Louisiana Department of Education
Mentor Teacher Training

Module 5:
Delivering High-Quality, Standards-Aligned Mathematics Instruction

Secondary Mathematics Cohort

Mentor Training Goals

1. Build strong relationships with their new and resident teachers.
2. Use quantitative data (i.e. student achievement results from summative and formative assessments) and qualitative data (i.e. student work and feedback, classroom observations, peer and supervisor feedback) to identify and prioritize new and resident teacher's strengths and areas for growth based on teacher **certification competencies and performance expectations**.
3. Implement a continuum of support and coaching techniques to build new and resident teacher knowledge and skill in content and content-specific pedagogy and self-analysis and reflection skills.
   - Set short- and long-term professional growth goals based on student results.
   - Create a logical sequence of coaching opportunities to build new/resident teacher's skill over time to reach goals.
   - Identify and leverage high-quality, evidence-based resources to support new and resident teachers' specific needs.
   - Lead effective feedback/coaching conversations grounded in student evidence (includes difficult conversations).
   - Track new and resident teacher progress.
   - Model and facilitate resident self-reflection and improvement.
4. Deepen content knowledge and content-specific pedagogy to support continued development of teacher competencies in the respective content areas.
5. Assess and develop mentor competencies in their discipline to identify strengths and gaps and to develop areas of need. These assessments are based on competencies and student results.

Module 5 Key Points

- Intentional planning and teaching of the EngageNY resources will promote the key shifts of focus, coherence, and rigor.
- It is necessary to establish a classroom culture that supports making sense of and persevering in solving problems.
- Intentional actions can make a tremendous impact on students' belief that they can succeed (in mathematics).

Module 5 Outcomes

Mentors will:

- Gain clarity on how the mathematics standards manifest in the secondary classroom when executed well in curriculum instruction.
- See effective instructional strategies in action that can be implemented directly in their classrooms.
- Identify implications for delivering high-quality, standards aligned mathematics instruction using EngageNY resources.
- Implement instructional strategies that foster conceptual understanding, procedural skill and fluency, and application skills with students.
- Determine teacher moves that promote a positive classroom culture and support perseverance in solving problems.
- Implement strategies that support a growth mindset with students.
- Revise and refine their thinking on what the big idea is.

Module 5 Agenda

8:30-12:00

- Review and revisit Modules 3 & 4
- Reflection on assignments
- Seeing the key shifts in action
- Exploring the EngageNY curriculum

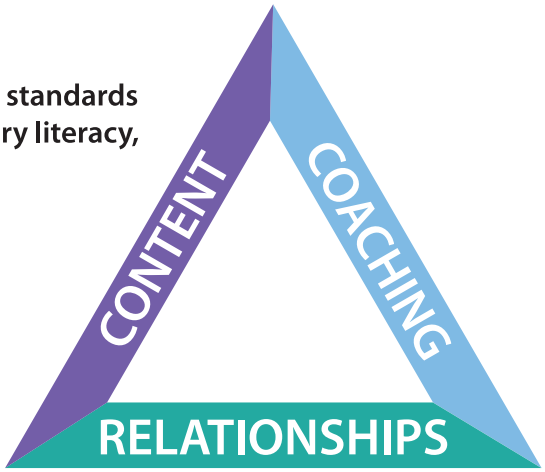
12:45-4:00

- Classroom culture
- Exploring coherence in the LSSM
- Teacher Preparation Competencies in mathematic

Agreements

- **M**ake the learning meaningful
- **E**ngage mentally and physically
- **N**otice opportunities to support the learning of others
- **T**ake responsibility for your own learning
- **O**wn the outcomes
- **R**espect the learning environment and the use of personal technology

- Student content-area standards (ELA, math, disciplinary literacy, other disciplines)
- State curriculum
- Teacher-preparation competencies
- Effective instruction



- Mentor distinction criteria
- Roles and responsibilities
- Coaching supports
- Coaching cycle
  - Assessing
  - Goal setting
  - Observing
  - Conferring
- Gradual release

- Partnership agreements
- Trust
- Growth-oriented (difficult) conversations
- Handling stress

Reflections on Module 3-4 assignments

Co-teaching reflection

1. What agreements did you establish prior to co-teaching? How did you decide those were the most important ones to establish?
2. What were the learning goals for you and your co-teacher for this co-teaching experience? How did you assess your success?
3. Which co-teaching method(s) did you use? Describe the factors you and your resident teacher/colleague considered to choose the co-teaching method you did.
4. What unanticipated surprises, issues, or challenges did you encounter and how did you and your co-teacher handle them? If you didn't have any, what might have contributed to the absence of them?
5. What will you do in future co-teaching situations, particularly with resident teachers, that will strength the opportunity for both teacher and student learning?

Key shifts in mathematics

1. Share evidence from your classroom that demonstrates your application of the math shifts. Describe how the practices influenced student learning.

Reflection conversation

1. Describe briefly the reflection conversation. With whom did you hold the conversation? What was the teacher's learning goal? What was the student learning goal?
2. Which stance (consult, collaborate, mediate) did you intend to take? Explain your reason for selecting this stance?
3. What were some prompts or questions you prepared/selected for the conversation? Analyze how two of those prompts or questions aligned with your selected stance.
4. What did you learn about your use of communication skills (listening, pausing, paraphrasing, and positive presuppositions) during the reflection conversation?
5. What did the teacher learn as a result from this reflection conversation?
6. What did you learn as a mentor from this reflection conversation?

Successes	Challenges	Insights

Math Content Session

Module 1: Setting the foundation for implementing the curriculum

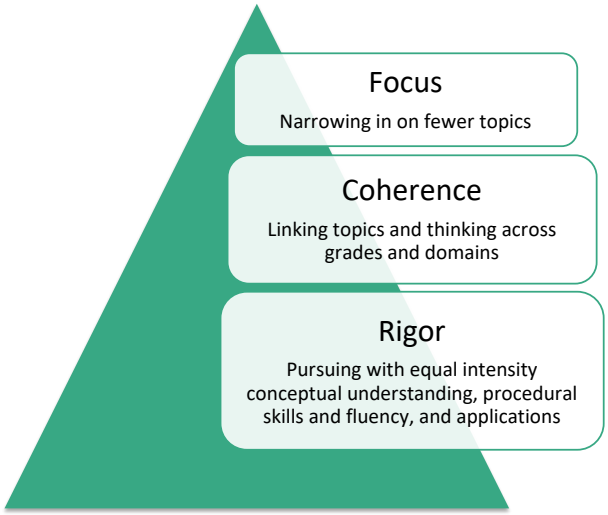
Session 3: Seeing the key shifts in action

Grades 6–9

Session outcomes

- Gain clarity on how the mathematics standards manifest in the secondary classroom when executed well in curriculum instruction.
- See effective instructional strategies in action that can be implemented directly in their classrooms.
- Identify implications for delivering high-quality, standards aligned mathematics instruction.

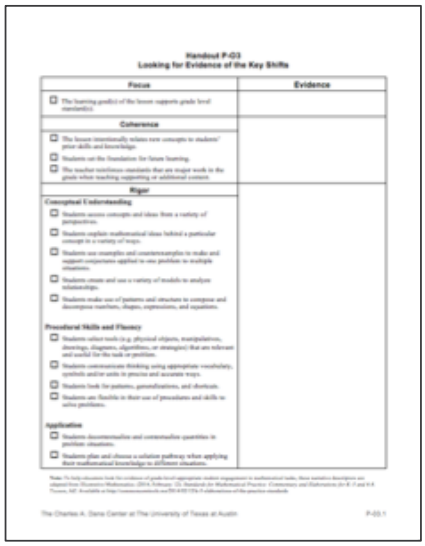
Key shifts of the LSSM



Evidence of the key shifts

Use the handout to:

- Check off student behaviors/actions that you observed.
- Record any observations you made that serve as evidence.



Handout P-03
Looking for Evidence of the Key Shifts

Focus	Evidence
☐ The learning goal(s) of the lesson supports grade level standard(s)	
Coherence	
☐ The lesson intentionally relates new concepts to students' prior skills and knowledge.	
☐ Students set the foundation for future learning.	
☐ The teacher reinforces standards that are major work in the grade when teaching supporting or additional content.	
Rigor	
Conceptual Understanding	
☐ Students access concepts and ideas from a variety of perspectives.	
☐ Students explain mathematical ideas behind a particular concept in a variety of ways.	
☐ Students use examples and counterexamples to make and support conjectures applied to one problem in multiple situations.	
☐ Students create and use a variety of models to analyze relationships.	
☐ Students make use of patterns and structure to compose and decompose numbers, shapes, expressions, and equations.	
Procedural Skills and Fluency	
☐ Students select tools (e.g. physical objects, manipulatives, drawings, diagrams, algorithms, or strategies) that are relevant and useful for the task or problem.	
☐ Students communicate thinking using appropriate vocabulary, symbols and/or units in precise and accurate ways.	
☐ Students look for patterns, generalizations, and shortcuts.	
☐ Students are flexible in their use of procedures and skills to solve problems.	
Application	
☐ Students decontextualize and contextualize quantities in problem situations.	
☐ Students plan and choose a solution pathway when applying their mathematical knowledge to different situations.	

Note: To help educators look for evidence of grade level appropriate student engagement in mathematical tasks, these statistics descriptors are adapted from Elements Mathematics, 2014, February. 32. Standards for Mathematical Practice: Consensus and Elaboration for K-7 and 8-8 Tucson, AZ. Available at http://commoncorestuff.com/DO-NOT-TOUCH-Collaboration-of-the-practice-standards

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Engaging in the math

Pauline mows a lawn at a constant rate. She mows a 35-square-foot lawn in 2.5 minutes.

How might we express Pauline's average rate for any number of minutes?

Let *y* represent the number of square feet and *t* represent the number minutes.

Talking about the math

Average Rate

Constant Rate

What is the same?

What is different?

Engaging in the math

Waters flows at a constant rate out of a faucet. It takes three minutes for 10.5 gallons to flow.

How can you express the number of gallons of water that flows in any number of minutes?

Convince us!

- Represent your answer in at least two different ways.
- Write two different ratios for this situation.

Engaging in the math – Practice with a partner

For exercise 1:

Partner A	Partner B
Create a TABLE for the situation	Create a GRAPH for the situation

Once you are both done, compare your table and graph. Then, turn and talk with your partner to discuss:

- What values did you use to generate your graph or table? Why?
- Where do you see the constant rate in each of the representations?

What linear equation can represent the number of pages typed in any given time interval? (Let p represent number of pages typed and t represent time in minutes.)

Engaging in the math – Practice with a partner

For exercise 2:

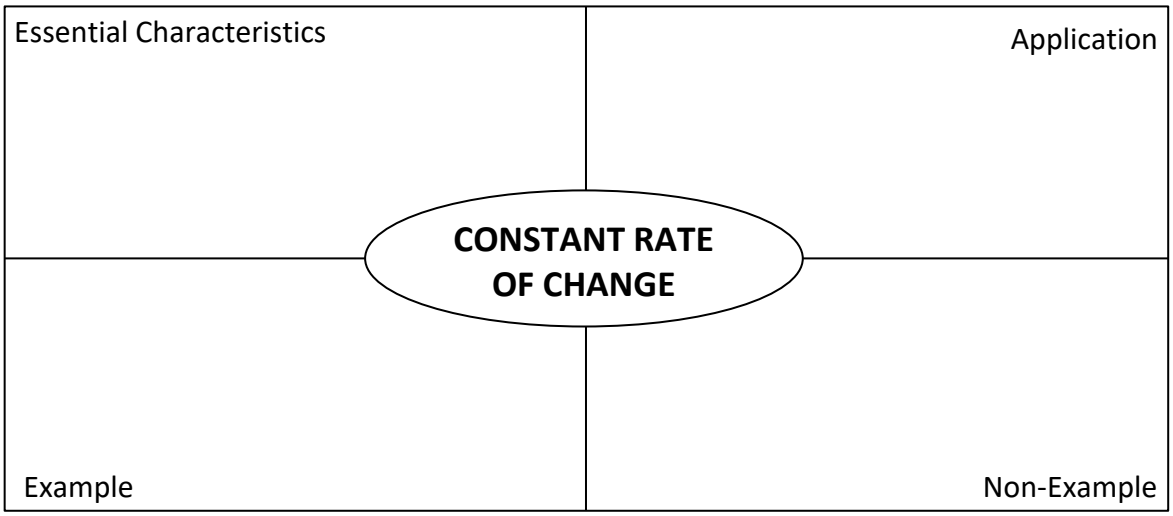
Partner A	Partner B
Create a GRAPH for the situation	Create a TABLE for the situation

Once you are both done, compare the table and graph, turn and talk with your partner:

- What values did you use to generate your graph or table? Why?
- Where do you see the constant rate in each of the representations?

What linear equation can represent the number of square feet Emily can paint in any given time interval? (Let *A* represent the area in square feet and *t* the number of minutes.)

Engaging in the math – Closing the lesson



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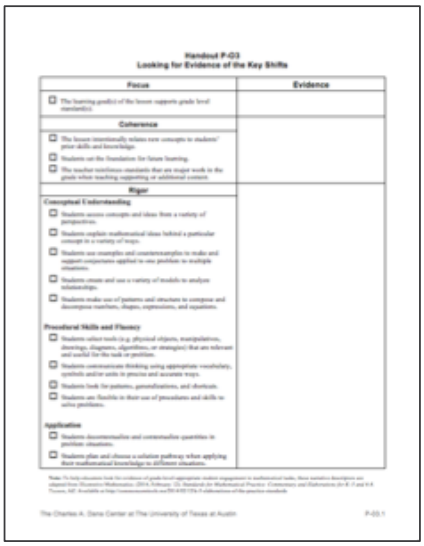

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Math Content Session

Module 1: Setting the foundation for implementing the Tier 1 curriculum

Session 4: Exploring the EngageNY Curriculum

Grades 6–9

Session Outcomes

- Identify implications for delivering high-quality, standards aligned mathematics instruction using EngageNY resources.
- Analyze EngageNY lesson components to ensure alignment to the standards.
- Implement instructional strategies that foster conceptual understanding, procedural skill and fluency, and application skills with students.

Mentor Teacher Module 5

What is alignment?

Alignment is more than…

- A chart
- A textbook correlation
- A scope and sequence
- A curriculum guide
- A testing plan

These imply alignment, but they do not achieve alignment.

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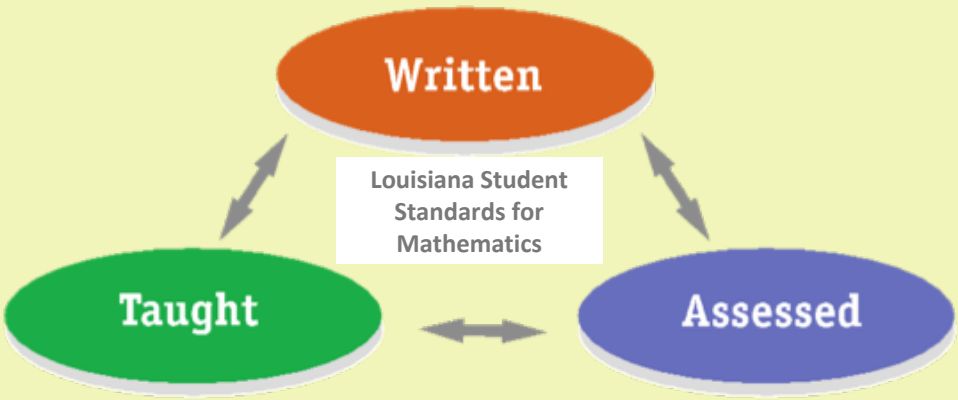
What is alignment?



(English, 2010)
Adapted from the work of Renwick English

Mentor Teacher Secondary ELA Module 5

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What is alignment?

Alignment means ***EVERY*** educator…

- Understands what is expected of students.
- Understands these expectations within the context of the K–12 program.
- Accepts responsibility for meeting these expectations.

Mentor Teacher Secondary ELA Module 5

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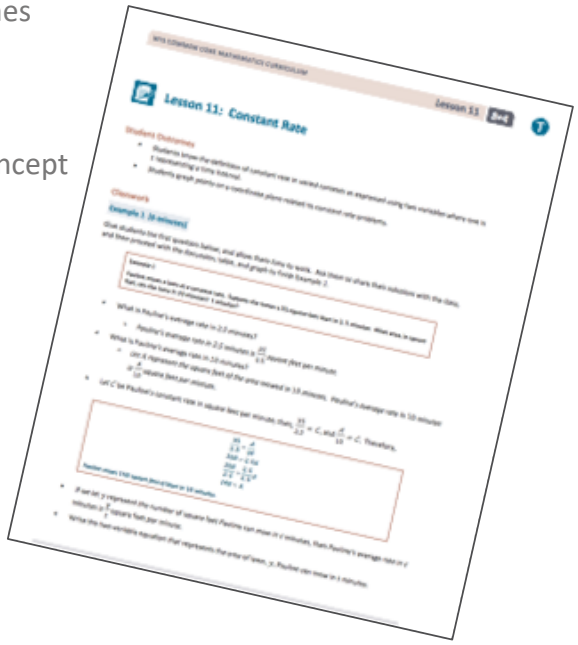
What are the main components of the Engage NY curriculum resource?

What is the purpose of each component?

- Student Outcomes
- Materials
- Classwork or Concept Development
- Exercises
- Closing
- Exit Ticket
- Problem Set



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Unpacking the EngageNY lesson

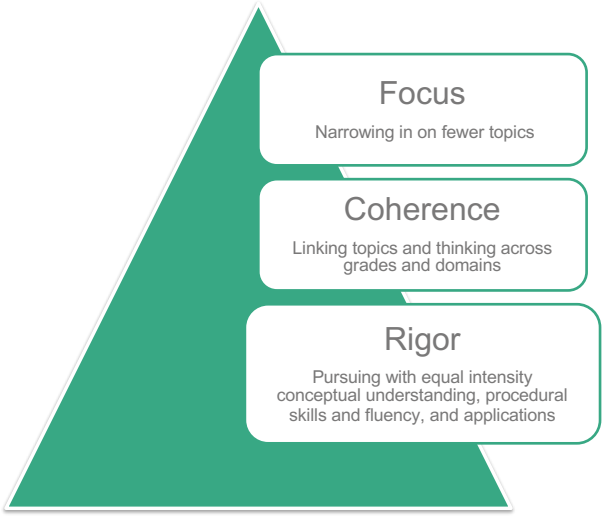
8.EE.B.5

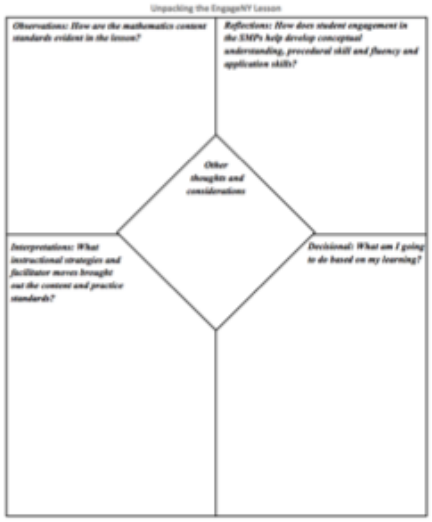
Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*

SMP 7

Look for and make use of structure.

Evidence of the key shifts





Session 4 reflection

Personal reflection

- How will unpacking EngageNY lessons for the standards and key shifts affect your planning and instruction?
- As a result of your learning in sessions 3 and 4, identify one *action* you will take to effectively plan for instruction.

Math Content Session

Module 1: Setting the foundation for implementing the curriculum

Session 5: Classroom culture

Grades 6–9

Session Outcomes

- Determine teacher moves that promote a positive classroom culture and support perseverance in solving problems.
- Reflect on their own beliefs about learning and how these beliefs influence students.
- Implement strategies that support a growth mindset with students.

"When students realize they can succeed in mathematics, they are transformed – you can see them raise their aspirations for school and life."

— Uri Treisman

University Distinguished Teaching Professor,
Professor of Mathematics, Professor of Public Affairs, and
Executive Director, The Charles A. Dana Center at
The University of Texas at Austin

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Quiz

Give yourself 4 points each time you answered AGREE on the gray squares.

Subtract 2 points for each time you answered AGREE on the white squares.

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Fixed versus malleable intelligence

Read Dweck's article, "The Perils and Promises of Praise," and consider the following:

What confirms your current knowledge and beliefs?

What questions does this research raise for you?

(Dweck, 2007)

Fixed versus malleable intelligence

Consequences of students who view intelligence as **fixed**	Consequences of students who view intelligence as **malleable**

Supporting a growth mindset

- Provide opportunities for collaboration and discourse.
- Encourage high, equitable levels of student participation.
- Encourage a broad range of competencies/approaches.
- Promote cooperative learning and shared accountability.
- Provide autonomy/choice in task, team, and processes.
- Model and monitor the climate for interactions among students.

What are the implications?

Based on your reading and discussion:

- How might this research inform the ways in which you interact with and teach your students?
- How might this research impact the culture of learning in your classroom?

Math Content

Module 2: Multiplicative Thinking: Ratios and Rates

Session 2: Exploring Coherence in the LSSM Grades 6–9

Session Outcomes

- Engage in a study of standards related to using multiplicative thinking to reason about rates and ratios to see how this idea develops across grades.
- Revise and refine their thinking on what the big idea is.
- Reflect on their study of the standards and how their instruction may be impacted by their new knowledge.

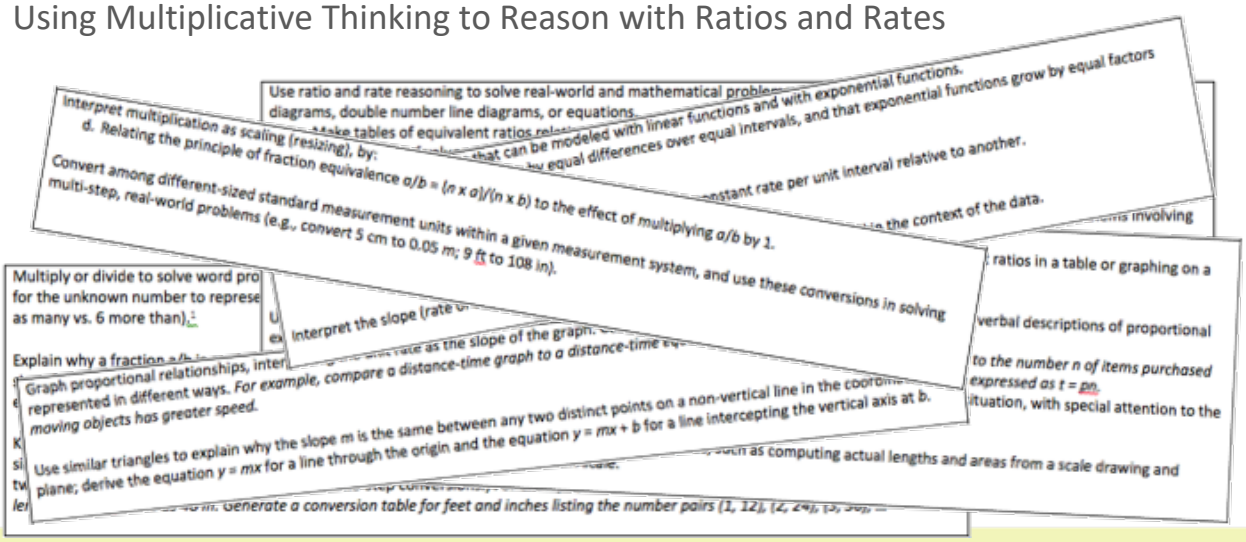
Exploring vertical alignment

Why study the vertical alignment of standards?

- Helps us understand our grade-level expectations
- Gives us information on student expectations from previous grades so that we can connect to prior learning and remediate, if necessary
- Gives us information on where students will be going with the material
- Gives us an idea of how the math develops and of our role within the larger system
- Provides insights on the type of instruction and experiences we need to provide our students

Vertical alignment

Using Multiplicative Thinking to Reason with Ratios and Rates



Vertical alignment

Vertical Alignment Chart

Big Idea: Using Multiplicative Thinking to Reason About Ratios and Rates

Grade/ Course	LSS	Content Standards Summary
Alg I	A1: F-LE.A.1	Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
	A1: S-ID.C.7	Interpret the slope (rate of change) ~~and the intercept (constant term)~~ of a linear model in the context of the data.
8	8.EE.B.5	Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*
	8.EE.B.6	Use similar triangles to explain why the slope *m* is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at *b*.
7	7.RP.A.2	Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.* d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r*) where *r* is the unit rate.
	7.G.A.1	Solve problems involving scale drawings of geometric figures, such as computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

Vertical alignment

6	6.RP.A.3	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what unit rate were lawns being mowed?* c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.
	6.EE.C.9	Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.*
5	5.NF.B.5	Interpret multiplication as scaling (resizing), by: d. Relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.
	5.MD.A.1	Convert among different-sized standard measurement units within a given measurement system, and use these conversions in solving multi-step, real-world problems (e.g., convert 5 cm to 0.05 m; 9 ft to 108 in).
4	4.OA.A.2	Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and/or equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison (Example: 6 times as many vs. 6 more than).[1]
	4.NF.A.1	Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. (Denominators are limited to 2, 3, 4, 5, 6, 8, 10, 12, and 100.)
	4.MD.A.1	Know relative sizes of measurement units within one system of units including ft, in; km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. (Conversions are limited to one-step conversions.) *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...*

Vertical alignment discussion questions

- What content changes occur from grade to grade?
- Where are concepts introduced, developed, and finalized?
- Does an idea or skill get more complex, and if so, how?
- How are the components of rigor (conceptual understanding, procedural skills and fluency, and application) evident in the standards?
- How do the components of rigor change as the big idea is developed across the grades?

Have your thoughts about the big idea changed after studying the standards?

How does studying the standards in this way benefit you as a teacher?

What are the implications for you as you plan instruction?

How does studying the standards in this way make evident the 3 key shifts of focus, coherence, and rigor?

Looking for Evidence of the Key Shifts

Focus	Evidence
☐ The learning goal(s) of the lesson supports grade-level standard(s).	
Coherence	
☐ The lesson intentionally relates new concepts to students' prior skills and knowledge. ☐ Students set the foundation for future learning. ☐ The teacher reinforces standards that are major work in the grade when teaching supporting or additional content.	
Rigor	
Conceptual Understanding ☐ Students access concepts and ideas from a variety of perspectives. ☐ Students explain mathematical ideas behind a particular concept in a variety of ways. ☐ Students use examples and counterexamples to make and support conjectures that are applied to one problem to multiple situations. ☐ Students create and use a variety of models to analyze relationships. ☐ Students make use of patterns and structure to compose and decompose numbers, shapes, expressions, and equations. **Procedural Skills and Fluency** ☐ Students select tools (e.g., physical objects, manipulatives, drawings, diagrams, algorithms, or strategies) that are relevant and useful for the task or problem. ☐ Students communicate thinking using appropriate vocabulary, symbols and/or units in precise and accurate ways. ☐ Students look for patterns, generalizations, and shortcuts. ☐ Students are flexible in their use of procedures and skills to solve problems. **Application** ☐ Students decontextualize and contextualize quantities in problem situations. ☐ Students plan and choose a solution pathway when applying their mathematical knowledge to different situations.	

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Engaging in the Math

Mowing Lawns

What do you know?	What do you want to know?	What might the question be?

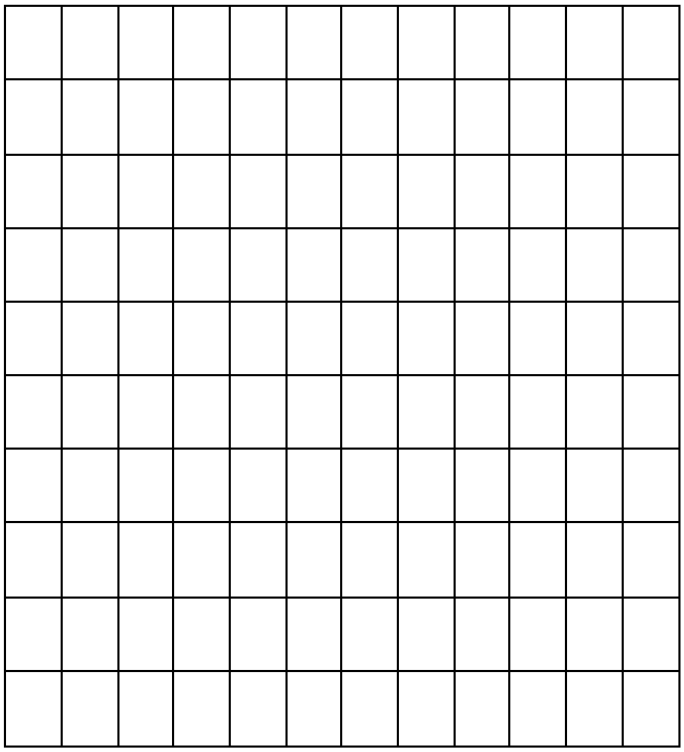
Work Space

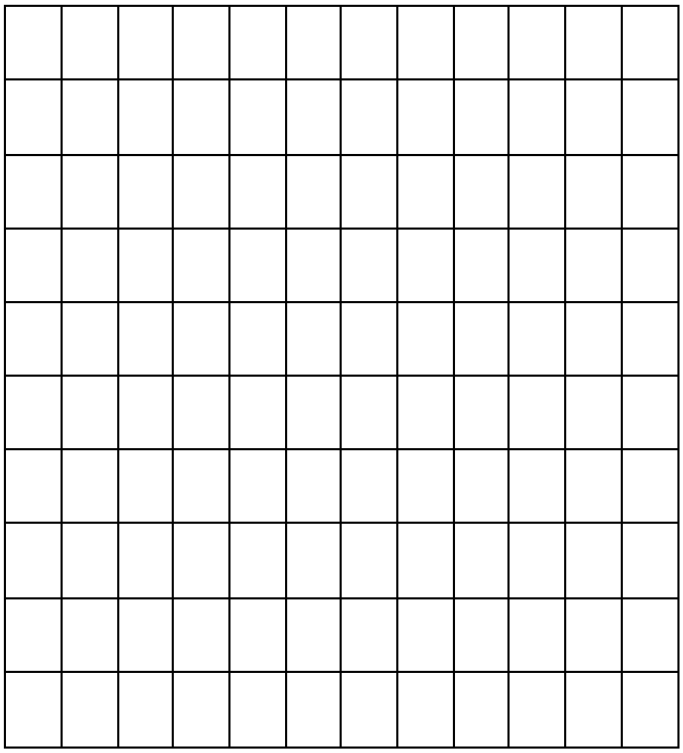
Flowing Water

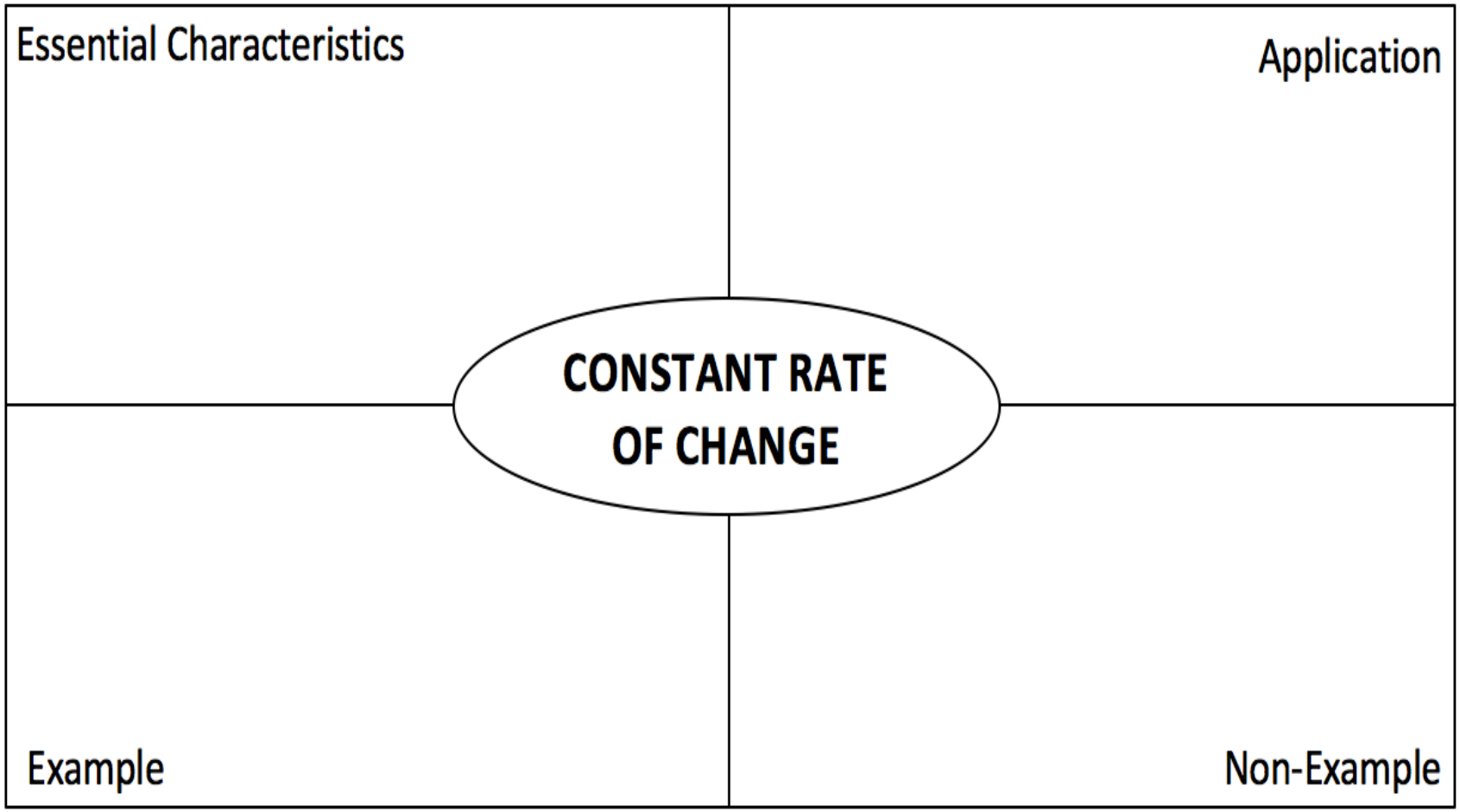
What do you know?	What information is important?	What might the question be?

Work Space

Partner Practice






Essential Characteristics
Application
CONSTANT RATE OF CHANGE
Example
Non-Example

Lesson 11: Constant Rate

Student Outcomes

- Students know the definition of constant rate in varied contexts as expressed using two variables where one is t representing a time interval.
- Students graph points on a coordinate plane related to constant rate problems.

Classwork

Example 1 (6 minutes)

Give students the first question below, and allow them time to work. Ask them to share their solutions with the class, and then proceed with the discussion, table, and graph to finish Example 1.

Example 1

Pauline mows a lawn at a constant rate. Suppose she mows a 35-square-foot lawn in 2.5 minutes. What area, in square feet, can she mow in 10 minutes? t minutes?

- What is Pauline's average rate in 2.5 minutes?
  - *Pauline's average rate in 2.5 minutes is $\frac{35}{2.5}$ square feet per minute.*
- What is Pauline's average rate in 10 minutes?
  - *Let A represent the square feet of the area mowed in 10 minutes. Pauline's average rate in 10 minutes is $\frac{A}{10}$ square feet per mintute.*
- Let C be Pauline's constant rate in square feet per minute; then, $\frac{35}{2.5} = C$, and $\frac{A}{10} = C$. Therefore,

$$\frac{35}{2.5} = \frac{A}{10}$$
$$350 = 2.5A$$
$$\frac{350}{2.5} = \frac{2.5}{2.5}A$$
$$140 = A$$

Pauline mows 140 square feet of lawn in 10 minutes.

- If we let y represent the number of square feet Pauline can mow in t minutes, then Pauline's average rate in t minutes is $\frac{y}{t}$ square feet per minute.
- Write the two-variable equation that represents the area of lawn, y, Pauline can mow in t minutes.

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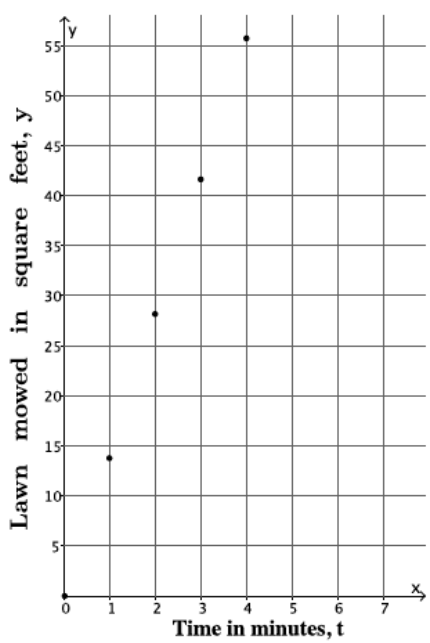
$$\frac{35}{2.5} = \frac{y}{t}$$
$$2.5y = 35t$$
$$\frac{2.5}{2.5}y = \frac{35}{2.5}t$$
$$y = \frac{35}{2.5}t$$

MP.7

- What is the meaning of $\frac{35}{2.5}$ in the equation $y = \frac{35}{2.5}t$?
  - *The number $\frac{35}{2.5}$ represents the constant rate at which Pauline can mow a lawn.*
- We can organize the information in a table.

t (time in minutes)	Linear Equation: $y = \frac{35}{2.5}t$	y (area in square feet)
0	$y = \frac{35}{2.5}(0)$	0
1	$y = \frac{35}{2.5}(1)$	$\frac{35}{2.5} = 14$
2	$y = \frac{35}{2.5}(2)$	$\frac{70}{2.5} = 28$
3	$y = \frac{35}{2.5}(3)$	$\frac{105}{2.5} = 42$
4	$y = \frac{35}{2.5}(4)$	$\frac{140}{2.5} = 56$

- On a coordinate plane, we will let the x-axis represent time t, in minutes, and the y-axis represent the area of mowed lawn in square feet. Then we have the following graph.



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- Because Pauline mows at a constant rate, we would expect the square feet of mowed lawn to continue to rise as the time, in minutes, increases.

Concept Development (6 minutes)

- In the last lesson, we learned about average speed and constant speed. Constant speed problems are just a special case of a larger variety of problems known as constant rate problems. Some of these problems were topics in Grade 7, such as water pouring out of a faucet into a tub, painting a house, and mowing a lawn.
- First, we define the average rate:

  Suppose V gallons of water flow from a faucet in a given time interval t (minutes). Then, the *average rate* of water flow in the given time interval is $\frac{V}{t}$ in gallons per minute.
- Then, we define the constant rate:

  Suppose the average rate of water flow is the same constant C for *any* given time interval. Then we say that the water is flowing at a *constant rate, C.*
- Similarly, suppose A square feet of lawn are mowed in a given time interval t (minutes). Then, the *average rate* of lawn mowing in the given time interval is $\frac{A}{t}$ square feet per minute. If we assume that the average rate of lawn mowing is the same constant, C, for *any* given time interval, then we say that the lawn is mowed at a constant rate, C.
- Describe the average rate of painting a house.
  - *Suppose A square feet of house are painted in a given time interval t (minutes). Then the average rate of house painting in the given time interval is $\frac{A}{t}$ square feet per minute.*
- Describe the constant rate of painting a house.
  - *If we assume that the average rate of house painting is the same constant, C, over any given time interval, then we say that the wall is painted at a constant rate, C.*
- What is the difference between average rate and constant rate?
  - *Average rate is the rate in which something can be done over a specific time interval. Constant rate assumes that the average rate is the same over any time interval.*
- As you can see, the way we define average rate and constant rate for a given situation is very similar. In each case, a transcription of the given information leads to an expression in two variables.

Example 2 (8 minutes)

Example 2

Water flows at a constant rate out of a faucet. Suppose the volume of water that comes out in three minutes is 10.5 gallons. How many gallons of water come out of the faucet in t minutes?

- Write the linear equation that represents the volume of water, V, that comes out in t minutes.

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Let C represent the constant rate of water flow.

$\frac{10.5}{3} = C$, *and* $\frac{V}{t} = C$; *then,* $\frac{10.5}{3} = \frac{V}{t}$.

$$\frac{10.5}{3} = \frac{V}{t}$$
$$3V = 10.5t$$
$$\frac{3}{3}V = \frac{10.5}{3}t$$
$$V = \frac{10.5}{3}t$$

MP.7

- What is the meaning of the number $\frac{10.5}{3}$ in the equation $V = \frac{10.5}{3}t$?
  - *The number $\frac{10.5}{3}$ represents the constant rate at which water flows from a faucet.*
- Using the linear equation $V = \frac{10.5}{3}t$, complete the table.

t (time in minutes)	Linear Equation: $V = \frac{10.5}{3}t$	V (in gallons)
0	$V = \frac{10.5}{3}(0)$	0
1	$V = \frac{10.5}{3}(1)$	$\frac{10.5}{3} = 3.5$
2	$V = \frac{10.5}{3}(2)$	$\frac{21}{3} = 7$
3	$V = \frac{10.5}{3}(3)$	$\frac{31.5}{3} = 10.5$
4	$V = \frac{10.5}{3}(4)$	$\frac{42}{3} = 14$

- On a coordinate plane, we will let the x-axis represent time t in minutes and the y-axis represent the volume of water. Graph the data from the table.

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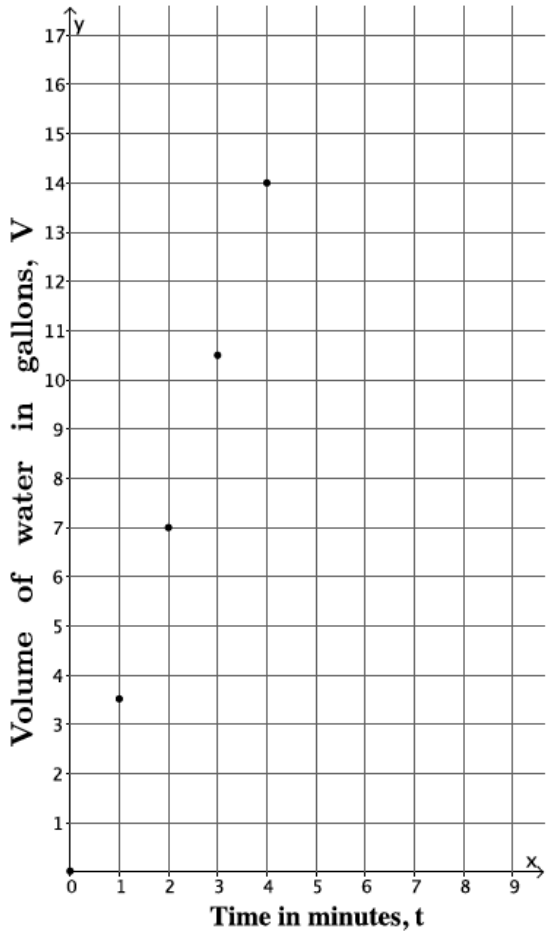

- Using the graph, about how many gallons of water do you think would flow after $1\frac{1}{2}$ minutes? Explain.
  - *After $1\frac{1}{2}$ minutes, between $3\frac{1}{2}$ and 7 gallons of water will flow. Since the water is flowing at a constant rate, we can expect the volume of water to rise between 1 and 2 minutes. The number of gallons that flow after $1\frac{1}{2}$ minutes then would have to be between the number of gallons that flow out after 1 minute and 2 minutes.*
- Using the graph, about how long would it take for 15 gallons of water to flow out of the faucet? Explain.
  - *It would take between 4 and 5 minutes for 15 gallons of water to flow out of the faucet. It takes 4 minutes for 14 gallons to flow; therefore, it must take more than 4 minutes for 15 gallons to come out. It must take less than 5 minutes because $3\frac{1}{2}$ gallons flow out every minute.*
- Graphing proportional relationships like these last two constant rate problems provides us more information than simply solving an equation and calculating one value. The graph provides information that is not so obvious in an equation.

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Exercises (15 minutes)

Students complete Exercises 1–3 independently.

Exercises

1. **Juan types at a constant rate. He can type a full page of text in $3\frac{1}{2}$ minutes. We want to know how many pages, p, Juan can type after t minutes.**

   a. **Write the linear equation in two variables that represents the number of pages Juan types in any given time interval.**

   Let C represent the constant rate that Juan types in pages per minute. Then,

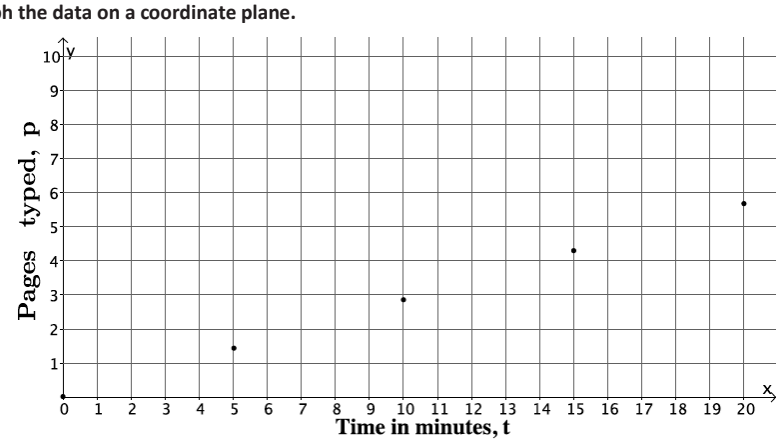
   $\frac{1}{3.5} = C$, *and* $\frac{p}{t} = C$; *therefore,* $\frac{1}{3.5} = \frac{p}{t}$.

$$\frac{1}{3.5} = \frac{p}{t}$$
$$3.5p = t$$
$$\frac{3.5}{3.5}p = \frac{1}{3.5}t$$
$$p = \frac{1}{3.5}t$$

   b. **Complete the table below. Use a calculator, and round your answers to the tenths place.**

t (time in minutes)	Linear Equation: $p = \frac{1}{3.5}t$	p (pages typed)
0	$p = \frac{1}{3.5}(0)$	0
5	$p = \frac{1}{3.5}(5)$	$\frac{5}{3.5} \approx 1.4$
10	$p = \frac{1}{3.5}(10)$	$\frac{10}{3.5} \approx 2.9$
15	$p = \frac{1}{3.5}(15)$	$\frac{15}{3.5} \approx 4.3$
20	$p = \frac{1}{3.5}(20)$	$\frac{20}{3.5} \approx 5.7$

   c. **Graph the data on a coordinate plane.**



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d. About how long would it take Juan to type a 5-page paper? Explain.

It would take him between 15 and 20 minutes. After 15 minutes, he will have typed 4.3 pages. In 20 minutes, he can type 5.7 pages. Since 5 pages is between 4.3 and 5.7, then it will take him between 15 and 20 minutes.

2. Emily paints at a constant rate. She can paint 32 square feet in 5 minutes. What area, A, in square feet, can she paint in t minutes?

a. Write the linear equation in two variables that represents the number of square feet Emily can paint in any given time interval.

Let C be the constant rate that Emily paints in square feet per minute. Then,

$\frac{32}{5} = C$, *and* $\frac{A}{t} = C$; *therefore,* $\frac{32}{5} = \frac{A}{t}$.

$$\frac{32}{5} = \frac{A}{t}$$
$$5A = 32t$$
$$\frac{5}{5}A = \frac{32}{5}t$$
$$A = \frac{32}{5}t$$

b. Complete the table below. Use a calculator, and round answers to the tenths place.

t (time in minutes)	Linear Equation: $A = \frac{32}{5}t$	A (area painted in square feet)
0	$A = \frac{32}{5}(0)$	0
1	$A = \frac{32}{5}(1)$	$\frac{32}{5} = 6.4$
2	$A = \frac{32}{5}(2)$	$\frac{64}{5} = 12.8$
3	$A = \frac{32}{5}(3)$	$\frac{96}{5} = 19.2$
4	$A = \frac{32}{5}(4)$	$\frac{128}{5} = 25.6$

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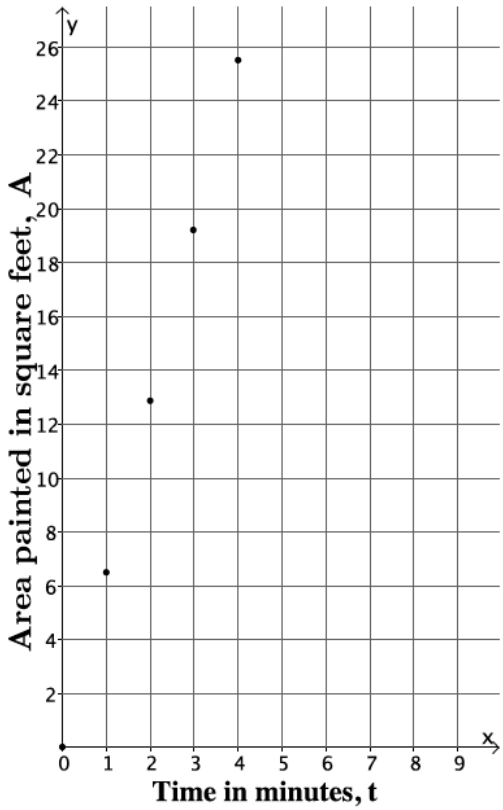
c. Graph the data on a coordinate plane.



d. About how many square feet can Emily paint in $2\frac{1}{2}$ minutes? Explain.

Emily can paint between 12.8 and 19.2 square feet in $2\frac{1}{2}$ minutes. After 2 minutes, she paints 12.8 square feet, and after 3 minutes, she will have painted 19.2 square feet.

3. Joseph walks at a constant speed. He walked to a store that is one-half mile away in 6 minutes. How many miles, m, can he walk in t minutes?

a. Write the linear equation in two variables that represents the number of miles Joseph can walk in any given time interval, t.

Let C be the constant rate that Joseph walks in miles per minute. Then,

$\frac{0.5}{6} = C$, *and* $\frac{m}{t} = C$; *therefore,* $\frac{0.5}{6} = \frac{m}{t}$.

$$\frac{0.5}{6} = \frac{m}{t}$$
$$6m = 0.5t$$
$$\frac{6}{6}m = \frac{0.5}{6}t$$
$$m = \frac{0.5}{6}t$$

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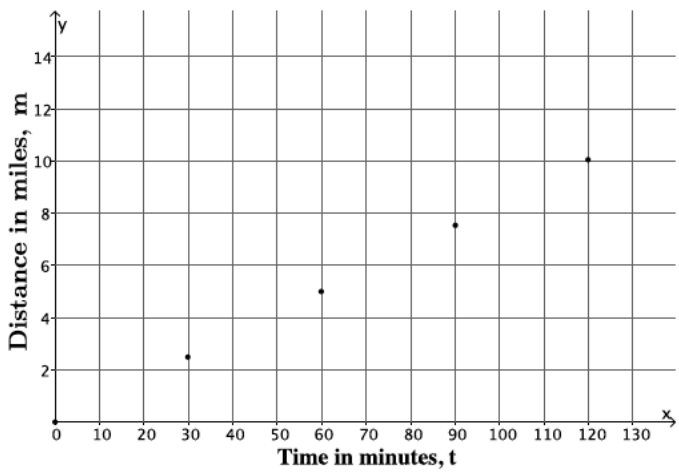
b. **Complete the table below. Use a calculator, and round answers to the tenths place.**

t (time in minutes)	Linear Equation: $m = \frac{0.5}{6}t$	m (distance in miles)
0	$m = \frac{0.5}{6}(0)$	0
30	$m = \frac{0.5}{6}(30)$	$\frac{15}{6} = 2.5$
60	$m = \frac{0.5}{6}(60)$	$\frac{30}{6} = 5$
90	$m = \frac{0.5}{6}(90)$	$\frac{45}{6} = 7.5$
120	$m = \frac{0.5}{6}(120)$	$\frac{60}{6} = 10$

c. **Graph the data on a coordinate plane.**



d. **Joseph's friend lives 4 miles away from him. About how long would it take Joseph to walk to his friend's house? Explain.**

It will take Joseph a little less than an hour to walk to his friend's house. Since it takes 30 minutes for him to walk 2.5 miles and 60 minutes to walk 5 miles, and 4 is closer to 5 than 2.5, it will take Joseph less than an hour to walk the 4 miles.

Closing (5 minutes)

Summarize, or ask students to summarize, the main points from the lesson:

- Constant rate problems appear in a variety of contexts like painting a house, typing, walking, or water flow.
- We can express the constant rate as a two-variable equation representing proportional change.
- We can graph the constant rate situation by completing a table to compute data points.

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Lesson Summary

When constant rate is stated for a given problem, then you can express the situation as a two-variable equation. The equation can be used to complete a table of values that can then be graphed on a coordinate plane.

Exit Ticket (5 minutes)

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Name _____ Date _____

Lesson 11: Constant Rate

Exit Ticket

Vicky reads at a constant rate. She can read 5 pages in 9 minutes. We want to know how many pages, p, Vicky can read after t minutes.

a. Write a linear equation in two variables that represents the number of pages Vicky reads in any given time interval.

b. Complete the table below. Use a calculator, and round answers to the tenths place.

t (time in minutes)	Linear Equation:	p (pages read)
0		
20		
40		
60		

c. About how long would it take Vicky to read 25 pages? Explain.

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Exit Ticket Sample Solutions

Vicky reads at a constant rate. She can read 5 pages in 9 minutes. We want to know how many pages, p, Vicky can read after t minutes.

a. Write a linear equation in two variables that represents the number of pages Vicky reads in any given time interval.

Let C represent the constant rate that Vicky reads in pages per minute. Then,

$\frac{5}{9} = C$, *and* $\frac{p}{t} = C$; *therefore,* $\frac{5}{9} = \frac{p}{t}$.

$$\frac{5}{9} = \frac{p}{t}$$
$$9p = 5t$$
$$\frac{9}{9}p = \frac{5}{9}t$$
$$p = \frac{5}{9}t$$

b. Complete the table below. Use a calculator, and round answers to the tenths place.

t (time in minutes)	Linear Equation: $p = \frac{5}{9}t$	p (pages read)
0	$p = \frac{5}{9}(0)$	0
20	$p = \frac{5}{9}(20)$	$\frac{100}{9} \approx 11.1$
40	$p = \frac{5}{9}(40)$	$\frac{200}{9} \approx 22.2$
60	$p = \frac{5}{9}(60)$	$\frac{300}{9} \approx 33.3$

c. About how long would it take Vicky to read 25 pages? Explain.

It would take her a little over 40 minutes. After 40 minutes, she can read about 22.2 pages, and after 1 hour, she can read about 33.3 pages. Since 25 pages is between 22.2 and 33.3, it will take her between 40 and 60 minutes to read 25 pages.

Problem Set Sample Solutions

Students practice writing two-variable equations that represent a constant rate.

1. A train travels at a constant rate of 45 miles per hour.

a. What is the distance, d, in miles, that the train travels in t hours?

Let C be the constant rate the train travels. Then, $\frac{45}{1} = C$, *and* $\frac{d}{t} = C$; *therefore,* $\frac{45}{1} = \frac{d}{t}$.

$$\frac{45}{1} = \frac{d}{t}$$
$$d = 45t$$

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b. How many miles will it travel in 2.5 hours?

$$\begin{aligned} d &= 45(2.5) \\ &= 112.5 \end{aligned}$$

The train will travel 112.5 miles in 2.5 hours.

2. Water is leaking from a faucet at a constant rate of $\frac{1}{3}$ gallons per minute.

a. What is the amount of water, w, in gallons per minute, that is leaked from the faucet after t minutes?

Let C be the constant rate the water leaks from the faucet in gallons per minute. Then,

$\frac{\frac{1}{3}}{1} = C$, *and* $\frac{w}{t} = C$; *therefore,* $\frac{\frac{1}{3}}{1} = \frac{w}{t}$.

$$\begin{aligned} \frac{\frac{1}{3}}{1} &= \frac{w}{t} \\ w &= \frac{1}{3}t \end{aligned}$$

b. How much water is leaked after an hour?

$$\begin{aligned} w &= \frac{1}{3}t \\ &= \frac{1}{3}(60) \\ &= 20 \end{aligned}$$

The faucet will leak 20 gallons in one hour.

3. A car can be assembled on an assembly line in 6 hours. Assume that the cars are assembled at a constant rate.

a. How many cars, y, can be assembled in t hours?

Let C be the constant rate the cars are assembled in cars per hour. Then,

$\frac{1}{6} = C$, *and* $\frac{y}{t} = C$; *therefore,* $\frac{1}{6} = \frac{y}{t}$.

$$\begin{aligned} \frac{1}{6} &= \frac{y}{t} \\ 6y &= t \\ \frac{6}{6}y &= \frac{1}{6}t \\ y &= \frac{1}{6}t \end{aligned}$$

b. How many cars can be assembled in a week?

A week is $24 \times 7 = 168$ hours. So, $y = \frac{1}{6}(168) = 28$. Twenty-eight cars can be assembled in a week.

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4. A copy machine makes copies at a constant rate. The machine can make 80 copies in $2\frac{1}{2}$ minutes.

a. Write an equation to represent the number of copies, n, that can be made over any time interval in minutes, t.

Let C be the constant rate that copies can be made in copies per minute. Then,

$\frac{80}{2\frac{1}{2}} = C$, *and* $\frac{n}{t} = C$; *therefore,* $\frac{80}{2\frac{1}{2}} = \frac{n}{t}$.

$$\frac{80}{2\frac{1}{2}} = \frac{n}{t}$$
$$2\frac{1}{2}n = 80t$$
$$\frac{5}{2}n = 80t$$
$$\frac{2}{5}\cdot\frac{5}{2}n = \frac{2}{5}\cdot 80t$$
$$n = 32t$$

b. Complete the table below.

t (time in minutes)	Linear Equation: $n = 32t$	n (number of copies)
0	$n = 32(0)$	0
0.25	$n = 32(0.25)$	8
0.5	$n = 32(0.5)$	16
0.75	$n = 32(0.75)$	24
1	$n = 32(1)$	32

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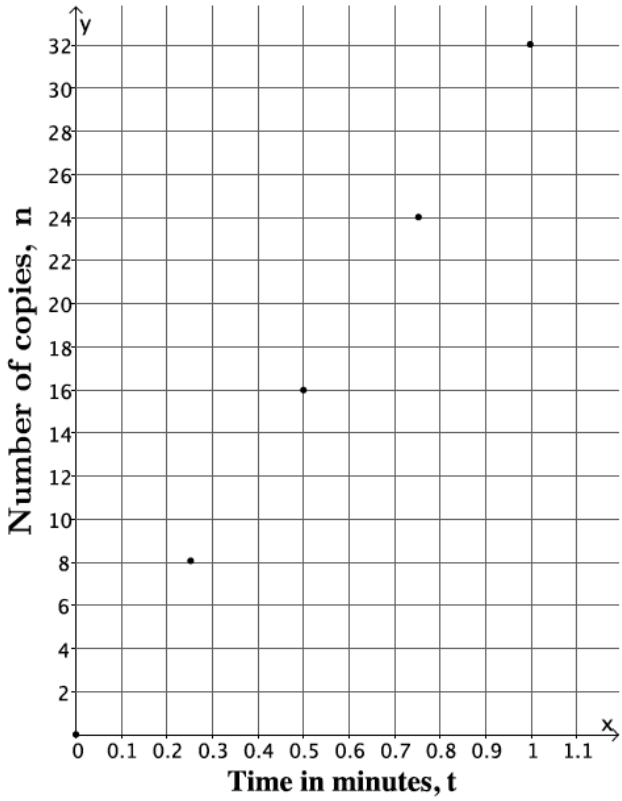
c. Graph the data on a coordinate plane.



d. The copy machine runs for 20 seconds and then jams. About how many copies were made before the jam occurred? Explain.

Since 20 seconds is approximately 0.3 of a minute, then the number of copies made will be between 8 and 16 because 0.3 is between 0.25 and 0.5.

5. Connor runs at a constant rate. It takes him 34 minutes to run 4 miles.

a. Write the linear equation in two variables that represents the number of miles Connor can run in any given time interval in minutes, t.

Let C be the constant rate that Connor runs in miles per minute, and let m represent the number of miles he ran in t minutes. Then,

$\frac{4}{34} = C$, *and* $\frac{m}{t} = C$; *therefore,* $\frac{4}{34} = \frac{m}{t}$.

$$\frac{4}{34} = \frac{m}{t}$$
$$34m = 4t$$
$$\frac{34}{34}m = \frac{4}{34}t$$
$$m = \frac{4}{34}t$$
$$m = \frac{2}{17}t$$

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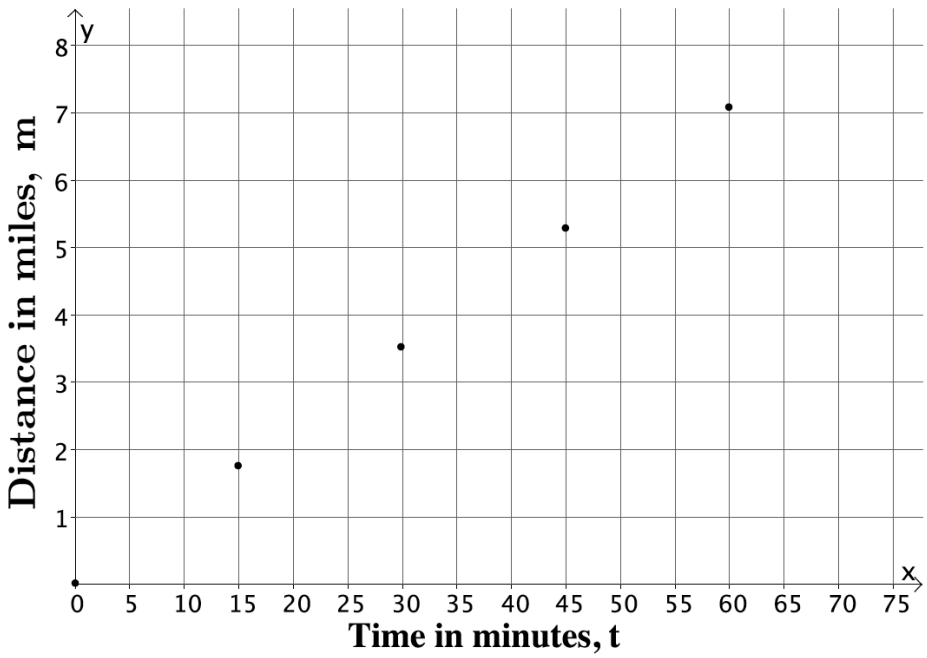
b. Complete the table below. Use a calculator, and round answers to the tenths place.

t (time in minutes)	Linear Equation: $m = \frac{2}{17}t$	m (distance in miles)
0	$m = \frac{2}{17}(0)$	0
15	$m = \frac{2}{17}(15)$	$\frac{30}{17} \approx 1.8$
30	$m = \frac{2}{17}(30)$	$\frac{60}{17} \approx 3.5$
45	$m = \frac{2}{17}(45)$	$\frac{90}{17} \approx 5.3$
60	$m = \frac{2}{17}(60)$	$\frac{120}{17} \approx 7.1$

c. Graph the data on a coordinate plane.



d. Connor ran for 40 minutes before tripping and spraining his ankle. About how many miles did he run before he had to stop? Explain.

Since Connor ran for 40 minutes, he ran more than 3.5 miles but less than 5.3 miles. Since 40 is between 30 and 45, then we can use those reference points to make an estimate of how many miles he ran in 40 minutes, probably about 5 miles.

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Diamond Reflection
Unpacking the EngageNY Lesson

Observations: How are the mathematics content standards evident in the lesson?

Reflections: How does student engagement in the SMPs help develop conceptual understanding, procedural skill and fluency, and application skills?

Other thoughts and considerations

Interpretations: What instructional strategies and facilitator moves brought out the content and practice standards?

Decisional: What am I going to do based on my learning?

What's Your Mindset?

Do NOT overthink these statements. Read each and put "agree" or "disagree" in the box to the left. This is a personal reflection, and results will not be shared. Please answer truthfully.

	My intelligence is something very basic about me that I can't really change.
	When I don't understand something, I like to slow down and try to figure it out.
	I am intimidated by academic challenges.
	I have been told by others that I am smart.
	Learning is fun.
	I often feel unmotivated to learn.
	When I don't do well in a subject, I think that I must not be very good at that particular subject.
	When I perform poorly academically, I do not get discouraged.
	When I don't understand something, I get very frustrated and want to give up.
	I shouldn't have to work as hard in subjects that I am naturally good at.

Quiz adapted from:
Baker-Kunst, S. (2010 January 13; originally posted 2008 March 6). The curse of the 'smart' student. *Extreme biology: Class connection*. Retrieved 2017 October 21 from https://extremebiology.wordpress.com/2010/01/13/the-curse-of-the-smart-student

For the next activity, please read the following article:

Dweck, C. S. (2007 October). The perils and promises of praise. *Educational leadership: Early intervention at every age [issue theme], 65(2)*, 34–39. Retrieved 2017 October 21 from http://www.ascd.org/publications/educational-leadership/oct07/vol65/num02/The-Perils-and-Promises-of-Praise.aspx

Vertical Alignment Chart

Big Idea: Using Multiplicative Thinking to Reason About Ratios and Rates

Grade/ Course	LSSM	Content Standards
Alg I	A1: F-LE.A.1	Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
	A1: S-ID.C.7	Interpret the slope (rate of change) ~~and the intercept (constant term)~~ of a linear model in the context of the data.
8	8.EE.B.5	Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*
	8.EE.B.6	Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.
7	7.RP.A.2	Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.* d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.
	7.G.A.1	Solve problems involving scale drawings of geometric figures, such as computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
6	6.RP.A.3	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

Vertical Alignment Chart

Grade/ Course	LSSM	Content Standards
		b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what unit rate were lawns being mowed?* c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.
	6.EE.C.9	Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.*
5	5.NF.B.5	Interpret multiplication as scaling (resizing), by: d. Relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.
	5.MD.A.1	Convert among different-sized standard measurement units within a given measurement system, and use these conversions in solving multi-step, real-world problems (e.g., convert 5 cm to 0.05 m; 9 ft to 108 in).
4	4.OA.A.2	Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and/or equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison (Example: 6 times as many vs. 6 more than).[1]
	4.NF.A.1	Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. (Denominators are limited to 2, 3, 4, 5, 6, 8, 10, 12, and 100.)
	4.MD.A.1	Know relative sizes of measurement units within one system of units including ft, in; km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. (Conversions are limited to one-step conversions.) *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), …*

Vertical Alignment Chart With Changes

Big Idea: Using Multiplicative Thinking to Reason About Ratios and Rates

Grade/ Course	LSS	Content Standards	Changes
Alg I	A1: F-LE.A.1 A1: S-ID.C.7	Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. Interpret the slope (rate of change) ~~and the intercept (constant term)~~ of a linear model in the context of the data.	***Changes from 8 to Algebra I:*** Students compare linear functions and exponential functions. Builds on interpreting slope to any linear model and in context with data.
8	8.EE.B.5 8.EE.B.6	Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.* Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.	***Changes from 7 to 8:*** Moves beyond graphing proportional relationships to interpreting unit rate as slope and comparing different proportional relationships using different representations. Similar triangles introduced and used to understand slope.
7	7.RP.A.2 7.G.A.1	Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.* d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate. Solve problems involving scale drawings of geometric figures, such as computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.	***Changes from 6 to 7:*** Students name proportional relationships by identifying equivalent ratios. Introduced constant of proportionality and connects to unit rates. Analyze the graphs of proportional relationships. Scale drawings introduced.
6	6.RP.A.3	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took*	***Changes from 5 to 6:*** Moves from equivalent fractions to equivalent ratios, creating tables of equivalent ratios and plotting pairs of values

Grade/ Course	LSS	Content Standards	Changes
	6.EE.C.9	*7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what unit rate were lawns being mowed?* c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities. Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.*	Unit rate introduced. Percents introduced. Conversions within measurement systems move to using ratio reasoning. Introduce variables and dependence relationships. Relationships are analyzed using graphs, tables, and equations.
5	5.NF.B.5 5.MD.A.1	Interpret multiplication as scaling (resizing), by: d. Relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1. Convert among different-sized standard measurement units within a given measurement system, and use these conversions in solving multi-step, real-world problems (e.g., convert 5 cm to 0.05 m; 9 ft to 108 in).	***Changes from 4 to 5:*** Moves from students understanding equivalent fractions to the effect of multiplying by 1. Extended work with conversion from one-step to multi-step and to problem solving. Stays within a measurement system. Equivalent measurements are recorded in a table, which is an informal ratio table.
4	4.OA.A.2 4.NF.A.1 4.MD.A.1	Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and/or equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison (Example: 6 times as many vs. 6 more than).[1] Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. (Denominators are limited to 2, 3, 4, 5, 6, 8, 10, 12, and 100.) Know relative sizes of measurement units within one system of units including ft, in; km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. (Conversions are limited to one-step conversions.) *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...*	

Teacher Preparation Competencies

In your learning teams,

- Review the math content knowledge and content-pedagogy *Teacher Preparation Competencies*.
- Circle the key content and skills included in each one.
- Explain how the content studied in Module 5 relates to the competencies. will assist you in supporting your resident teacher to refine his or her content knowledge and content-specific pedagogy and to meet the competencies.

Partner Task

In turn, one assumes the role of the mentor and the other the role of the resident. The mentor explains to the resident how the math content relates to achievement of the competencies. In other words, why does the resident need to know the content you have learned to meet these competencies? Use specific examples.

The partner in the resident role then summarizes what the mentor says and offers feedback on the clarity, specificity, etc. of the explanation.

Switch roles and repeat.

Mathematics Teacher Competencies

INTRODUCTION

The mathematics teacher preparation competencies identify essential knowledge and skills for teacher candidates seeking certification in birth to kindergarten, pre-kindergarten – third grade, elementary grades 1-5, elementary grades 1-5 integrated to merged, mathematics grades 4-8, mathematics grades 4-8 integrated to merged, mathematics 6-12, and mathematics 6-12 integrated to merged.

CONTENT KNOWLEDGE COMPETENCIES

A. With respect to the academic standards for mathematics for the certification grade band and neighboring grade bands, the teacher candidate, at minimum, demonstrates the following competencies to plan for instruction, teach, and assess student learning:

1. Provides exact, explicit definitions of mathematical ideas and concepts using appropriate mathematical language.
2. Provides precise, accurate, useful descriptions of algorithms and procedures, including descriptions of the accuracy of alternative procedures or algorithms.
3. Exhibits an integrated, functional grasp of mathematical concepts and procedures.
4. Explains concepts and executes procedures flexibly, accurately, efficiently, and appropriately.
5. Models the mathematical dispositions and habits of mind described in the Practice Standards, including precision of language, logical thought, reflection, explanation, and justification.
6. Recognizes and uses the connections between the topics identified in the relevant standards and problems arising in real-world applications.
7. Portrays mathematics as sensible, useful, and worthwhile.

CONTENT PEDAGOGY COMPETENCIES

A. The teacher candidate applies knowledge of mathematical topics and their relationships within and across mathematical domains to identify key mathematical ideas and select or design mathematically sound lesson sequences and units of study that develop students' conceptual understanding, procedural skill and fluency, and ability to solve real-world and mathematical problems.

1. The teacher candidate appropriately sequences content for instruction within a lesson or unit of study and plans appropriate scaffolding to provide opportunities for students to access and master grade-level standards.
2. The teacher candidate anticipates student misconceptions which may arise during a lesson or unit of study, identifies key points in the lesson or unit to check for misconceptions, and identifies appropriate instructional strategies to respond to misconceptions, including but not limited to questioning, whole group discussion, problem sets, instructional tools and representations that make the mathematics of the lesson explicit.
3. The teacher candidate selects or designs standards-based tasks that use varied strategies, including but not limited to real-life applications, manipulatives, models, diagrams/pictures, that present opportunities for instruction and assessment.
4. The teacher candidate selects or designs practice sets that include scaffolding and differentiation of mathematical content to provide opportunities for students to develop and demonstrate mastery.
5. The teacher candidate identifies appropriate student groupings, for example pairs or small groups, to develop students' conceptual understanding, skill, and fluency with mathematical content as well as independent mathematical thinking.

B. The teacher candidate applies understanding of students' mathematical language development to provide regular opportunities during instruction for students to explain their understanding both in writing and orally through classroom conversations.

1. The teacher candidate explains the connection between students' informal language to precise mathematical language to develop students' ability to use precise mathematical language in their explanations and discussions.

C. The teacher candidate applies understanding of the intersection of mathematical content and mathematical practices to provide regular, repeated opportunities for students to exhibit the math practices while engaging with the mathematical content of the lesson, including but not limited to the following:

1. Uses appropriate prompting and questioning that allow students to refine their mathematical thinking and build upon one another's understanding of the mathematical content of the lesson.
2. Poses challenging problems that offer opportunities for productive struggle, encouraging reasoning, problem solving, and perseverance in solving problems in the face of initial difficulty;
3. Facilitates student conversations in which students are encouraged to discuss each other's thinking in order to clarify or improve their own mathematical understanding;
4. Provides opportunities for students to choose and use appropriate tools when solving a problem; and
5. Prompts students to explain and justify work and provides feedback that guides students to produce revised explanations and justifications.

D. The teacher candidate applies knowledge of mathematical topics and their relationships within and across mathematical domains to select or design and use a range of ongoing classroom assessments, including but not limited to diagnostic, formal and informal, formative and summative, oral and written, which determine students' mastery of grade-level standards in order to inform and adjust planning and instruction.

1. The teacher candidate identifies errors, gaps, and inconsistencies in students' knowledge, skills, and mathematical reasoning to remediate or scaffold students' learning during lesson implementation, using, but not limited to, the following strategies:
   a. oral and written explanations of the elements and structures of mathematics and the meaning of procedures, analogies, and real life experiences;
   b. manipulatives, models, and pictures or diagrams; and
   c. problem sets.
2. The teacher candidate uses trends in assessment results to plan lesson structure and sequence, instructional strategies, remediation and enrichment opportunities for students.

Module 5 Review

In your team, review the outcomes (p. 4) for Module 5. Share how you anticipate applying what you learned as a mentor.

Exit slip

Use an index card on your table. Write a brief note.

3 ideas I am taking away
2 actions I'll take based on what I learned
1 question I have about what I learned