphs to help students discover the properties of a graphed function.
- Share another graph with its table. Ask students to predict if the example is or is not a function and their reasoning.
- Share additional graphs without a table, ask students to predict if the example is or is not a function and their reasoning.
- Challenge students to notice the similarities and differences between those that are functions compared to those that are not functions.
- Discuss the concept dependent (y-coordinate) and independent variables (x-coordinates) when discussing the graphs.

Teacher note: If students have difficulty noticing that a graph is not a function when there are multiple y-values for one x-value (vertical line test), share one of the non-function graphs and create a table of values that corresponds to the graph. See example below:

x	y
5	1

not a function.
(SMP#1)

Students will attend to precision as they communicate precisely with others and try to use clear mathematical language when discussing the graphs.
(SMP#6)



5	5
5	9

Culminating Question/Summary: Given a graph, how do you determine that it is a function?

Formative Assessment:

Use formative assessment example attached below.

Closure

Have students complete the following sentence:

A function is ...

Supporting Information

Interventions/Enrichments

- Students with Disabilities/Struggling Learners

For students who need additional instruction visualizing functions from a table of values, use the mapping examples in the appendix.

Vocabulary – ELL and Students with Disabilities/Struggling Learners need to know and understand the following

<ul style="list-style-type: none"> • ELL • Gifted and Talented 	<p>terms to be successful with these activities: <i>independent, dependent, input, output, vertical, x-axis, y-axis, table of values, function table, domain, and range.</i></p> <p>This website uses two-operations and also $f(x)$ - a good extension activity for a function machine: http://www.glencoe.com/sites/common_assets/mathematics/im1/concepts_in_motion/interactive_labs/M2_05/M2_05_dev_100.html</p>
Technology	<ul style="list-style-type: none"> • At least 1 computer with internet access • LED projector • Smart Board or White Board or something comparable
Resources (must be available to all stakeholders)	See appendix for additional resources.

Activity One: Is it a function?

	Yes	No												
<p>1)</p> <table border="1"> <thead> <tr> <th>Input</th> <th>output</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>5</td> </tr> <tr> <td>1</td> <td>6</td> </tr> <tr> <td>2</td> <td>7</td> </tr> <tr> <td>3</td> <td>8</td> </tr> <tr> <td>4</td> <td>9</td> </tr> </tbody> </table>	Input	output	0	5	1	6	2	7	3	8	4	9		
Input	output													
0	5													
1	6													
2	7													
3	8													
4	9													
<p>3)</p> <table border="1"> <thead> <tr> <th>x</th> <th>y</th> </tr> </thead> <tbody> <tr> <td>5</td> <td>-5</td> </tr> <tr> <td>6</td> <td>-6</td> </tr> <tr> <td>7</td> <td>-7</td> </tr> <tr> <td>8</td> <td>-8</td> </tr> <tr> <td>9</td> <td>-9</td> </tr> </tbody> </table>	x	y	5	-5	6	-6	7	-7	8	-8	9	-9		
x	y													
5	-5													
6	-6													
7	-7													
8	-8													
9	-9													
<p>5)</p> <table border="1"> <thead> <tr> <th>x</th> <th>y</th> </tr> </thead> <tbody> <tr> <td>-12</td> <td>12</td> </tr> <tr> <td>12</td> <td>12</td> </tr> <tr> <td>-24</td> <td>24</td> </tr> <tr> <td>25</td> <td>25</td> </tr> <tr> <td>-30</td> <td>30</td> </tr> </tbody> </table>	x	y	-12	12	12	12	-24	24	25	25	-30	30		
x	y													
-12	12													
12	12													
-24	24													
25	25													
-30	30													
<p>7)</p> <table border="1"> <thead> <tr> <th>Input</th> <th>output</th> </tr> </thead> <tbody> <tr> <td>$\frac{1}{8}$</td> <td>$\frac{1}{4}$</td> </tr> <tr> <td>$\frac{1}{6}$</td> <td>$\frac{1}{3}$</td> </tr> <tr> <td>$\frac{1}{2}$</td> <td>1</td> </tr> <tr> <td>$\frac{2}{3}$</td> <td>$1\frac{1}{3}$</td> </tr> </tbody> </table>	Input	output	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$	1	$\frac{2}{3}$	$1\frac{1}{3}$				
Input	output													
$\frac{1}{8}$	$\frac{1}{4}$													
$\frac{1}{6}$	$\frac{1}{3}$													
$\frac{1}{2}$	1													
$\frac{2}{3}$	$1\frac{1}{3}$													

	Yes	No												
<p>2)</p> <table border="1"> <thead> <tr> <th>x</th> <th>y</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>0</td> </tr> <tr> <td>1</td> <td>1</td> </tr> <tr> <td>1</td> <td>-1</td> </tr> <tr> <td>4</td> <td>2</td> </tr> <tr> <td>4</td> <td>-2</td> </tr> </tbody> </table>	x	y	0	0	1	1	1	-1	4	2	4	-2		
x	y													
0	0													
1	1													
1	-1													
4	2													
4	-2													
<p>4)</p> <table border="1"> <thead> <tr> <th>input</th> <th>output</th> </tr> </thead> <tbody> <tr> <td>2</td> <td>4</td> </tr> <tr> <td>2</td> <td>8</td> </tr> <tr> <td>2</td> <td>9</td> </tr> <tr> <td>2</td> <td>11</td> </tr> <tr> <td>2</td> <td>15</td> </tr> </tbody> </table>	input	output	2	4	2	8	2	9	2	11	2	15		
input	output													
2	4													
2	8													
2	9													
2	11													
2	15													
<p>6)</p> <table border="1"> <thead> <tr> <th>Domain</th> <th>range</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>-3</td> </tr> <tr> <td>1</td> <td>-1</td> </tr> <tr> <td>2</td> <td>1</td> </tr> <tr> <td>3</td> <td>3</td> </tr> </tbody> </table>	Domain	range	0	-3	1	-1	2	1	3	3				
Domain	range													
0	-3													
1	-1													
2	1													
3	3													
<p>8)</p> <table border="1"> <thead> <tr> <th>x</th> <th>y</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>5</td> </tr> <tr> <td>2</td> <td>5</td> </tr> <tr> <td>3</td> <td>5</td> </tr> <tr> <td>4</td> <td>5</td> </tr> <tr> <td>5</td> <td>5</td> </tr> </tbody> </table>	x	y	1	5	2	5	3	5	4	5	5	5		
x	y													
1	5													
2	5													
3	5													
4	5													
5	5													

9)

Domain	Range
2	4
2	5
3	6
3	8
4	10

10)

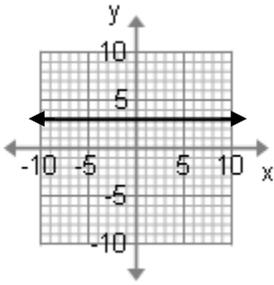
Input	output
0	0
$\frac{1}{4}$	$\frac{1}{2}$
$\frac{1}{4}$	$-\frac{1}{2}$
$\frac{1}{16}$	$\frac{1}{4}$
$\frac{1}{16}$	$-\frac{1}{4}$

Activity Two: Is it a function?

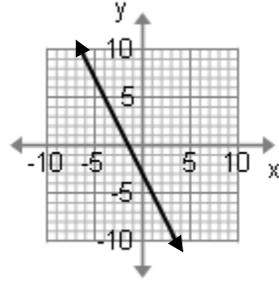
	Yes	No												
<p>1)</p> <table border="1"> <thead> <tr> <th>Input</th> <th>output</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>5</td> </tr> <tr> <td>1</td> <td>6</td> </tr> <tr> <td>2</td> <td>7</td> </tr> <tr> <td>3</td> <td>8</td> </tr> <tr> <td>4</td> <td>9</td> </tr> </tbody> </table>	Input	output	0	5	1	6	2	7	3	8	4	9		
Input	output													
0	5													
1	6													
2	7													
3	8													
4	9													
<p>3)</p> <table border="1"> <thead> <tr> <th>x</th> <th>y</th> </tr> </thead> <tbody> <tr> <td>5</td> <td>-5</td> </tr> <tr> <td>6</td> <td>-6</td> </tr> <tr> <td>7</td> <td>-7</td> </tr> <tr> <td>8</td> <td>-8</td> </tr> <tr> <td>9</td> <td>-9</td> </tr> </tbody> </table>	x	y	5	-5	6	-6	7	-7	8	-8	9	-9		
x	y													
5	-5													
6	-6													
7	-7													
8	-8													
9	-9													
<p>5)</p>														

	Yes	No												
<p>2)</p> <table border="1"> <thead> <tr> <th>input</th> <th>output</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>0</td> </tr> <tr> <td>1</td> <td>1</td> </tr> <tr> <td>1</td> <td>-1</td> </tr> <tr> <td>4</td> <td>2</td> </tr> <tr> <td>4</td> <td>-2</td> </tr> </tbody> </table>	input	output	0	0	1	1	1	-1	4	2	4	-2		
input	output													
0	0													
1	1													
1	-1													
4	2													
4	-2													
<p>4)</p> <table border="1"> <thead> <tr> <th>x</th> <th>y</th> </tr> </thead> <tbody> <tr> <td>2</td> <td>4</td> </tr> <tr> <td>2</td> <td>8</td> </tr> <tr> <td>2</td> <td>9</td> </tr> <tr> <td>2</td> <td>11</td> </tr> <tr> <td>2</td> <td>15</td> </tr> </tbody> </table>	x	y	2	4	2	8	2	9	2	11	2	15		
x	y													
2	4													
2	8													
2	9													
2	11													
2	15													
<p>6)</p>														

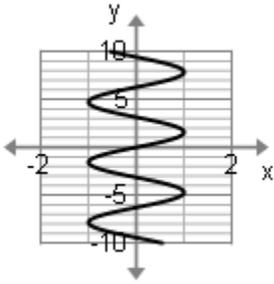
7)



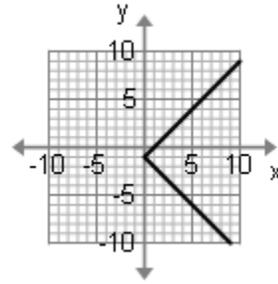
8)



9)

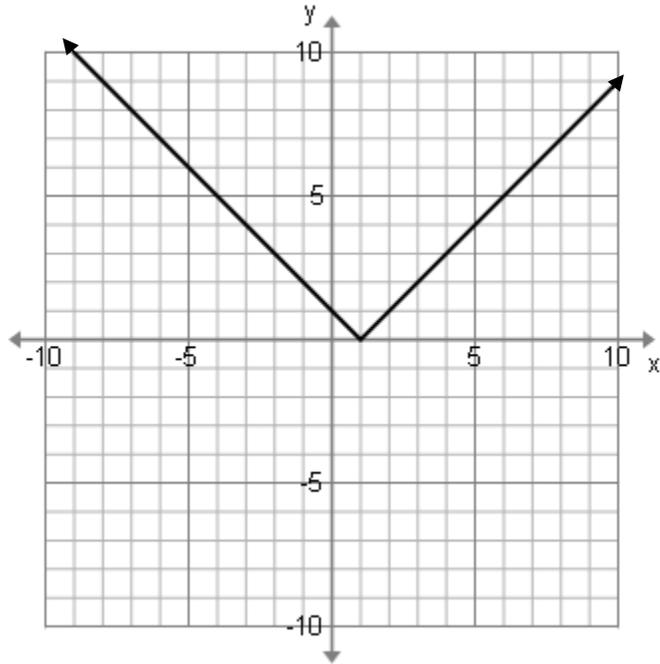


10)



Activity Two Formative Assessment Two:

Given the graph below, Maria says this is an example of a function. Do you agree or disagree with her, justify your answer.

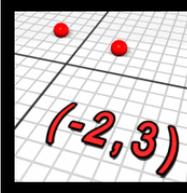


Appendix

<http://www.mathplayground.com/functionmachine.html>

Function Machine

Investigate the relationship between input and output values.



Beginner Enter the rule or function using simple math symbols.

Advanced Enter the function as a linear equation.

$x \times 5$
 $\div 3$
 $x \times 2 + 1$

$y = 5x$
 $y = x/3$
 $y = 2x+1$

Beginner:

Main Screen
Instructions

What's the Function?

You Decide The Input

Computer Decides Input

x	y
INPUT	OUTPUT

Set Maximum Number 50

Choose an Activity Level 1

Reset the Board

Main Screen
Instructions

4
1

What's the Function?

It's Your Turn

Use the number pad to enter the function. For example:
+2 or x3+1

To check your answer, just press the GO button.



x	y
INPUT	OUTPUT
2	-1
4	1

Set Maximum Number 50

Choose an Activity Level 1

Reset the Board

Advanced:

Main Screen
Instructions

What's the Function?

You Decide The Input

Computer Decides Input

x	y
INPUT	OUTPUT

Set Maximum Number 50

Choose an Activity Level 1

Reset the Board

Main Screen
Instructions

4
32

What's the Function?

It's Your Turn

Use the number pad to enter the function. For example:
 $y=3x+4$ or $y=x/2$

To check your answer, just press the GO button.

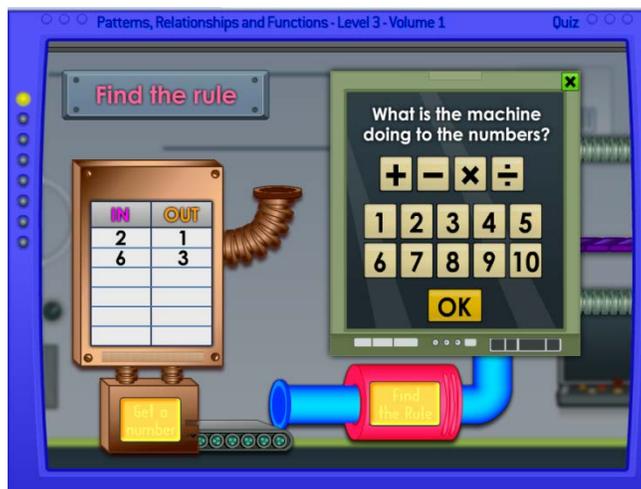
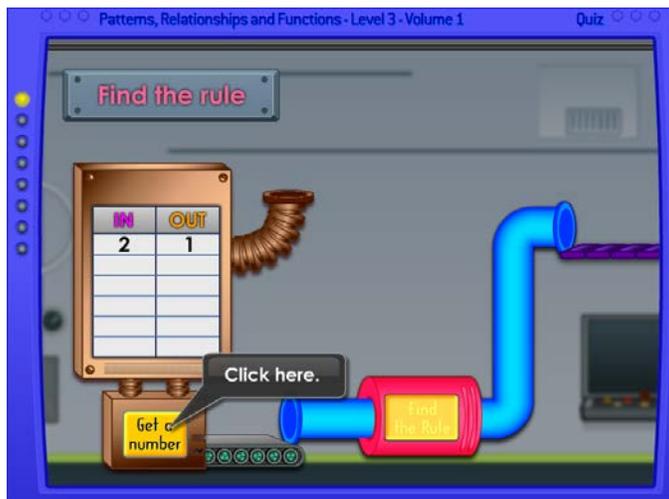


x	y
INPUT	OUTPUT
2	16
4	32

Set Maximum Number 50

Choose an Activity Level 1

Reset the Board



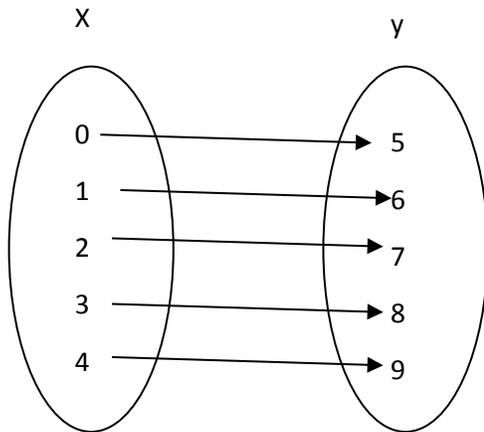
Other function machine websites:

Beginner machine: <http://pbskids.org/cyberchase/math-games/stop-creature/>

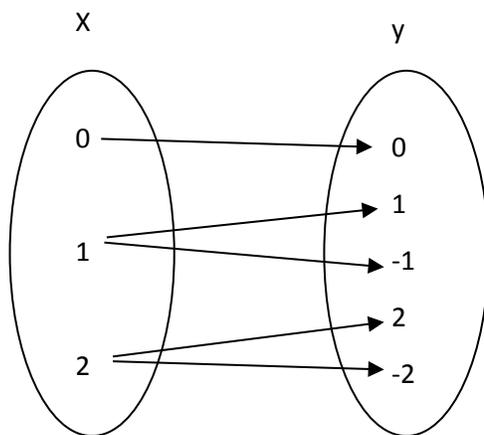
Includes graph with function machine:
http://hotmath.com/util/hm_flash_movie.html?movie=/learning_activities/interactivities/function_machine.swf&return_to=Algebra%20Activities%20-%20Math%20Activities%20-%20Hotmath&title=The%20Function%20Machine

Does not include a table with the machine:
<http://www.shodor.org/interactivate/activities/FunctionMachine/>

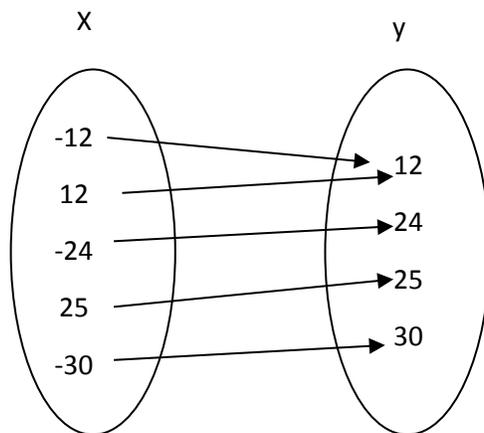
Mapping Examples



Input	output
0	5
1	6
2	7
3	8
4	9



x	y
0	0
1	1
1	-1
4	2
4	-2



x	y
-12	12
12	12
-24	24
25	25
-30	30

Lesson Plan: 8.G.A.3 Effects of Transformations on a Coordinate Plane

(This lesson should be adapted, including instructional time, to meet the needs of your students.)

Background Information	
Content/Grade Level	Geometry/Grade 8
Unit/Cluster	Understand congruence and similarity using physical models, transparencies, or geometry software.
Essential Questions/Enduring Understandings Addressed in the Lesson	<p>How do transformations affect the coordinates of a point, line segment, and/or polygon in a coordinate plane?</p> <p>What is the difference between rigid and non-rigid transformations?</p> <p>What is the relationship between an image and pre-image on a coordinate plane?</p> <p>Transformations create congruent or similar figures in a coordinate plane.</p>
Standards Addressed in This Lesson	<p>8.G.A.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.</p> <p>It is critical that the Standards for Mathematical Practices are incorporated in ALL lesson activities throughout the unit as appropriate. It is not the expectation that all eight Mathematical Practices will be evident in every lesson. The Standards for Mathematical Practices make an excellent framework on which to plan your instruction. Look for the infusion of the Mathematical Practices throughout this unit.</p>
Lesson Topic	Transformations on a coordinate plane
Relevance/Connections	<p>8.G.A.1. Verify experimentally the properties of rotations, reflections, and translations:</p> <ol style="list-style-type: none"> a. Lines are taken to lines, and line segments to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines. <p>8.G.A.2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.</p>
Student Outcomes	Students will be able to discover the effects of transformations on two-dimensional figures using coordinates.

Prior Knowledge Needed to Support This Learning	<p>6.G.A.3 Draw Polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply the techniques in the context of solving real-world and mathematical problems.</p> <p>6.NS.C.6b Understand signs of numbers in order pairs as indicating locations in quadrants of the coordinate planes; recognize when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.</p>
Method for determining student readiness for the lesson	Use the warm-up as a formative assessment to determine the level of understanding.

Learning Experience		
<i>Component</i>	<i>Details</i>	<i>Which Standards for Mathematical Practice(s) does this address? How is the Practice used to help students develop proficiency?</i>
Warm Up	<p>Warm-up attachment#1 consists of basic vocabulary needed for the lessons. Students are to fill in the vocabulary necessary to make the statements true.</p> <p>Warm-up attachment #2 consists of basic graphing needed for the lessons. Students will name and plot coordinates on a coordinate plane.</p> <p>Warm-up attachment #3/Pre-assessment: Students will graph and label coordinates, determine quadrants and answer and extension question based on rotations.</p>	
Motivation	<p>To help students get excited about transformations, students will be able to play an interactive game as a class. Students will produce transformations to capture as many bugs as possible.</p> <p>Game link: http://calculationnation.nctm.org</p> <p>Directions: Click “Guest pass,” Click “Play Games”, scroll down to “Flip-n-Slide” game. Click “Challenge Yourself”</p> <p>Play and discover as a class for no more than 5 minutes.</p> <p>Discussion question:</p>	

Learning Experience		
<i>Component</i>	<i>Details</i>	<i>Which Standards for Mathematical Practice(s) does this address? How is the Practice used to help students develop proficiency?</i>
	What do you need to learn to excel at the following game? (Create a checklist to post in the classroom to use in conjunction with the closure activity.)	
<p>Activity 1</p> <p>UDL Components</p> <ul style="list-style-type: none"> • Multiple Means of Representation • Multiple Means for Action and Expression • Multiple Means for Engagement <p>Key Questions</p> <p>Formative Assessment</p> <p>Summary</p>	<p>UDL Components:</p> <ul style="list-style-type: none"> • <u>Principle I: Representation</u> is present in this activity. Students use prior knowledge as they plot coordinates to discover the rules for transformations. • <u>Principle II: Expression</u> is present in this activity. Students can use geo-boards to perform the tasks. • <u>Principle III: Engagement</u> is present in this activity. Students have opportunities for collaboration, peer tutoring & support in their small groups as they complete the activity. <p>Directions:</p> <ul style="list-style-type: none"> • Divide your class into small groups with 3 or 4 students depending on class size. If graph paper is needed use attachments 8 and 9. <p>Task I</p> <ul style="list-style-type: none"> • Assign the group Task 1: Patterns of TRANSLATIONS (attachment #4) • Have students complete Parts A-D for their assigned task. <ul style="list-style-type: none"> ▪ Students can use Geo-boards, and/or graph paper as a resource if needed. • Once complete, have students share their discoveries in their group. • Whole class discussion questions/statements led by teacher <ul style="list-style-type: none"> ▪ In your discoveries, what transformation(s) leave the x-value the same and why? y-value and why? ▪ What would happen if both the x and y values changed? 	<p>Students make sense of problems and persevere in solving them as they work through the “Patterns of Translations” activity. (SMP #1)</p> <p>Students construct viable arguments and critique the reasoning of others as they share their findings in the Jigsaw activity. (SMP#3)</p> <p>Students attend to precision as they translate and reflect figures on the coordinate plane. (SMP#6)</p>

Learning Experience

<i>Component</i>	<i>Details</i>	<i>Which Standards for Mathematical Practice(s) does this address? How is the Practice used to help students develop proficiency?</i>
	<p>Task II</p> <ul style="list-style-type: none"> • Assign each group Task 2: Patterns of REFLECTIONS (attachment #5) • Have students complete Parts A-D for their assigned task. <ul style="list-style-type: none"> ▪ Students can use Geo-boards, and/or graph paper as a resource if needed. • Once complete, have students share their discoveries in their groups. • Whole class discussion questions/statements led by teacher <ul style="list-style-type: none"> ▪ What generalizations do you have for a reflection over the x axis? Why? ▪ What generalizations do you have for a reflection over the y axis? Why? <p>Task III</p> <ul style="list-style-type: none"> • Assign each group Task 3: Patterns of ROTATIONS (attachment #6) • Have students complete Parts A-D for their assigned task. <ul style="list-style-type: none"> ▪ Students can use Geo-boards, and/or graph paper as a resource if needed. • Once complete, have students share their discoveries in their groups. • Whole class discussion questions/statements led by teacher <ul style="list-style-type: none"> ▪ What do you know about a 90° rotation? Explain. ▪ What do you know about a 180° rotation? Explain. ▪ What do you know about a 360° rotation? Explain. ▪ What predictions do you have for a rotation around a point other than the origin? <p>Task IV</p> <ul style="list-style-type: none"> • Have each group complete the final task as a group or individually as a formative assessment. (attachment #7) 	

Learning Experience		
<i>Component</i>	<i>Details</i>	<i>Which Standards for Mathematical Practice(s) does this address? How is the Practice used to help students develop proficiency?</i>
	<p>Optional Full Class activity or individual with computers: Transformation Golf: http://www.mathsonline.co.uk/nonmembers/gamesroom/transform/golfrans.html</p> <p>Students will play a round of golf using a series of transformations to get the golf ball in the hole without going over par.</p>	
<p>Activity 2</p> <p>UDL Components</p> <ul style="list-style-type: none"> Multiple Means of Representation Multiple Means for Action and Expression Multiple Means for Engagement <p>Key Questions</p> <p>Formative Assessment Summary</p>	<p>UDL Components:</p> <ul style="list-style-type: none"> <u>Principle I: Representation</u> is present in this activity. Students use prior knowledge as they discover the rules for dilations. <u>Principle II: Expression</u> is present in this activity. Students will use visual diagrams to discover the rules for dilations. <u>Principle III: Engagement</u> is present in this activity. Students have opportunities for collaboration, peer tutoring & support in their small groups as they complete the activity. <p><i>What is dilation?</i></p> <p>Complete Activity Two: Dilation Sort Chart attachment #10 (Note: this activity could be completed using an interactive whiteboard)</p> <p>Suggested conversations:</p> <ul style="list-style-type: none"> Sort the first two dilations for the students – yes it is dilated and no it is not dilated. Have the students work with a partner or a group to discuss the similarities and differences between the two figures to help students discover the properties of dilations. Share one dilation at a time. Ask students to predict if the example is or is not dilated and their reasoning. 	<p>Students make sense of problems and persevere in solving them as they work through the “Dilation Sort Chart.” (SMP #1)</p> <p>Students construct viable arguments and critique the reasoning of others while defining dilations. (SMP#3)</p> <p>Students attend to precision as they decide which figures are and are not dilations. (SMP#6)</p>

Learning Experience

<i>Component</i>	<i>Details</i>	<i>Which Standards for Mathematical Practice(s) does this address? How is the Practice used to help students develop proficiency?</i>
	<ul style="list-style-type: none"> • Challenge students to notice the similarities and differences between those that are dilated compared to those that are not dilated. • Have students justify the answer they chose to one or more of the examples. <p>Culminating Question/Summary: What defines dilation?</p> <p>Formative Assessment: Option 1: Give students several examples. Students need to determine if the figures are dilated or not and justify.</p> <p>Possible Examples:</p> <div style="display: flex; justify-content: space-around; align-items: center;">   </div> <p>Option 2: Students create an example of a series of figures that are dilated and not dilated.</p>	
<p>Activity 3</p> <p>UDL Components</p> <ul style="list-style-type: none"> • Multiple Means of Representation 	<p>UDL Components:</p> <ul style="list-style-type: none"> • <u>Principle I: Representation</u> is present in this activity. Students use prior knowledge as they discover the rules for transformations. • <u>Principle II: Expression</u> is present in this activity. Students will use different tools to display their visuals. 	<p>Student will make sense of problems and persevere in solving them as they determine where the snowflake patterns should be placed.</p>

Learning Experience		
<i>Component</i>	<i>Details</i>	<i>Which Standards for Mathematical Practice(s) does this address? How is the Practice used to help students develop proficiency?</i>
<ul style="list-style-type: none"> • Multiple Means for Action and Expression • Multiple Means for Engagement Key Questions Formative Assessment Summary	<ul style="list-style-type: none"> • <u>Principle III: Engagement</u> is present in this activity. Students have opportunities for collaboration, peer tutoring & support in their small groups as they complete the activity. See attached document on instructions. (attachment #11) Use attachments #12-14 for this activity.	(SMP#1) Students will attend to precision as they understand where each symbol in the design should be placed and label it appropriately. (SMP#6) Students will look for and express regularity in repeated reasoning as they continually evaluate the reasonableness of their placement of drawing should be on the snowflakes. (SMP#8)
Closure	Either as a class or students individually in a computer lab, have students play the interactive game Flip-N-Slide from Calculation Nation by NCTM. Students will take their understanding of describing the effects of transformations to complete the task of capturing as many bugs as possible through a series of transformation. Follow up-question: During the motivation the teacher asked “What do you need to learn to excel at the following game?” Ask students, did they learn everything they needed to excel at the game. UDL Principle III: Provide Multiple Means of Engagement: Self Regulation. Game link: http://calculationnation.nctm.org Directions: Click “Play Games”, scroll down to “Flip-n-Slide game. Click “Challenge Yourself”	

Supporting Information

<p>Interventions/Enrichments</p> <ul style="list-style-type: none"> • Students with Disabilities/Struggling Learners • ELL • Gifted and Talented 	<p>Students with Disabilities/Struggling Learners: Give students a template with polygons and have them cut them out to translate, rotate and reflect these two-dimensional figures. Ask them to place these on the coordinate plane and write the coordinate points of the vertices. Then finish the activity as stated in the lesson.</p> <p>ELL and Struggling Learners: Vocabulary –need to know and understand the following terms to be successful with these activities: <i>coordinate plane, coordinates, dilation, image, ordered pair, origin, pre-image, quadrant, reflections, rotations, transformations, translations, x-axis, x-value, y-axis, y-value</i></p> <p>Gifted and Talented: Translate the same figure two or more times. Rotate figures around points other than the origin. Reflect figures over lines or line segments other than the x-axis and the y-axis. Research when dilations are used other than in a mathematics class.</p>
<p>Materials</p>	<p>Graph Paper Geo-boards</p>
<p>Technology</p>	<p>Computer Projector Interactive WhiteBoard</p>
<p>Resources</p>	<p>Calculation Nation Flip-N-Slide Interactive Game http://calculationnation.nctm.org Transformation Golf: http://www.mathsonline.co.uk/nonmembers/gamesroom/transform/golftrans.html</p>

8.G.3 Warm-up Attachment#1

Name _____

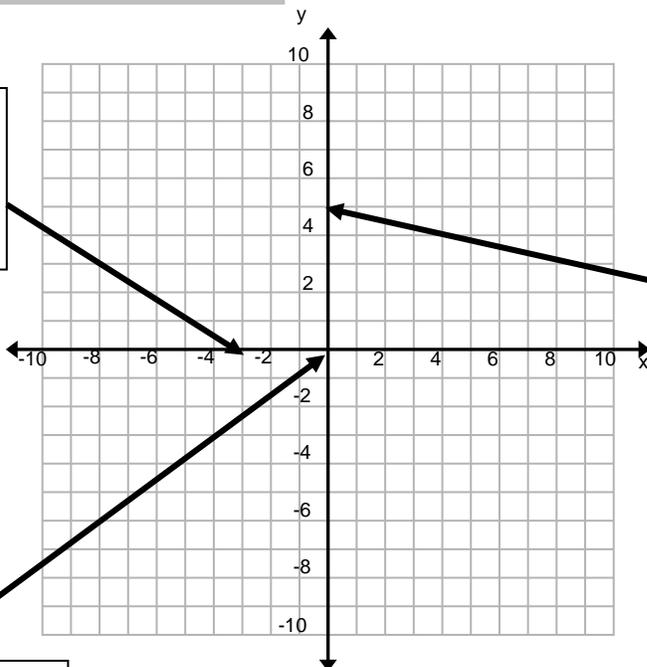
1) An _____ is a pair of numbers such as $(6,-3)$ used to locate a point in the coordinate plane.

2) The _____ is the first number, and it corresponds to a location on the _____

3) The _____ is the second number, and it corresponds to a location on the _____

4) The horizontal number line is called the _____

5) The vertical number line is called the _____



6) The _____ is the point at which the number lines intersect.

7) The ordered pair for the origin is:
(__, __)

8.G.3 Warm-up Attachment #1 Answer Key

Name _____

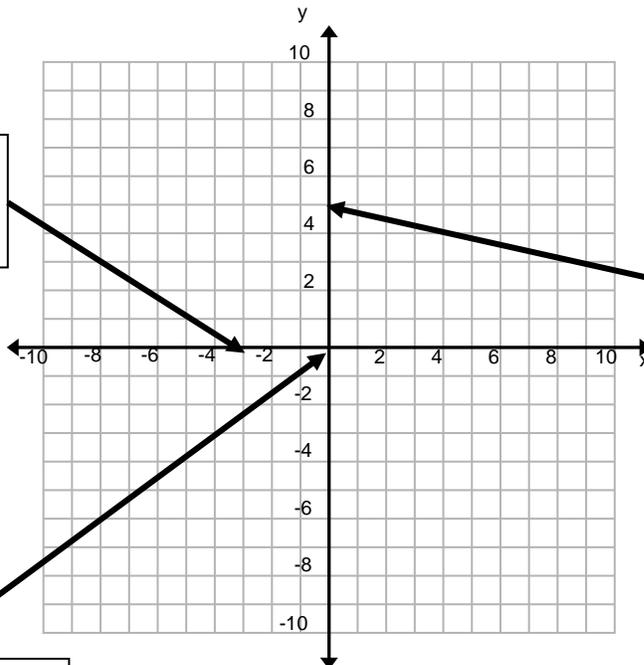
1) An **ordered pair** is a pair of numbers such as $(6,-3)$ used to locate a point in the coordinate plane.

2) The **x-value** is the first number, and it corresponds to a location on the **x-axis**.

3) The **y-value** is the second number, and it corresponds to a location on the **y-axis**.

4) The horizontal number line is called the **x-axis**.

5) The vertical number line is called the **y-axis**.



6) The **origin** is the point at which the number lines intersect.

7) The ordered pair for the origin is: $(0,0)$

8.G.3 Warm-up Attachment#2

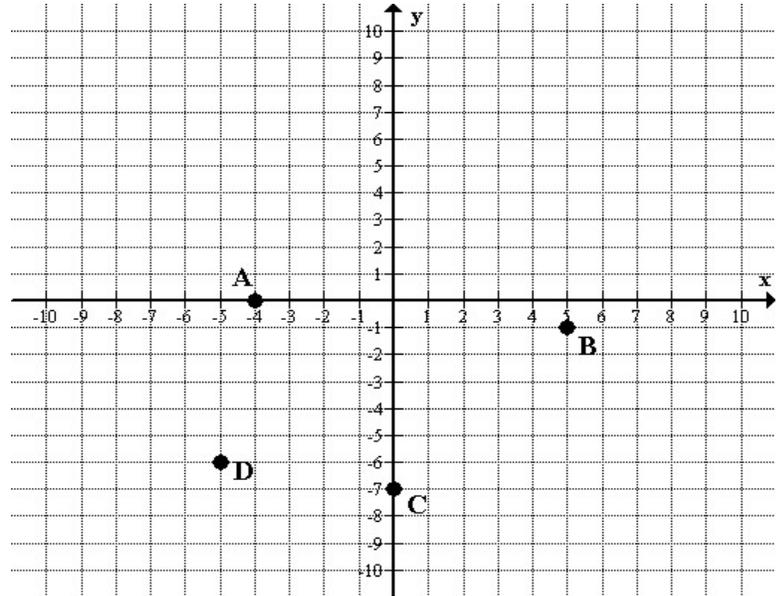
Name _____

Name the ordered pairs for each point graphed on the given coordinate plane.

1. A (,)
2. B (,)
3. C (,)
4. D (,)

Graph and label each point at the given location on the given coordinate plane.

5. E. (-6, -8)
6. F. (-5, 7)
7. G. (0, 5)
8. H. (6,0)



8.G.3 Warm-up Attachment #2 Answer Key

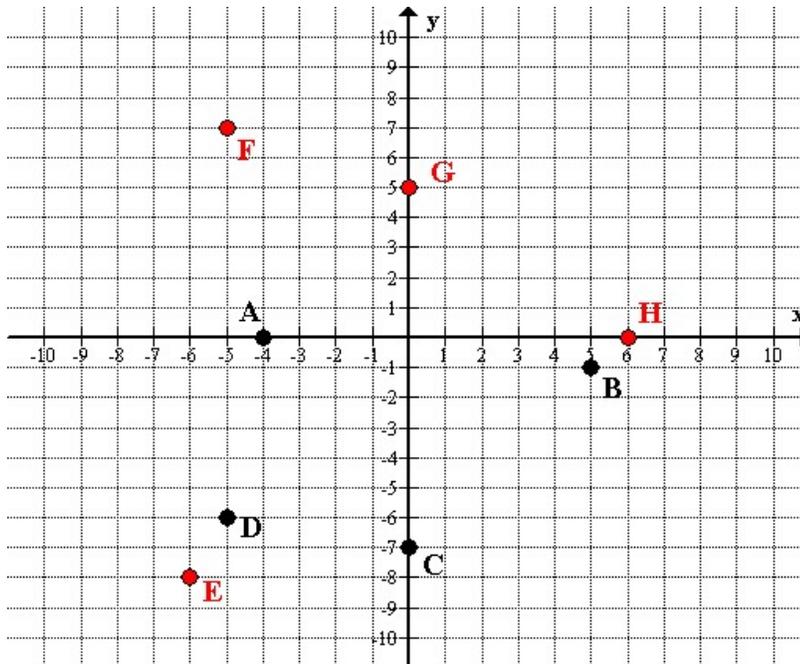
Name _____

Name the ordered pairs for each point graphed on the given coordinate plane.

- 1 A (,)
- 2 B (,)
- 3 C (,)
- 4 D (,)

Graph and label each point at the given location on the given coordinate plane.

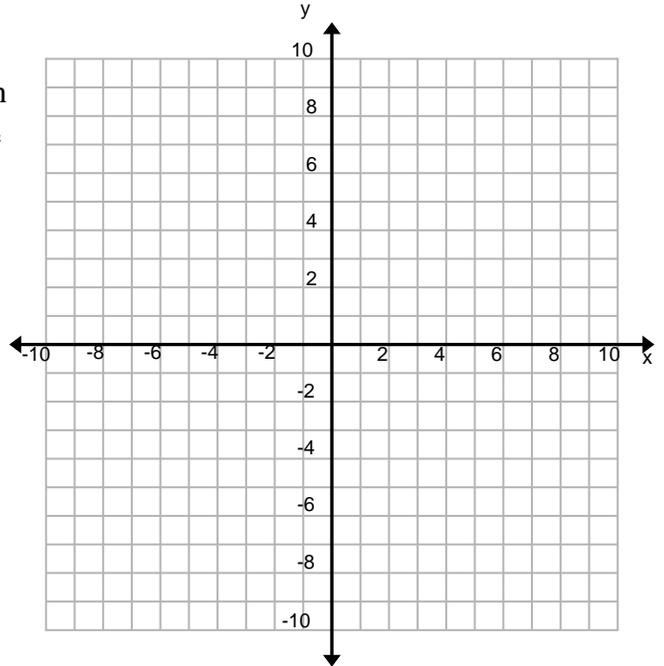
- 5 E. (-6, -8)
- 6 F. (-5, 7)
- 7 G. (0, 5)
- 8 H. (6,0)



8.G.3 Warm-up Attachment #3

Name _____

1. Graph, label, and connect the following points in order to create a polygon on the given coordinate plane
 - a. Point A (-5, 6)
 - b. Point B (-8, 4)
 - c. Point C (-3, -4)
 - d. Point D (-1, 0)



Name the quadrant where Point A will be located after the given transformation:

2. Reflection across the y-axis _____
3. Translation down 10 units _____
4. Rotation of 180° clockwise about the origin _____

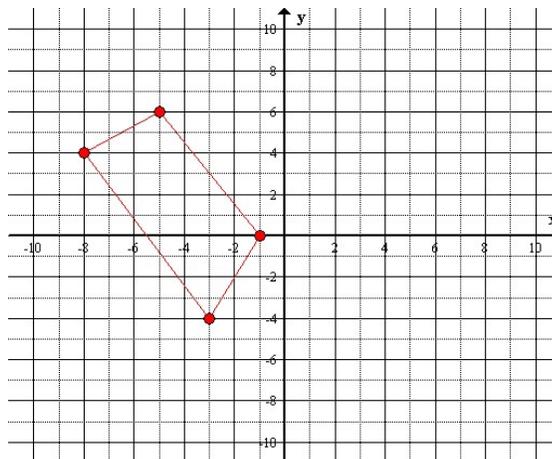
Extension:

5. Rotation of 360° clockwise about the origin _____
Explain your thinking.

8.G.3 Warm-up Attachment #3 Answer Key

Name _____

- Graph, label, and connect the following points in order to create a polygon on the given coordinate plane
 - Point A (-5, 6)
 - Point B (-8, 4)
 - Point C (-3, -4)
 - Point D (-1, 0)



Name the quadrant where Point A will be located after the given transformation (point resets after each transformation):

- Reflection across the y-axis **Quadrant 1**
- Translation down 10 units **Quadrant 3**
- Rotation of 180° clockwise about the origin **Quadrant 4**

Extension:

- Rotation of 360° clockwise about the origin **Quadrant 2**
Explain your thinking. **For every 90 degree clockwise rotation, Point A would change quadrant locations. It would go from quadrant 2 to quadrant 1, then 4, then 3, before returning to quadrant 2. A 360° rotation is equivalent to the number of degrees in a circle, therefore making a full rotation.**

Task 1: Patterns of TRANSLATIONS

Directions: In your small group, complete the graphs in Parts A, B, and C. Use this information to discuss and answer the questions in Part D.

PART A:

- Graph, label, and connect the following points to create a triangle.
 $A(3, 5)$ $B(4, 2)$ $C(1, 1)$
- The triangle is in Quadrant _____. This triangle is the **pre-image**.
- Color the triangle blue.
- Translate the blue triangle 6 units down. Color this translation red. The red triangle is the **image**.
- Identify the coordinates of this translation in the table below.

Blue Triangle	$A(3, 5)$	$B(4, 2)$	$C(1, 1)$
Red Triangle			

PART B:

- Graph, label, and connect the following points to create a quadrilateral.
 $D(-5, 5)$ $E(-2, 4)$ $F(-1, 2)$ $G(-4, 1)$
- The quadrilateral is in Quadrant _____. This quadrilateral is the **pre-image**.
- Color the quadrilateral blue.
- Translate the blue quadrilateral 5 units right. Color this translation red. The red quadrilateral is the **image**.
- Identify the coordinates of this translation in the table below.

Blue Quadrilateral	$D(-5, 5)$	$E(-2, 4)$	$F(-1, 2)$	$G(-4, 1)$
Red Quadrilateral				

PART C:

- Graph, label, and connect the following points to create a pentagon.
 $H(-3, -1)$ $J(-1, -3)$ $K(-2, -6)$ $L(-4, -6)$ $M(-5, -3)$
- The pentagon is in Quadrant _____. This pentagon is the **pre-image**.
- Color the pentagon blue.
- Translate the blue pentagon 4 units right and 5 units up. Color this translation red. The red pentagon is the **image**.
- Identify the coordinates of this translation in the table below.

Blue Pentagon	$H(-3, -1)$	$J(-1, -3)$	$K(-2, -6)$	$L(-4, -6)$	$M(-5, -3)$
Red Pentagon					

PART D:

- In which graphs did the x-values change from the pre-image to the image? _____
- What do the problems have in common? _____

- What rule could we use to find the new image coordinates of a similar problem if we do not have a graph? _____

Try out your rule:

Translate the point 6 units to the left.

	x-coordinate	y-coordinate		x-coordinate	y-coordinate
<i>pre-image</i>	5	-2	<i>pre-image</i>	x	y
<i>image</i>			<i>image</i>		

4) In which graphs did the y-values change from the pre-image to the image?

5) What do the problems have in common? _____

6) What rule could we use to find the new image coordinates of a similar problem if we do not have a graph? _____

Try out your rule:

Translate the point 3 units down.

	x-coordinate	y-coordinate
<i>pre-image</i>	5	-2
<i>image</i>		

	x-coordinate	y-coordinate
<i>pre-image</i>	x	y
<i>image</i>		

Task 1: Patterns of TRANSLATIONS

Directions: In your small group, complete the graphs in Parts A, B, and C. Use this information to discuss and answer the questions in Part D.

PART A:

6. Graph, label, and connect the following points to create a triangle.

$$A (3, 5) \quad B (4, 2) \quad C (1, 1)$$

7. The triangle is in Quadrant 1. This triangle is the **pre-image**.
8. Color the triangle blue.
9. Translate the blue triangle 6 units down. Color this translation red. The red triangle is the **image**.
10. Identify the coordinates of this translation in the table below.

Blue Triangle	$A (3, 5)$	$B (4, 2)$	$C (1, 1)$
Red Triangle	$A'(3,-1)$	$B'(4,-4)$	$C'(1,-5)$

PART B:

- 1) Graph, label, and connect the following points to create a quadrilateral.

$$D (-5, 5) \quad E (-2, 4) \quad F (-1, 2) \quad G (-4, 1)$$

- 2) The quadrilateral is in Quadrant 2. This quadrilateral is the **pre-image**.
- 3) Color the quadrilateral blue.
- 4) Translate the blue quadrilateral 5 units right. Color this translation red. The red quadrilateral is the **image**.
- 5) Identify the coordinates of this translation in the table below.

Blue Quadrilateral	$D (-5, 5)$	$E (-2, 4)$	$F (-1, 2)$	$G (-4, 1)$
Red Quadrilateral	$D'(0,5)$	$E'(3,4)$	$F'(4,2)$	$G'(1,1)$

PART C:

6. Graph, label, and connect the following points to create a pentagon.
 $H(-3, -1)$ $J(-1, -3)$ $K(-2, -6)$ $L(-4, -6)$ $M(-5, -3)$
7. The pentagon is in Quadrant 3. This pentagon is the **pre-image**.
8. Color the pentagon blue.
9. Translate the blue pentagon 4 units right and 5 units up. Color this translation red. The red pentagon is the **image**.
10. Identify the coordinates of this translation in the table below.

Blue Pentagon	$H(-3, -1)$	$J(-1, -3)$	$K(-2, -6)$	$L(-4, -6)$	$M(-5, -3)$
Red Pentagon	$H'(1,4)$	$J'(3,2)$	$K'(2,-1)$	$L'(0,-1)$	$M'(-1,2)$

PART D:

- 7) In which graphs did the x-values change from the pre-image to the image? B and C
- 8) What do the problems have in common? they move to the right
- 9) What rule could we use to find the new image coordinates of a similar problem if we do not have a graph? add or subtract from the x value

Try out your rule:

Translate the point 6 units to the left.

	x-coordinate	y-coordinate		x-coordinate	y-coordinate
<i>pre-image</i>	5	-2	<i>pre-image</i>	x	y
<i>image</i>	-1	-2	<i>image</i>	$x-6$	y

10) In which graphs did the y-values change from the pre-image to the image? **A and C**

11) What do the problems have in common? **they move up or down**

12) What rule could we use to find the new image coordinates of a similar problem if we do not have a graph? **add or subtract to the y value**

Try out your rule:

Translate the point 3 units down.

	x-coordinate	y-coordinate
<i>pre-image</i>	5	-2
<i>image</i>	5	-5

	x-coordinate	y-coordinate
<i>pre-image</i>	x	y
<i>image</i>	x	$y-3$

Task 2: Patterns of REFLECTIONS

Directions: In your small group, complete the graphs in Parts A, B, and C. Use this information to discuss and answer the questions in Part D.

PART A:

- 1) Graph, label, and connect the following points to create a triangle.

$$A (3, 5) \quad B (4, 2) \quad C (1, 1)$$

- 2) The triangle is in Quadrant _____. This triangle is the **pre-image**.
- 3) Color the triangle blue.
- 4) Reflect the blue triangle over the x -axis. Color this reflection red. The red triangle is the **image**.
- 5) Identify the coordinates of this reflection in the table below.

Blue Triangle	$A (3, 5)$	$B (4, 2)$	$C (1, 1)$
Red Triangle			

PART B:

- 1) Graph, label, and connect the following points to create a quadrilateral.

$$D (-5, 5) \quad E (-2, 4) \quad F (-1, 2) \quad G (-4, 1)$$

- 2) The quadrilateral is in Quadrant _____. This quadrilateral is the **pre-image**.
- 3) Color the quadrilateral blue.
- 4) Reflect the blue quadrilateral over the x -axis. Color this reflection red. The red quadrilateral is the **image**.
- 5) Identify the coordinates of this reflection in the table below.

Blue Quadrilateral	$D (-5, 5)$	$E (-2, 4)$	$F (-1, 2)$	$G (-4, 1)$
Red Quadrilateral				

PART C:

- 1) Graph, label, and connect the following points to create a pentagon.

$$H(-3, -1) \quad J(-1, -3) \quad K(-2, -6) \quad L(-4, -6) \quad M(-5, -3)$$

- 2) The pentagon is in Quadrant _____. This pentagon is the **pre-image**.
- 3) Color the pentagon blue.
- 4) Reflect the blue pentagon over the *x*-axis. Color this reflection red. The red pentagon is the **image**.
- 5) Identify the coordinates of this reflection in the table below.

Blue Pentagon	$H(-3, -1)$	$J(-1, -3)$	$K(-2, -6)$	$L(-4, -6)$	$M(-5, -3)$
Red Pentagon					

PART D:

- 1) In which graphs did the *x*-values change from the pre-image to the image? _____
- 2) What do the problems have in common? _____

- 3) What rule could we use to find the new image coordinates of a similar problem if we do not have a graph? _____

Try out your rule:

Reflect the pre-image over the *x*-axis.

<i>pre-image</i>	5	-2	<i>pre-image</i>	<i>x</i>	<i>y</i>
<i>image</i>			<i>image</i>		

Patterns of REFLECTIONS (*continued*)

4) In which graphs did the y-values change from the pre-image to the image?

5) What do the problems have in common? _____

6) What rule could we use to find the new image coordinates of a similar problem if we do not have a graph? _____

Try out your rule:

Reflect the pre-image over the x -axis.

	x-coordinate	y-coordinate
<i>pre-image</i>	5	-2
<i>image</i>		

	x-coordinate	y-coordinate
<i>pre-image</i>	x	y
<i>image</i>		

Task 2: Patterns of REFLECTIONS

Directions: In your small group, complete the graphs in Parts A, B, and C. Use this information to discuss and answer the questions in Part D.

PART A:

- 1) Graph, label, and connect the following points to create a triangle.

$$A (3, 5) \quad B (4, 2) \quad C (1, 1)$$

- 2) The triangle is in Quadrant 1. This triangle is the **pre-image**.
- 3) Color the triangle blue.
- 4) Reflect the blue triangle over the x-axis. Color this reflection red. The red triangle is the **image**.
- 5) Identify the coordinates of this reflection in the table below.

Blue Triangle	$A (3, 5)$	$B (4, 2)$	$C (1, 1)$
Red Triangle	$A'(3,-5)$	$B'(4,-2)$	$C'(1,-1)$

PART B:

- 1) Graph, label, and connect the following points to create a quadrilateral.

$$D (-5, 5) \quad E (-2, 4) \quad F (-1, 2) \quad G (-4, 1)$$

- 2) The quadrilateral is in Quadrant 2. This quadrilateral is the **pre-image**.
- 3) Color the quadrilateral blue.
- 4) Reflect the blue quadrilateral over the x-axis. Color this reflection red. The red quadrilateral is the **image**.
- 5) Identify the coordinates of this reflection in the table below.

Blue Quadrilateral	$D (-5, 5)$	$E (-2, 4)$	$F (-1, 2)$	$G (-4, 1)$
Red Quadrilateral	$D'(-5,-5)$	$E'(-2,-4)$	$F'(-1,-2)$	$G'(-4,-1)$

PART C:

- 1) Graph, label, and connect the following points to create a pentagon.

$$H(-3, -1) \quad J(-1, -3) \quad K(-2, -6) \quad L(-4, -6) \quad M(-5, -3)$$

- 2) The pentagon is in Quadrant 3. This pentagon is the **pre-image**.
- 3) Color the pentagon blue.
- 4) Reflect the blue pentagon over the x-axis. Color this reflection red. The red pentagon is the **image**.
- 5) Identify the coordinates of this reflection in the table below.

Blue Pentagon	$H(-3, -1)$	$J(-1, -3)$	$K(-2, -6)$	$L(-4, -6)$	$M(-5, -3)$
Red Pentagon	$H'(3,1)$	$J'(-1,3)$	$K'(-2,6)$	$L'(-4,6)$	$M'(-5,3)$

PART D:

- 4) In which graphs did the x -values change from the pre-image to the image?
none
- 5) What do the problems have in common? they went over the x axis
- 6) What rule could we use to find the new image coordinates of a similar problem if we do not have a graph? x stays the same if reflected over the x axis and y becomes the opposite

Try out your rule:

Reflect the pre-image over the x -axis.

<i>pre-image</i>	5	-2	<i>pre-image</i>	x	y
<i>image</i>	5	2	<i>image</i>	x	$-y$

Patterns of REFLECTIONS (*continued*)

- 7) In which graphs did the y-values change from the pre-image to the image?
all
- 8) What do the problems have in common? reflected over the x axis
- 9) What rule could we use to find the new image coordinates of a similar problem if we do not have a graph? y value becomes the opposite value if reflected over the x axis

Try out your rule:

Reflect the pre-image over the x-axis.

	x-coordinate	y-coordinate
<i>pre-image</i>	5	-2
<i>image</i>	5	2

	x-coordinate	y-coordinate
<i>pre-image</i>	x	y
<i>image</i>	x	$-y$

Task 3: Patterns of ROTATIONS**Part A:**

Directions: In your small group, complete the graphs in Parts A, B, and C. Use this information to discuss and answer the questions in Part D.

- 1) Graph and label point A with coordinates (3, 5).
- 2) The point is in Quadrant _____. This point is the **pre-image**.
- 3) Rotate the point 90° counterclockwise around the origin.
- 4) Color this point blue. The blue point is the **image**.
- 5) What are the coordinates of the image? (_____, _____)
- 6) Now rotate the original point 90° clockwise around the origin.
- 7) Color this point red. The red point is another **image**.
- 8) What are the coordinates of this new image? (_____, _____)

Part B:

- 1) Graph, label, and connect the following points to create a segment.

$$B (-5, 5) \quad C (-2, 4)$$

- 2) The segment is in Quadrant _____. This segment is the **pre-image**.
- 3) Rotate the segment 90° counterclockwise around the origin.
- 4) What are the coordinates of this rotation?

Segment	$B (-5, 5)$	$C (-2, 4)$
90° counterclockwise		

- 5) Color this rotation blue. The blue segment is the **image**.
- 6) Now rotate the original segment 90° clockwise around the origin.
- 7) What are the coordinates of this rotation?

Segment	$B (-5, 5)$	$C (-2, 4)$
---------	---------------	---------------

90° clockwise		
------------------	--	--

- 8) Color this rotation red. The red segment is another **image**.

Part C:

- 1) Graph, label, and connect the following points to create a triangle.

$$D(-4, -1) \quad E(-1, -2) \quad F(-5, -5)$$

- 2) The triangle is in Quadrant _____. This triangle is the **pre-image**.
 3) Rotate the triangle 180° counterclockwise around the origin.
 4) What are the coordinates of this rotation?

Triangle	$D(-4, -1)$	$E(-1, -2)$	$F(-5, -5)$
180° counterclockwise			

- 5) Color this rotation blue. The blue triangle is the **image**.
 6) Now rotate the original triangle 180° clockwise around the origin.
 7) What are the coordinates of this rotation?

Triangle	$D(-4, -1)$	$E(-1, -2)$	$F(-5, -5)$
180° clockwise			

- 8) Color this triangle red. The red triangle is another **image**.

Part D:

- 1) When rotating clockwise 90° , what patterns do you notice the coordinates from the pre-image to the image? _____
- 2) When rotating counter-clockwise 90° , what patterns do you notice the coordinates from the pre-image to the image? _____
- 3) What rule could we use to find the new image coordinates of a similar problem if we do not have a graph? _____

*Try out your rule:***Rotate the pre-image 90° clockwise.**

<i>pre-image</i>	5	-2	<i>pre-image</i>	<i>x</i>	<i>y</i>
<i>image</i>			<i>image</i>		

- 4) When rotating clockwise 180° , what patterns do you notice the coordinates from the pre-image to the image? _____
- 5) When rotating counter-clockwise 180° , what patterns do you notice the coordinates from the pre-image to the image? _____
- 6) What rule could we use to find the new image coordinates of a similar problem if we do not have a graph? _____

*Try out your rule:***Rotate the pre-image 180° counterclockwise.**

	x-coordinate	y-coordinate
<i>pre-image</i>	<i>x</i>	<i>y</i>
<i>image</i>		

	x-coordinate	y-coordinate
<i>pre-image</i>	5	-2
<i>image</i>		

Task 3: Patterns of ROTATIONS**Part A:**

Directions: In your small group, complete the graphs in Parts A, B, and C. Use this information to discuss and answer the questions in Part D.

- 9) Graph and label point A with coordinates (3, 5).
- 10) The point is in Quadrant 1. This point is the **pre-image**.
- 11) Rotate the point 90° counterclockwise around the origin.
- 12) Color this point blue. The blue point is the **image**.
- 13) What are the coordinates of the image? (-5 , 3)
- 14) Now rotate the original point 90° clockwise around the origin.
- 15) Color this point red. The red point is another **image**.
- 16) What are the coordinates of this new image? (5 , -3)

Part B:

- 1) Graph, label, and connect the following points to create a segment.

$$B (-5, 5) \quad C (-2, 4)$$

- 2) The segment is in Quadrant 3. This segment is the **pre-image**.
- 3) Rotate the segment 90° counterclockwise around the origin.
- 4) What are the coordinates of this rotation?

Segment	$B (-5, 4)$	$C (-2, 4)$
90° counterclockwise	$B'(-4,-5)$	$C'(-4,-2)$

- 5) Color this rotation blue. The blue segment is the **image**.
- 6) Now rotate the original segment 90° clockwise around the origin.
- 7) What are the coordinates of this rotation? $B'(4,5)$ $C'(4,2)$

Segment	$B (-5, 4)$	$C (-2, 4)$
---------	---------------	---------------

90° clockwise	$B'(4,5)$	$C'(4,2)$
------------------	-----------	-----------

- 8) Color this rotation red. The red segment is another **image**.

Part C:

- 9) Graph, label, and connect the following points to create a triangle.

$$D(-4, -1) \quad E(-1, -2) \quad F(-5, -5)$$

- 10) The triangle is in Quadrant 3. This triangle is the **pre-image**.

- 11) Rotate the triangle 180° counterclockwise around the origin.

- 12) What are the coordinates of this rotation?

Triangle	$D(-4, -1)$	$E(-1, -2)$	$F(-5, -5)$
180° counterclockwise	$D'(4,1)$	$E'(1,2)$	$F'(5,5)$

- 13) Color this rotation blue. The blue triangle is the **image**.

- 14) Now rotate the original triangle 180° clockwise around the origin.

- 15) What are the coordinates of this rotation?

Triangle	$D(-4, -1)$	$E(-1, -2)$	$F(-5, -5)$
180° clockwise	$D'(4,1)$	$E'(1,2)$	$F'(5,5)$

- 16) Color this triangle red. The red triangle is another **image**.

Part D:

- 7) When rotating clockwise 90° , what patterns do you notice the coordinates from the pre-image to the image? $(-5,4) \rightarrow (4,5)$ and $(-2,4) \rightarrow (4,2)$
- 8) When rotating counter-clockwise 90° , what patterns do you notice the coordinates from the pre-image to the image? $(-5,4) \rightarrow (-4,-5)$ and $(-2,4) \rightarrow (-4,-2)$
- 9) What rule could we use to find the new image coordinates of a similar problem if we do not have a graph? When you rotate clockwise 90° , you switch the x and y and make the y become the opposite sign. When you rotate counter-clockwise 90° , you switch the x and y and make the x become the opposite sign.

Try out your rule:

Rotate the pre-image 90° clockwise.

<i>pre-image</i>	5	-2	<i>pre-image</i>	x	y
<i>image</i>	-2	-5	<i>image</i>	y	$-x$

- 10) When rotating clockwise 180° , what patterns do you notice the coordinates from the pre-image to the image? $(5,-2) \rightarrow (-5,2)$
- 11) When rotating counter-clockwise 180° , what patterns do you notice the coordinates from the pre-image to the image? $(5,-2) \rightarrow (-5,2)$
- 12) What rule could we use to find the new image coordinates of a similar problem if we do not have a graph? When you rotate either clockwise or counter-clockwise you switch the signs of to the opposite for x and y .

Try out your rule:

Rotate the pre-image 180° counterclockwise.

	x-coordinate	y-coordinate
<i>pre-image</i>	x	y
<i>image</i>	$-x$	$-y$

	x-coordinate	y-coordinate
<i>pre-image</i>	5	-2
<i>image</i>	-5	2

FINAL TASK: Patterns of TRANSLATIONS, ROTATIONS & REFLECTIONS

Directions: In your small group, complete the graphs in Parts A, B, and C. Use this information to discuss and answer the questions in Part D.

PART A:

- Graph, label, and connect the following points to create a triangle.
 $A(2, 5)$ $B(4, 3)$ $C(1, 2)$
- The triangle is in Quadrant _____. This triangle is the **pre-image**.
- Color the triangle blue.
- Translate the blue triangle 4 units down. Color this translation red. The red triangle is the **image**.
- Identify the coordinates of this translation in the table below.

Blue Triangle	$A(2, 5)$	$B(4, 3)$	$C(1, 2)$
Red Triangle			

PART B:

- Graph, label, and connect the following points to create a quadrilateral.
 $D(-2, 5)$ $E(-1, 4)$ $F(-3, 1)$ $G(-5, 3)$
- The quadrilateral is in Quadrant _____. This quadrilateral is the **pre-image**.
- Color the quadrilateral blue.
- Reflect the blue quadrilateral over the x -axis. Color this reflection red. The red quadrilateral is the **image**.
- Identify the coordinates of this reflection in the table below.

Blue Quadrilateral	$D(-2, 5)$	$E(-1, 4)$	$F(-3, 1)$	$G(-5, 3)$
Red Quadrilateral				

Part C:

17) Graph, label, and connect the following points to create a triangle.

$$D (-2, -1) \quad E (-3, -2) \quad F (-5, -1)$$

18) The triangle is in Quadrant _____. This triangle is the **pre-image**.

19) Rotate the triangle 180° counterclockwise around the origin.

20) What are the coordinates of this rotation?

Triangle	$D (-2, -1)$	$E (-3, -2)$	$F (-5, -1)$
180° counterclockwise			

21) Color this rotation blue. The blue triangle is the **image**.

22) Now rotate the original triangle 180° clockwise around the origin.

23) What are the coordinates of this rotation?

Triangle	$D (-2, -1)$	$E (-3, -2)$	$F (-5, -1)$
180° clockwise			

24) Color this triangle red. The red triangle is another **image**.

FINAL TASK: Patterns of TRANSLATIONS, ROTATIONS & REFLECTIONS

Directions: In your small group, complete the graphs in Parts A, B, and C. Use this information to discuss and answer the questions in Part D.

PART A:

6. Graph, label, and connect the following points to create a triangle.
 $A(2, 5)$ $B(4, 3)$ $C(1, 2)$
7. The triangle is in Quadrant 1. This triangle is the **pre-image**.
8. Color the triangle blue.
9. Translate the blue triangle 4 units down. Color this translation red. The red triangle is the **image**.
10. Identify the coordinates of this translation in the table below.

Blue Triangle	$A(2, 5)$	$B(4, 3)$	$C(1, 2)$
Red Triangle	$A'(2, 1)$	$B'(4, -1)$	$C'(1, -2)$

PART B:

- 1) Graph, label, and connect the following points to create a quadrilateral.
 $D(-2, 5)$ $E(-1, 4)$ $F(-3, 1)$ $G(-5, 3)$
- 2) The quadrilateral is in Quadrant 2. This quadrilateral is the **pre-image**.
- 3) Color the quadrilateral blue.
- 4) Reflect the blue quadrilateral over the x-axis. Color this reflection red. The red quadrilateral is the **image**.
- 5) Identify the coordinates of this reflection in the table below.

Blue Quadrilateral	$D(-2, 5)$	$E(-1, 4)$	$F(-3, 1)$	$G(-5, 3)$
Red Quadrilateral	$D'(-2, -5)$	$E'(-1, -4)$	$F'(-3, -1)$	$G'(-5, -3)$

Part C:

25) Graph, label, and connect the following points to create a triangle.

$$D (-2, -1) \quad E (-3, -2) \quad F (-5, -1)$$

26) The triangle is in Quadrant 3. This triangle is the **pre-image**.

27) Rotate the triangle 180° counterclockwise around the origin.

28) What are the coordinates of this rotation?

Triangle	$D (-2, -1)$	$E (-3, -2)$	$F (-5, -1)$
180° counterclockwise	$D'(2,1)$	$E'(3,2)$	$F'(5,1)$

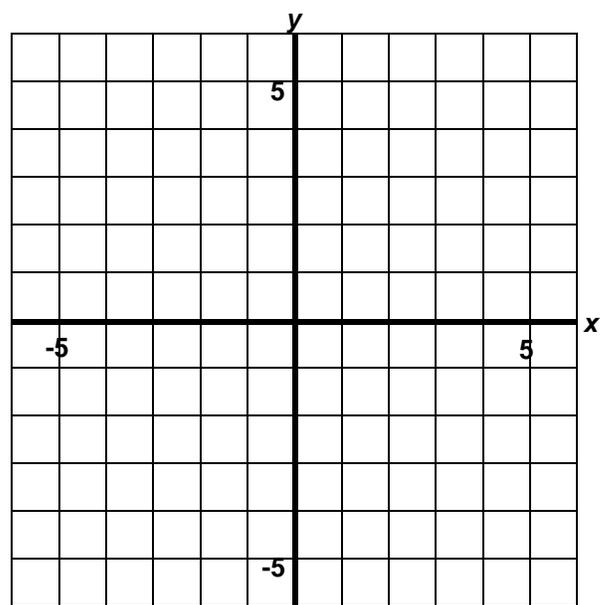
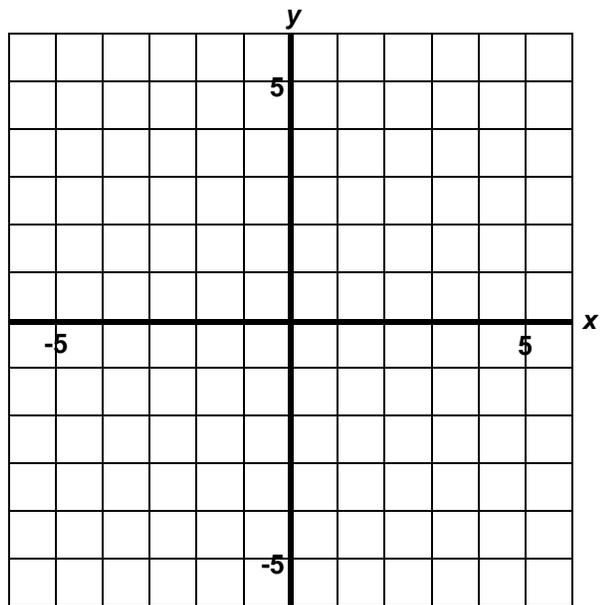
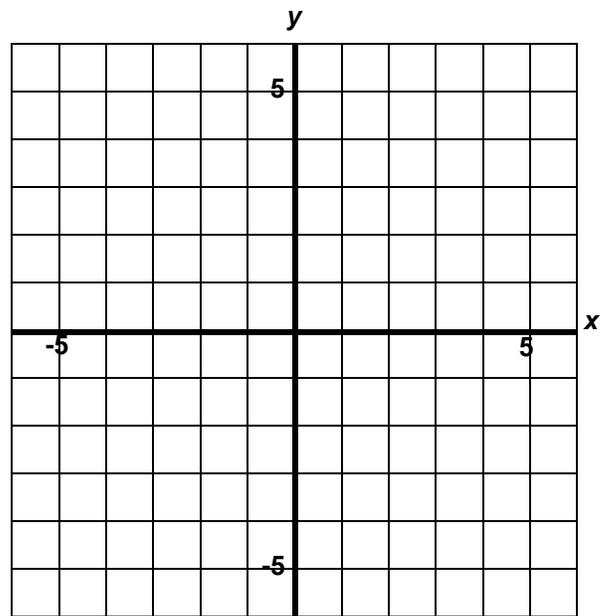
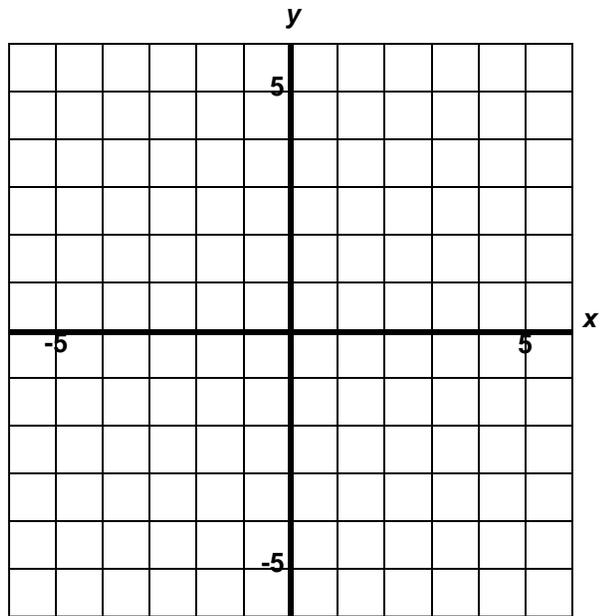
29) Color this rotation blue. The blue triangle is the **image**.

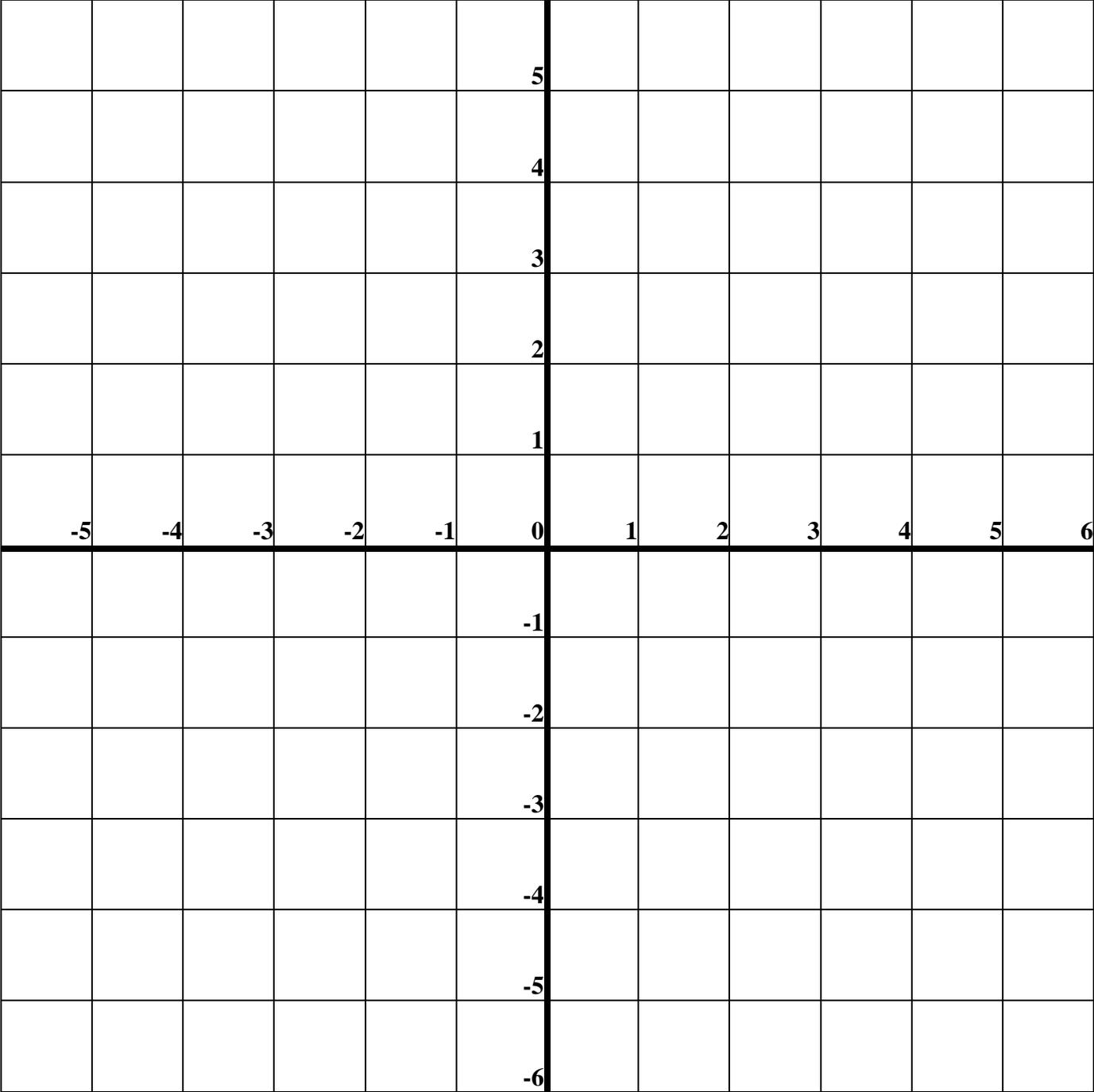
30) Now rotate the original triangle 180° clockwise around the origin.

31) What are the coordinates of this rotation?

Triangle	$D (-2, -1)$	$E (-3, -2)$	$F (-5, -1)$
180° clockwise	$D'(2,1)$	$E'(3,2)$	$F'(5,1)$

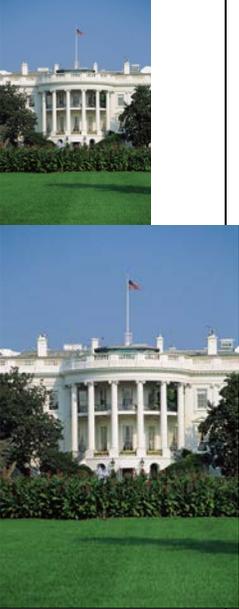
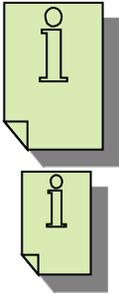
Color this triangle red. The red triangle is another **image**.



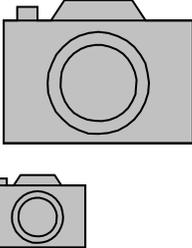


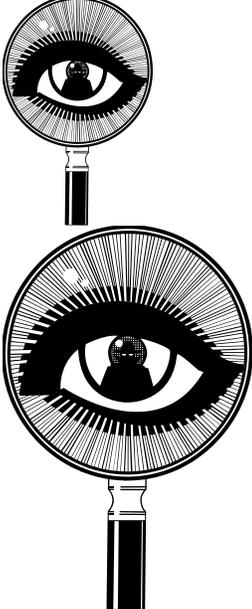
Activity Two: Is it dilated?

Attachment # 10

	Yes	No
1) 		
3) 		
5) 		

	Yes	No
2) 		
4) 		
6) 		

	Yes	No
7) 		
9) 		

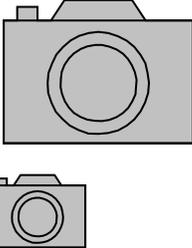
	Yes	No
8) 		
10) 		

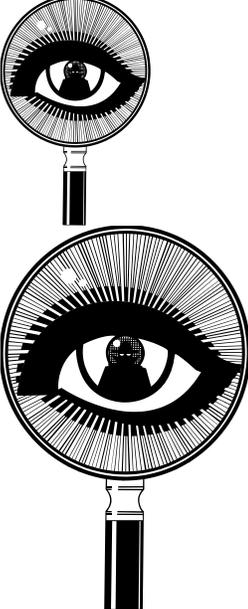
Activity Two: Is it dilated?

	Yes	No
1) 		X
3) 	X	
5) 	X	

Answer Attachment #10

	Yes	No
2) 	X	
4) 		X
6) 		X

	Yes	No
7) 		X
9) 	X	

	Yes	No
8) 	X	
10) 		X

Activity Three: *Teacher Directions*

Predicting Snowflakes

Attachment #11

Question 1

Teacher Demonstration

- A. Take a **square** piece of paper and model making a snowflake the same or different than the template in question 1. On a $\frac{1}{4}$ sheet, mark your design with a pencil. Cut the design. Do not open it up yet.
- B. Ask students to predict what $\frac{1}{2}$ of the paper will look like when you open the paper to $\frac{1}{2}$ sheet. Have them draw it in the $\frac{1}{2}$ sheet box on the handout.
- C. Open the paper to $\frac{1}{2}$ sheet and evaluate predictions.
- D. Ask students to predict what the whole snowflake will look like. Have them draw it in the whole sheet box on the handout.
- E. Open the full sheet and evaluate predictions.
- F. Discuss the transformations that occurred by cutting all quadrants at one time. (*Reflecting over the y-axis and x-axis.*)

Questions 2 – 5

Determining Snowflake Image Coordinates: Guided Practice/ Independent Practice

- A. Students will reflect the pre-image (handout) according to the directions on the student page to create a snowflake.
- B. Reflect the pre-image over the y-axis, then the x-axis, and lastly the y-axis again. Point A is labeled in the pre-image. Students need to label point
- C. In the process of the transformations, students will conclude with A''' or B''' in their images. Students should be able to locate A or B in all of the images.
- D. Students are to write the ordered pair for A''' or B''' once the final reflection is completed.

Enrichment:

Complex Snowflakes

- A. Students take an $\frac{1}{8}$ triangle design and reflect it to make an entire snowflake.
- B. The pre-image is intended to be the bottom eighth of quadrant IV.

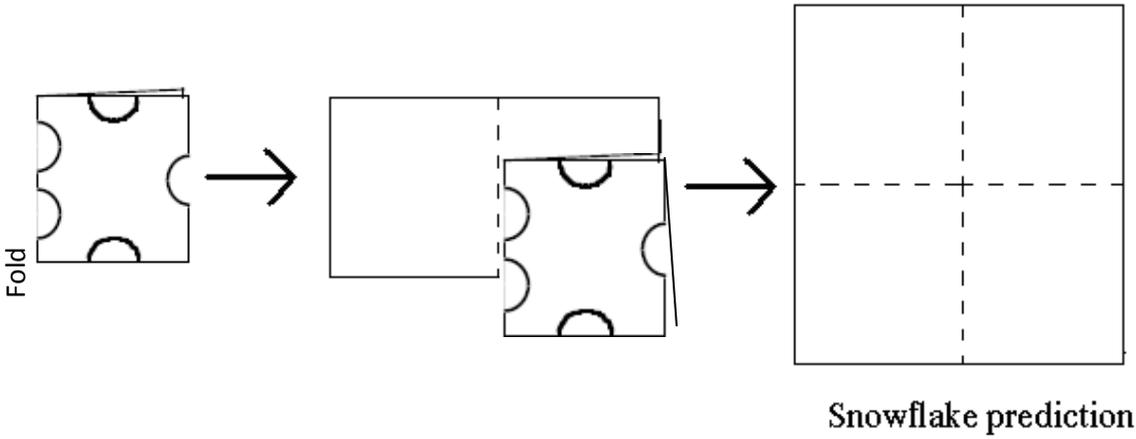
Assessment:

- A. Two different versions of an assessment are available. Version A gives a pre-image and transformation directions to find the image. Students are to find the image. Version B gives the image and the transformation directions that were used to find the image. Students are to work backwards to find the pre-image.
- B. Consider using both versions and differentiating which version groups/students receive based on progress in class.

Activity Three: Predicting Snowflakes

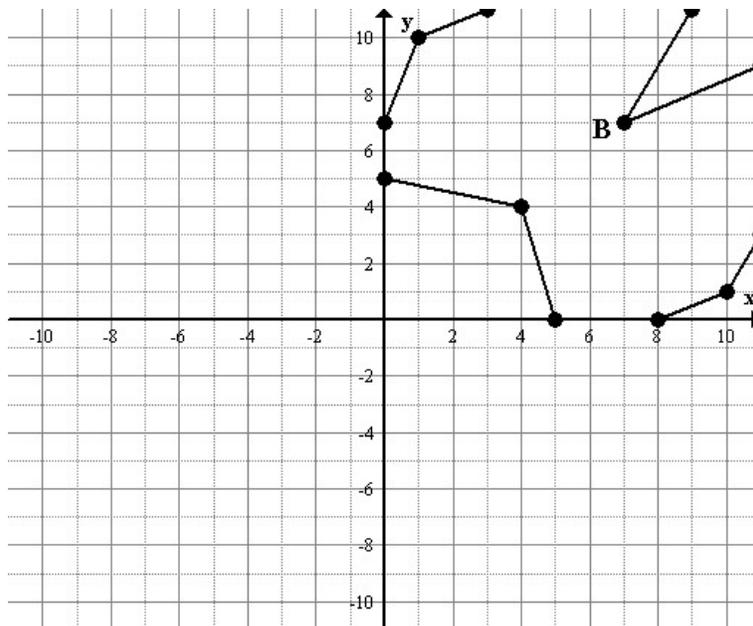
(What type of transformation are you using?)

1.



2. Using the partial snowflake design below. Complete the following transformations to complete the snowflake.

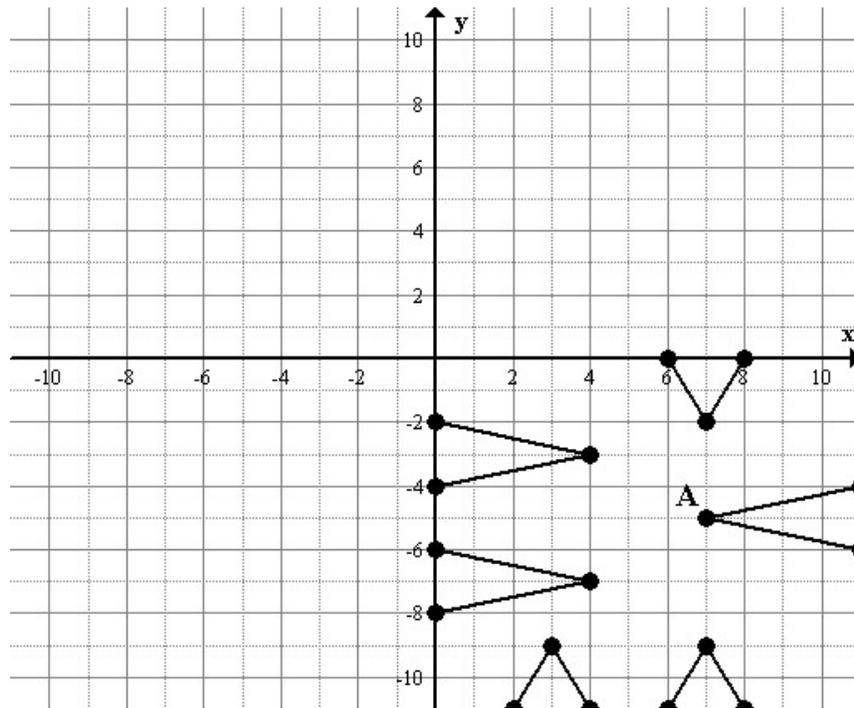
- a. Reflect over the y-axis
- b. Reflect over the x-axis
- c. Reflect over the y-axis



3. What is the location of point B after the three given transformations? Explain how you determined your answer.

Attachment #12

4. Using the partial snowflake design below. Complete the following transformations to complete the snowflake.
- Reflect over the x-axis
 - Reflect over the y-axis
 - Reflect over the x-axis

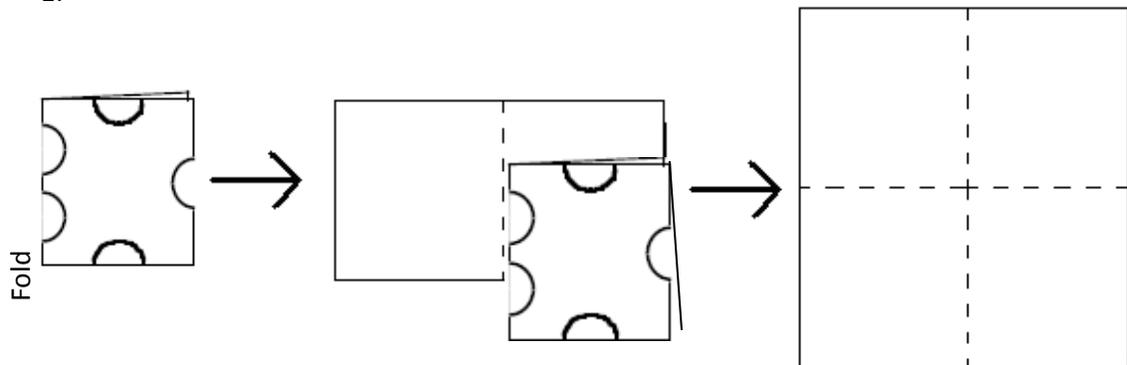


5. What is the location of point A after the three given transformations? Explain how you determined your answer.

Activity Three: Predicting Snowflakes

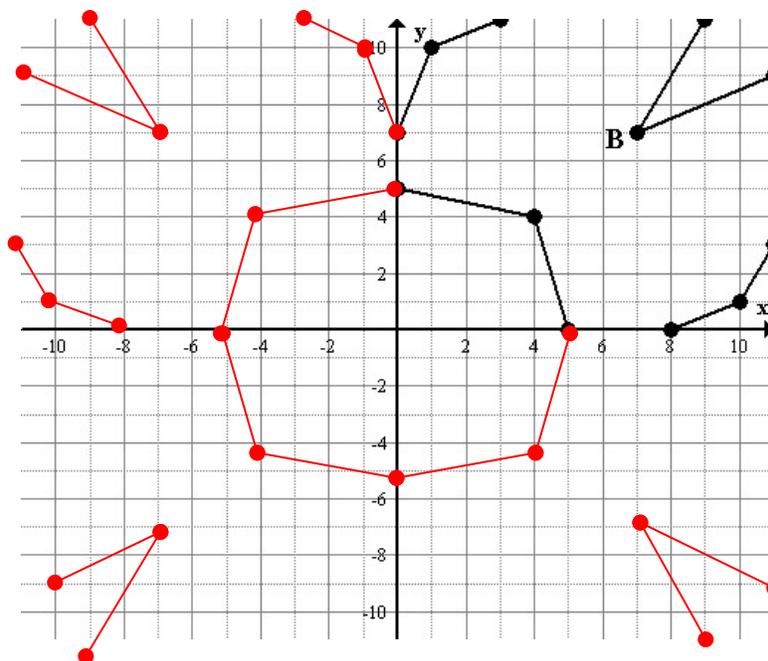
(What type of transformation are you using?)

1.



Snowflake prediction

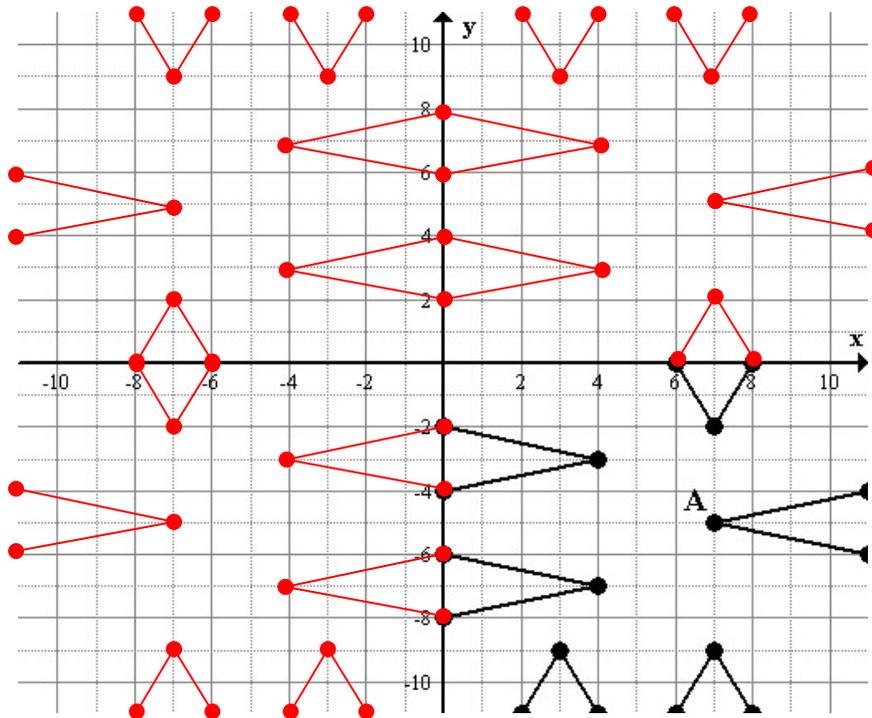
2. Using the partial snowflake design below. Complete the following transformations to complete the snowflake.
 - a. Reflect over the y-axis
 - b. Reflect over the x-axis
 - c. Reflect over the y-axis



3. What is the location of point B after the three given transformations? Explain how you determined your answer.

Answers Attachment #12

4. Using the partial snowflake design below. Complete the following transformations to complete the snowflake.
- Reflect over the x-axis
 - Reflect over the y-axis
 - Reflect over the x-axis

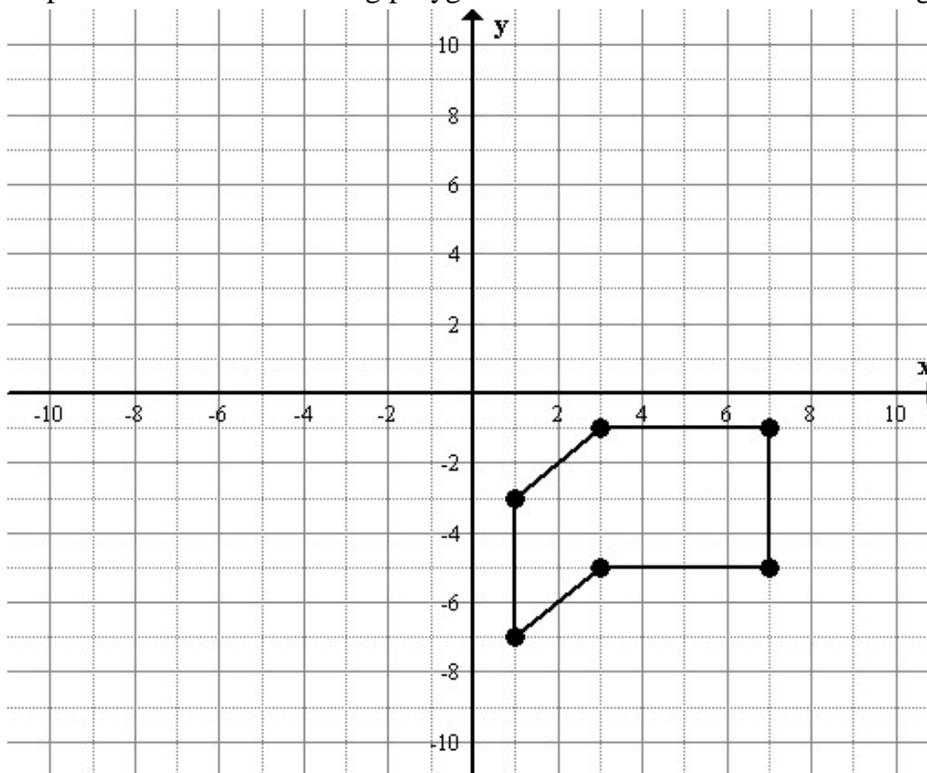


5. What is the location of point A after the three given transformations? Explain how you determined your answer.

Activity Three Assessment
Version A

Attachment # 13

Directions: Graph and label the following polygon after each transformation on the graph provided.

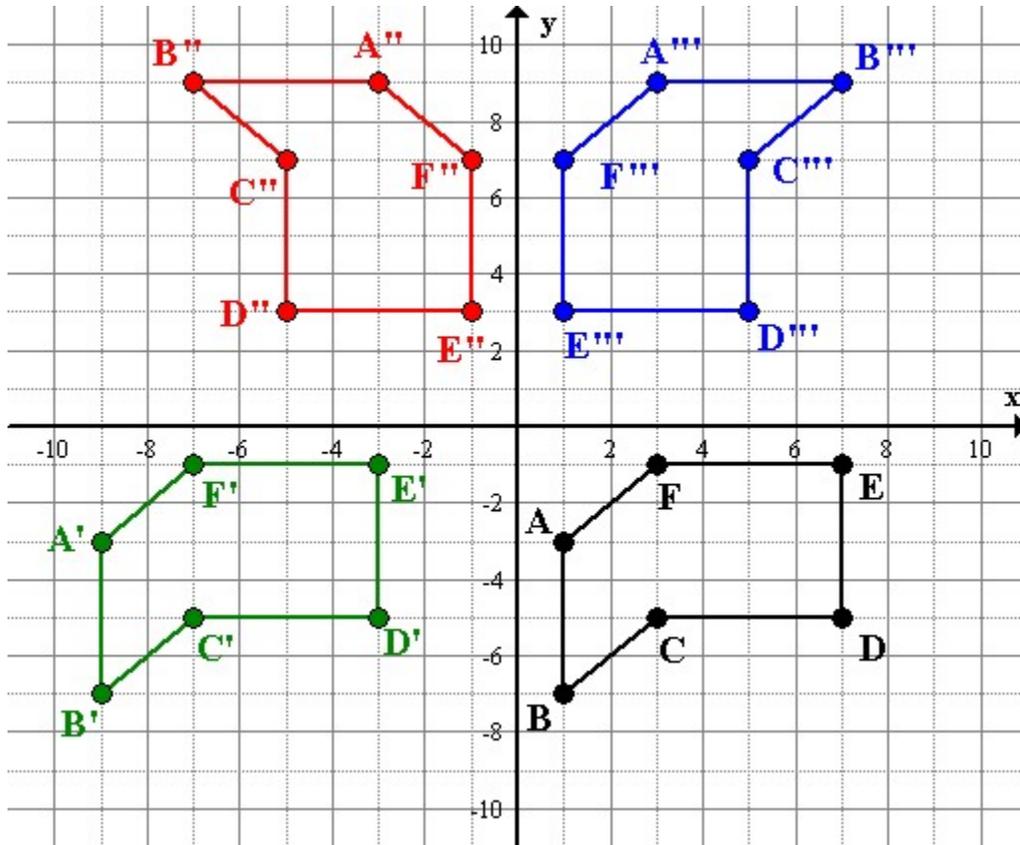


- 1) Translate the following object 10 spaces to the left.
- 2) Rotate the polygon 90° clockwise.
- 3) Reflect the polygon across the y-axis.

Activity Three Assessment
Version A

Answer Key Attachment #13

Directions: Graph and label the following polygon after each transformation on the graph



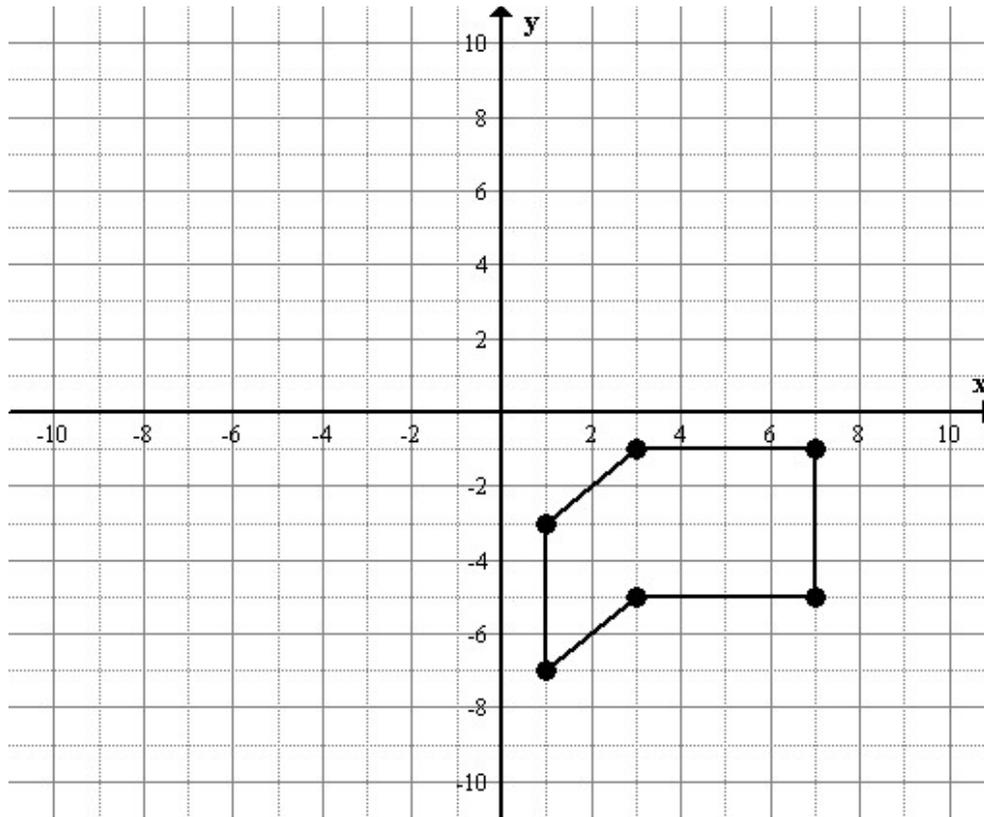
provide.

- 1) Translate the following object 10 spaces to the left.
- 2) Rotate the polygon 90° clockwise.
- 3) Reflect the polygon across the y-axis.

**Activity Three Assessment
Version B**

Attachment #14

Directions: Determine the coordinates of the pre-image given the following transformations that result in the image provided below.



- 1) Reflect across the y-axis
- 2) Rotate the polygon 90° counter-clockwise
- 3) Translate the object 10 spaces to the right.

A (,)

B (,)

C (,)

D (,)

E (,)

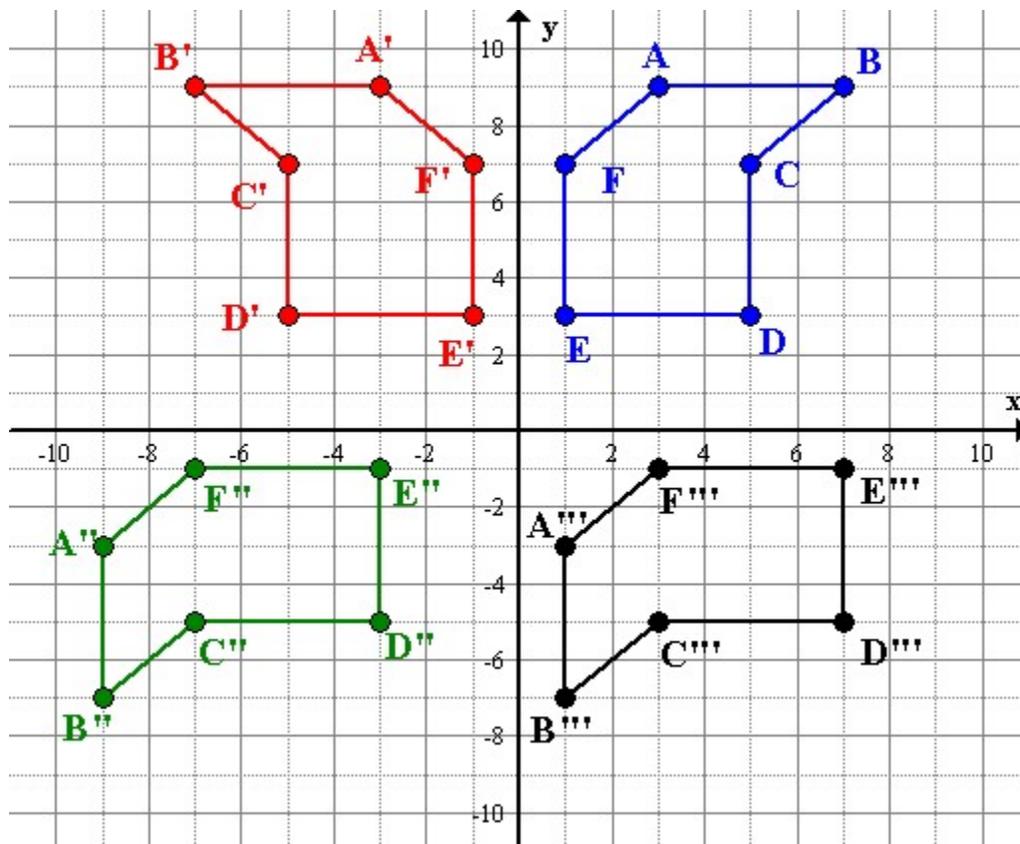
F (,)

Activity Three Assessment
Version B

Answer Key Attachment #14

Directions: Determine the coordinates of the pre-image given the following transformations that result in the image provided below.

- 1) Reflect across the y-axis
- 2) Rotate the polygon 90° counter-clockwise
- 3) Translate the object 10 spaces to the right.



A (3, 9)

B (7, 9)

C (5, 7)

D (5, 3)

E (1, 3)

F (1, 7)