# Do Teachers Know What Their Students Know?

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**Do teachers know what their students know? That was one of the questions that motivated our study of student and teacher understanding of aspects of lines, their characteristics and representations (grades 8 – 10).** We chose this as our content focus because of its centrality to the study of algebra, and because success in Algebra I is considered to open the gate in the pipeline to the study of more advanced mathematics, science, and technology. Since the early 1990s and the beginning of the *Algebra for All* movement, student difficulties with algebra and algebraic thinking have been studied and well documented. Despite numerous remediation attempts, failure in Algebra I continues to escalate, with some school districts reporting failure rates of 54 to 84 percent, with higher percentages associated with socio-economically disadvantaged students.

In efforts to gain insight into the nature of difficulties with algebra, some researchers study students' understanding of its fundamental concepts. Others study teachers' understanding of the source and nature of those difficulties. Based on our prior research with high school students and informal interviews with students (Greenes, Chang & BenChaim 2007), we suspected that teachers may not be fully aware of the types of mathematical difficulties their students have with aspects of linear functions, and that their predictions of difficult problems for students may not match students' performance or identifications of types of difficulty.

We used a variation of the Mini Algebra Diagnostic Test (MDT) (Postelnicu, 2011), a nine-item assessment of aspects of linearity and modified the form for students and teachers to include a request to rank order the problems from most to least difficulty (after completing the items) and to provide a rationale for the problem ranked "most difficult." Students ranked problems based on what was most challenging for them. Teachers ranked based on what would be most challenging for their students.

The MDT and ranking form were administered to 1561 grades 8 – 10 students and their 27 mathematics teachers. Subsequent to the assessment, student and teacher tests were scored and their problem rankings tabulated. Comments about most difficult problems were recorded. Students' MDT scores were compared with their problem rankings and with their teachers' problem rankings. One-to-one interviews were conducted with 20 teachers and 40 students, two students from each interviewed teacher; one with an above-average score on the MDT and the other with a below-average score.

**Does teacher ranking of problems by difficulty correspond to student performance on the MDT? The short answer is "No!"**

The most difficult problems for students were those requiring identification of the slope of a line from its graph. That difficulty persisted across grade levels and types of mathematics course (e.g., Algebra I, Advanced Algebra, Geometry). Teacher rankings were almost the inverse of student performance --teachers were not aware of their students' difficulties, particularly with the concept of slope. By contrast, student rankings more close matched their performance.

From analyses of student and teacher comments on the MDT and their interviews, we gleaned the following about what makes a problem challenging. For teachers, challenge is determined by: 1) "newness" of the problem type or its manner of presentation ("Students haven't seen the material in this way," "My students can solve it but the wording is not clear"); 2) length of the text presentation or the inclusion of numerous sub-problems ("Lots of reading"); and 3) nature of the response required ("Solving is not difficult for the students; however, they do have difficulty explaining how they decide things."). Few comments focused on slope or other mathematical concepts or skills required for solving the problems.

Students were better able to identify their difficulties and the mathematical nature of those difficulties than were their teachers. Students described difficulties of understanding slope in the context of distance-rate-time problems, of how to measure rise and run, of interpreting a point on a graph if it did not lie on the intersection of grid lines, and of computing slope involving decimal fractions.

Teachers' weak understanding of student difficulties with important topics like slope is even more problematic considering the great amount of instructional time devoted to teaching the concept. "I beat that topic to death," commented one teacher, surprised that students still had difficulties with slope after "being taught" how to solve slope problems for 2 months. Of our 27 teachers, most were stunned by their students' performance, particularly the fact that difficulties experienced by eighth graders persisted through grades 9 and 10, for most students, and all were eager to learn new instructional approaches to enable student understanding and retention. That led to our recommendations and our current work.

**A major concern in education is how to best address the individual needs of students.** We all agree that remediating what students already know is not a good use of their time or ours. That kind of remediation may lead to boredom, or worse, contribute to total disinterest in and dislike for mathematics. Likewise, giving little attention to difficult to learn topics may lead to student failure, resulting in fewer students in the pipeline to college and STEM careers.

Thus, based on this work (and now continuing with middle school students and their teachers), we recommend that in-service professional development programs use a concept-study approach for the education and updating of teachers. In concept-study groups, teachers 1) identify major concepts and how those become more robust/complex with increasing grade level; 2) select one concept to be the focus of the assessment design; 3) develop a set of problems to evaluate aspects (sub concepts) of that concept along with a problem-difficulty ranking form; and 3) administer that assessment to math teachers and students in their school(s). Subsequent to administration and scoring of the problems, teachers 1) identify the major difficulties experienced by students; 2) compare student difficulties with those identified by them (teachers); and 3) talk to the students since they are more successful at describing when they get stuck than tests reveal. Finally, based on analyses of these results, rethink how to teach the topics/concepts/skills with which students have difficulty. Doing more of the same, doesn't work.

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