Learning Slope via Rate not Rote

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Why change?

- According to a survey by Valentina Postelnicu and Carole Greenes (2011–2012), the most difficult problems for students are those that require students to identify the slope of a line from its graph.
- "Students described difficulties of understanding slope in the context of distance-rate-time problems, of how to measure rise and run, of interpreting a point on a graph if it did not lie on the intersection of grid lines, and of computing slope involving decimal fractions" (Postelnicu and Greenes 2011–2012).

Postelnicu, Valentina, and Carole Greenes. 2011–2012. "Do Teachers Know What Their Students Know?" *NCSM Newsletter* (Winter).

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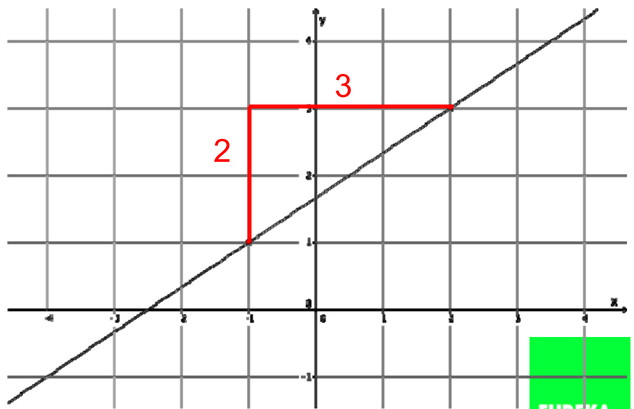
But it’s so EASY!

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$



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There has to be a better way.

Key components:

- Ratios (an association of two or more quantities)
- Equivalent Ratios
- Rate (unit rate in particular)
- Proportional Relationships
- Similar Triangles

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CCSSM Grade 6

- **6.RP.A.1**

Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.
- For a ratio *a:b*, know the associated ratios *b:a*, *a:(a + b), and* b*:(b + a).*

- **6.RP.A.2**

Understand the concept of a unit rate *a/b* associated with a ratio *a:b* with *b* ≠ 0.
- A unit rate is the numerical value of the rate.
- The unit rate is also the *value of the ratio*.
- For example: $10 for every 4 packs is the ratio 10: 4, which has the value 10/4 = 2.5.

- **6.RP.A.3A**

Use ratio and rate reasoning to solve real-world problems.
- Make tables of equivalent ratios, and plot the pairs of values on the coordinate plane.

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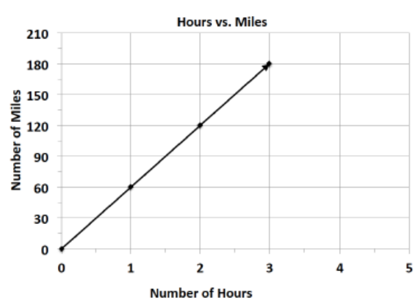
Grade 6 Sample Problem

A car is traveling at a constant rate. After 2 hours, the car has driven 120 miles.
- Complete the table.
- Graph the proportional relationship.
- Identify the unit rate.

h	0	1	2	3
m			120	



The rate is 60 miles per hour; therefore, the unit rate is 60.

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CCSSM Grade 7

• 7.RP.A.2A

Recognize and represent proportional relationships between two quantities.
- Test for equivalent ratios in a table or by graphing on a coordinate plane; is the graph a straight line through the origin?

• 7.RP.A.2B

Recognize and represent proportional relationships between two quantities.
- Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions.

• 7.RP.A.2C

Represent proportional relationships by equations.

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Grade 7 Sample Problem

A car is traveling at a constant rate. After 2 hours, the car has driven 120 miles.

- Identify the constant of proportionality.
- Write an equation that allows you to determine how many miles the car has driven after any number of hours.

The constant of proportionality expresses the multiplicative relationship between each x-value and its corresponding y-value.

$+1$ $+1$ $+1$

$\boldsymbol{h}$	**0**	**1**	**2**	**3**
$\boldsymbol{m}$	**0**	**60**	**120**	**180**

$+60$ $+60$ $+60$

If a proportional relationship is described by the set of ordered pairs that satisfies the equation $y = kx$, where k is a positive constant, then k is called the constant of proportionality.

Using m for miles and h for hours, $m = 60h$.

The constant of proportionality is the unit rate $\frac{m}{h}$ which is $\frac{60}{1}$ or 60.

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CCSSM Grade 8

- **8.EE.B.5**

Graph proportional relationships, interpreting the unit rate as the slope of the graph.

- **8.EE.B.6**

Use similar triangles to explain why the slope *m* is the same between any two distinct points on a non-vertical line; derive the equations $y = mx$ and $y = mx + b$.

- **8.G.A.2 & 8.G.A.4**

- Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rigid motions.
- Understand that a two-dimensional figure is similar to another if the second can be obtained by the first by a sequence of rigid motions and dilations.

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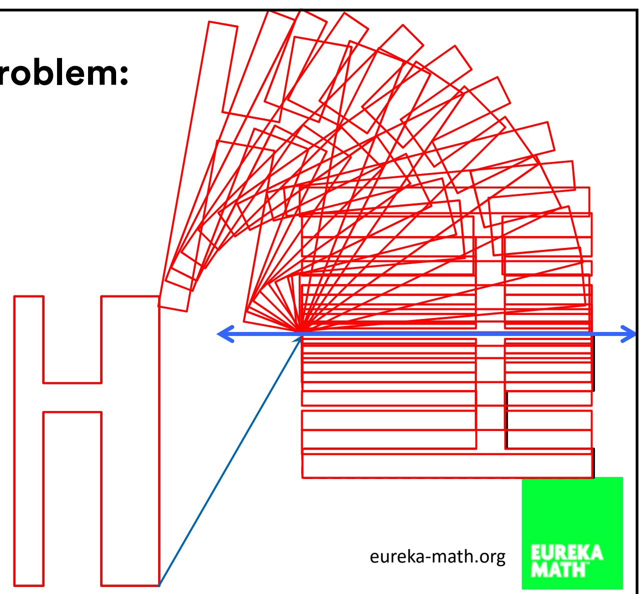
Grade 8 Sample Problem: Congruence

- Show that two identical figures are congruent. Describe the sequence of basic rigid motions that maps one figure onto another.



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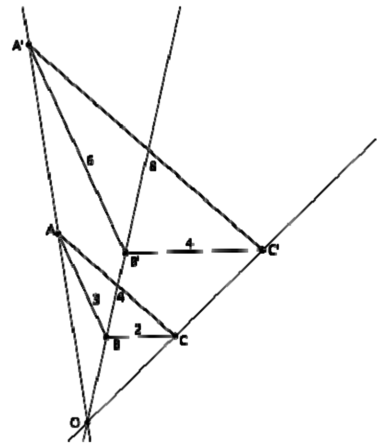
Grade 8 Sample Problem: Similarity

Are the two triangles shown similar? If so, describe the similarity transformation that takes one triangle to the other.

$$\frac{A'B'}{AB} = \frac{B'C'}{BC} = \frac{A'C'}{AC} = r$$

$$\frac{6}{3} = \frac{8}{4} = \frac{4}{2} = 2$$



$\triangle ABC \sim \triangle A'B'C'$

A dilation from center O by scale factor 2 maps $\triangle ABC$ onto $\triangle A'B'C'$.

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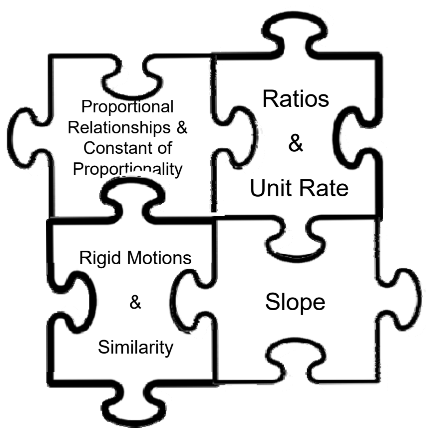

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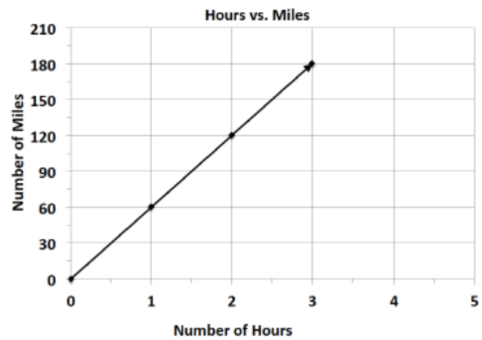
Slope as a Unit Rate

Recall the Grade 6 Sample Problem. Notice that the unit rate is 60.

Using what we know about slope, we see that 60 is also the slope of the line.

How do we make this connection to an arbitrary line in the plane?



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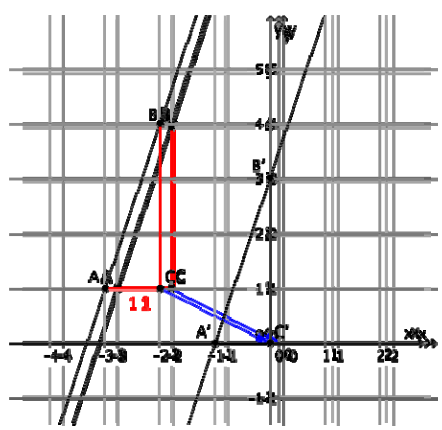
Slope as a Unit Rate

When the horizontal distance between two points on a line is equal to one, then the slope of the line can be found using translation.

Do you think this strategy works for a line whose slope is negative?



$m_{AB} = 3$

Symbolically, the slope of the line is $\frac{|BC|}{|AC|}$ when $|AC| = 1$.

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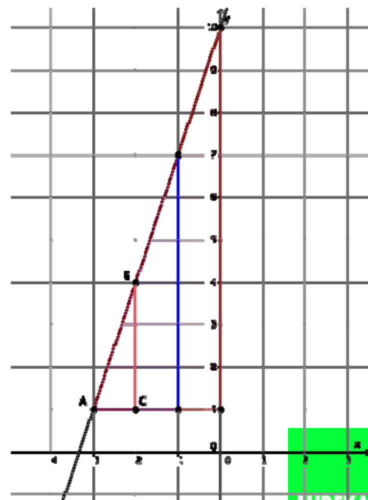
The Role of Equivalent Ratios

When we compare the value of equivalent ratios, we see that they are equal to the same constant; the constant of proportionality.

$$\frac{3}{1}=\frac{6}{2}=\frac{9}{3}=3$$

We can now extend our understanding of slope;

$m=\frac{|BC|}{|AC|}$ when $|AC|$ is a nonzero integer.



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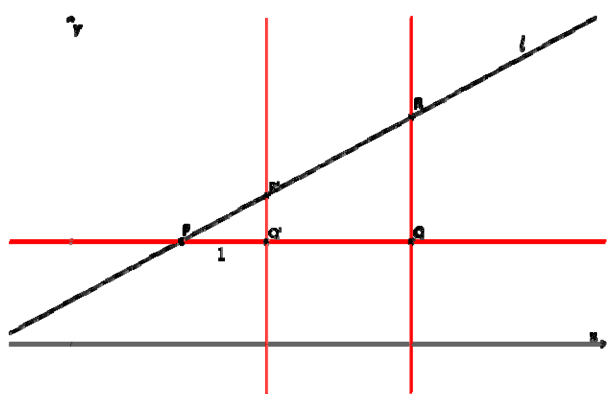
Deriving the Slope Formula Using Similar Triangles

Now we begin to generalize. In this case, we are looking at arbitrary points in the plane that lie on line l.

We want to say that

$m=\frac{|RQ|}{|PQ|}$ for any $|PQ|$.



$\triangle PQR \sim \triangle PQ'R'$; therefore, $\frac{|R'Q'|}{|RQ|}=\frac{|PQ'|}{|PQ|}$.

By construction, $|PQ'|=1$; therefore, $|R'Q'|=m$.

By substitution, $\frac{m}{|RQ|}=\frac{1}{|PQ|} \Rightarrow m=\frac{|RQ|}{|PQ|}$.

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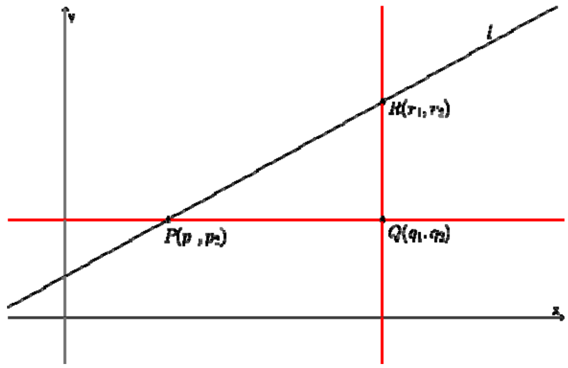
Deriving the Slope Formula Using Similar Triangles

Now we generalize even further, using the actual coordinates of the points.

If we can show for points $R(r_1, r_2)$ and $P(p_1, p_2)$ that $m = \frac{r_2 - p_2}{r_1 - p_1}$, we can substitute the points (x_1, y_1) and (x_2, y_2) to get the usual slope formula.



What is the length of RQ? $|RQ| =$

What is the length of PQ? $|PQ| =$

For line l, $m = \frac{|RQ|}{|PQ|}$, by substitution $m = \frac{r_2 - q_2}{q_1 - p_1}$.

What is true about r_1 and q_1? What is true about p_2 and q_2?

By substitution again, $m = \frac{r_2 - p_2}{r_1 - p_1}$.

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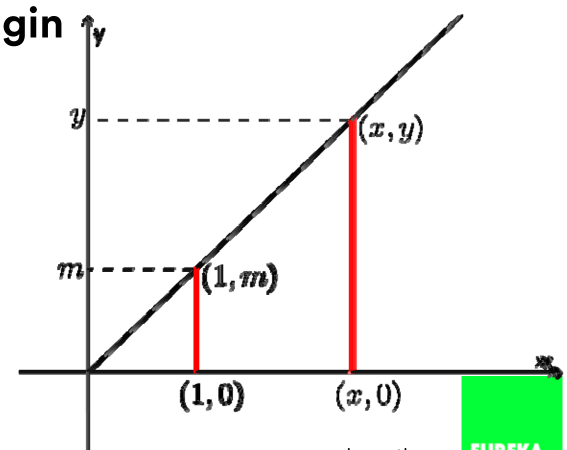
Deriving the Equation of a Line Through the Origin

By similar triangles,

$$\frac{y-0}{m-0} = \frac{x}{1}.$$

$$\frac{y}{m} = \frac{x}{1} \implies y = mx$$



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Deriving the Equation of a Line

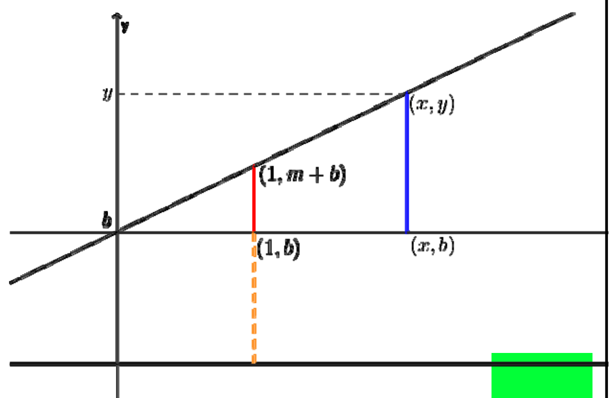
By similar triangles,

$$\frac{y-b}{m+b-b} = \frac{x}{1}.$$

$$\frac{y-b}{m} = \frac{x}{1}$$

$$\implies \quad y - b = mx$$

$$\implies \quad y = mx + b$$



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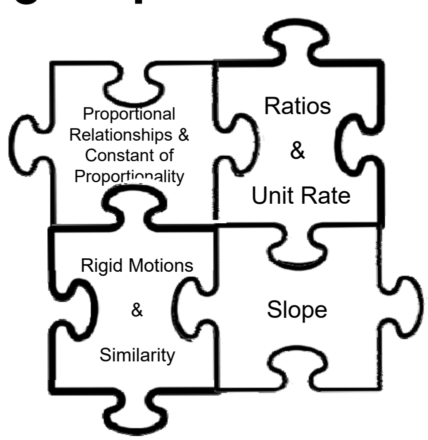

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Questions/Comments

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- Hung Hsi-Wu. This approach to slope was taken from his 2013 MPDI notes.
- Larry Francis. For his fantastic animation skills: https://www.youtube.com/watch?v=O2XPy3ZLU7Y
- Valentina Postelnicu. For sharing her enlightening article.

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