Module 2: Arithmetic Operations Including Division of Fractions

Lesson	Course Level Content Standards	Standards from other Grades	Action	Notes/Rationale for Action
2.1		3.OA.B.6, 5.NF.B.7a, 5.NF.B.7c	R	• Reserve these Lessons to be used wth students who need a review of previous grade level concepts prior to engaging with Grade 6 concepts.
2.2		3.OA.B.6, 5.NF.B.7b, 5.NF.B.7c	R	
2.3	6.NS.A.1		O	
2.4	6.NS.A.1		O	
2.5	6.NS.A.1		E	• These Lessons focus on creating story contexts for problems involving division of fractions which extends beyond the explicit expectation of 6.NS.A.1. Although the focus of the Lesson is on creating story contexts, the Problem Sets include more problems than not aligned to the explicit expectations of 6.NS.A.1 and the decision to use these Lessons should be made at the teacher level.
2.6	6.NS.A.1		E	
2.7	6.NS.A.1		O	
2.8	6.NS.A.1		O	
2.9	6.NS.B.3*		O	• This Lesson focusses on fluently adding and subtracting multi-digit decimals using the standard algorithm for each operation which will lead to mastery of 6.NS.B.3.
2.10	6.NS.B.3*		O	• These Lessons include fluently multiplying multi-digit decimals using the standard algorithm which will lead to mastery of 6.NS.B.3.
2.11	6.NS.B.3*		O	
2.12	6.NS.B.2		O	• It should be noted that these Lessons assume students know the standard algorithm for division which may not be the case if teachers taught within the boundaries of the Standards as the standard algorithm for division is not the expectation until Grade 6, 6.NBT.B.2.
2.13	6.NS.B.2		O	
2.14	6.NS.B.2, 6.NS.B.3*		O	

R = optional for remediation; E = optional for enrichment; O = on grade level

Lesson	Course Level Content Standards	Standards from other Grades	Action	Notes/Rationale for Action
2.15	6.NS.B.2, 6.NS.B.3		E	• It should be noted that this Lesson assumes students know the standard algorithm for division which may not be the case if teachers taught within the boundaries of the Standards as the standard algorithm for division is not the expectation until Grade 6, 6.NBT.B.2. • This Lesson focusses on developing and using mental math strategies for division. Although this is not an explicit expectation of any Grade 6 standard, it may prove to be advantageous for students long term. The decision to use this Lesson should be made at the teacher level.
2.16			E	• This Lesson focusses on generalizing rules for adding and multiplying even and odd numbers which is not an explicit expectation of any Grade 6 standard.
2.17			E	• This Lesson focusses on developing divisibility rules for 3 and 9 which is not an explicit expectation of any Grade 6 standard.
2.18	6.NS.B.4		O	
2.19	6.NS.B.4		E	• This Lesson focusses on understanding and applying Euclid's algorithm to find the greatest common factor (GCF) of two whole numbers which is not an explicit expectation of 6.NS.B.4. Additionally, this Lesson goes beyond the explicit limitations of the target standard, 6.NS.B. 4, by asking students to find the GCF of two whole numbers greater than 100.

R = optional for remediation; E = optional for enrichment; O = on grade level

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Module 3: Rational Numbers

Lesson	Course Level Content Standards	Action	Notes/Rationale for Action
3.1	6.NS.C.6*, 6.NS.C.6a, 6.NS.C.6c*	O	• This Lesson focusses on understanding a rational number as a point on the number line extend number line diagrams familiar from previous grades to represent points on the line with negative number coordinates which will lead to mastery of 6.NS.C.6. • This Lesson includes finding and positioning integers and other rational numbers on a horizontal or vertical number line diagram which will lead to mastery of 6.NS.C.6c.
3.2	6.NS.C.5, 6.NS.C.6c*	O	• These Lessons include finding and positioning integers on a horizontal or vertical number line diagram which will lead to mastery of 6.NS.C.6c.
3.3	6.NS.C.5, 6.NS.C.6c*	O	
3.4	6.NS.C.5, 6.NS.C.6a, 6.NS.C.6c*	O	
3.5	6.NS.C.5, 6.NS.C.6a, 6.NS.C.6c*	O	
3.6	6.NS.C.5, 6.NS.C.6a, 6.NS.C.6c*	O	• This Lesson focusses on finding and positioning integers and other rational numbers on a horizontal or vertical number line diagram which will lead to mastery of 6.NS.C.6c.
3.7	6.NS.C.6c*, 6.NS.C.7b*	O	• These Lessons include finding and positioning integers and other rational numbers on a horizontal or vertical number line diagram which will lead to mastery of 6.NS.C.6c. • These Lessons include explaining statements of order for rational numbers in real-world contexts which will lead to mastery of 6.NS.C.6c. • It should be noted that, although these Lessons do not include any inequalities, they do develop the understanding called for in 6.NS.C.7a and, as a result, should prove to be advantageous for students in their pursuit to master 7.NS.C.7.
3.8	6.NS.C.6c*, 6.NS.C.7b*	O	
3.9	6.NS.C.6c*, 6.NS.C.7b*	O	
3.10	6.NS.C.7a, 6.NS.C.7b	O	
3.11	6.NS.C.7c, 6.NS.C.7d	O	
3.12	6.NS.C.7a, 6.NS.C.7c	O	
3.13	6.NS.C.7b, 6.NS.C.7c, 6.NS.C.7d	O	

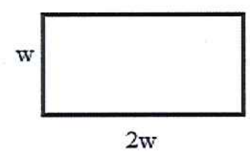
R = optional for remediation; E = optional for enrichment; O = on grade level

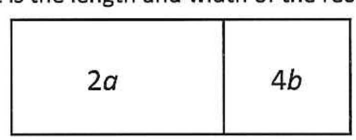
3

Lesson	Course Level Content Standards	Action	Notes/Rationale for Action
3.14	6.NS.C.6c*	O	• This Lesson includes finding and positioning pairs of integers on a coordinate plane which will lead to mastery of 6.NS.C.6c.
3.15	6.NS.C.6b*, 6.NS.C.6c*	O	• This Lesson includes understanding signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane which will lead to mastery of 6.NS.C.6b. • This Lesson includes finding and positioning pairs of integers on a coordinate plane which will lead to mastery of 6.NS.C.6c.
3.16	6.NS.C.6b, 6.NS.C.6c*	O	• This Lesson includes finding and positioning pairs of integers on a coordinate plane which will lead to mastery of 6.NS.C.6c.
3.17	6.NS.C.6b, 6.NS.C.6c*	O	• This Lesson focusses on finding and positioning pairs of integers and other rational numbers on a coordinate plane which will lead to mastery of 6.NS.C.6c.
3.18	6.NS.C.8*	O	• This Lesson focusses on using coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate which will lead to mastery of 6.NS.C.8.
3.19	6.NS.C.8*	O	• This Lesson focusses on solving mathematical problems by graphing points in all four quadrants of the coordinate plane which will lead to mastery of 6.NS.C.8.

R = optional for remediation; E = optional for enrichment; O = on grade level

<table>
<tr><td colspan="2">Expressions and Equations (EE)
Use properties of operations to generate equivalent expressions.</td></tr>
<tr><td colspan="2">Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: coefficients, like terms, distributive property, factor</td></tr>
<tr><td>Louisiana Standard</td><td>Explanations and Examples</td></tr>
<tr><td>7.EE.A.1. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients to include multiple grouping symbols (e.g., parentheses, brackets, and braces).</td><td>Examples:

Write an equivalent expression for $3(x + 5) - 2$.
Solution: $3x + 13$
Suzanne says the two expressions $2(3a - 2) + 4a$ and $10a - 2$ are equivalent? Is she correct? Explain why or why not?
Solution: no. $10a - 4$ is not equivalent to $10a-2$
Write an equivalent expression for: $3a + 12$.
Solution: $3(a + 4)$
A rectangle is twice as long as its width. Write an expression to represent its perimeter.


Solution: 6w
An equilateral triangle has a perimeter of $6x + 15$. What is the length of each side of the triangle?
Solution: $2x = 5$
What is the length and width of the rectangle below?


Solution: $2(a + 2b)$
</td></tr>
</table>

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Name ______________________________ Date ______________

1. Gloria says the two expressions $\frac{1}{4}(12x + 24) - 9x$ and $-6(x + 1)$ are equivalent. Is she correct? Explain how you know.

2. A grocery store has advertised a sale on ice cream. Each carton of any flavor of ice cream costs \$3.79.

   a. If Millie buys one carton of strawberry ice cream and one carton of chocolate ice cream, write an algebraic expression that represents the total cost of buying the ice cream.

   b. Write an equivalent expression for your answer in part (a).

   c. Explain how the expressions are equivalent.

Addition/Subtraction of Whole Numbers - *Standards progression*

KEY: Conceptual Standard | Conceptual/Procedural Standard | Procedural Standard

4			Fluently add multi-digit whole numbers using the standard algorithm. (4.NBT.B.4)	Fluently subtract multi-digit whole numbers using the standard algorithm. (4.NBT.B.4)
3			Fluently add within 1000 using strategies and algorithms based on place value, properties, and relationships. (3.NBT.A.2)	Fluently subtract within 1000 using strategies and algorithms based on place value, properties, and relationships. (3.NBT.A.2)
2	Fluently add within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. (2.NBT.B.5)	Fluently subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. (2.NBT.B.5)	Add w/in 1000 using concrete or visual models and other strategies (2.NBT.B.7)	Subtract w/in 1000 using concrete or visual models and other strategies (2.NBT.B.7)
			Explain why addition and subtraction strategies work, using place value and the properties of operation. (2.NBT.B.9)	
			Understand three-digit numbers are composed of hundreds, tens, and ones. (2.NBT.A.1)	
1	Add within 100 using concrete or visual models properties, and relationships. (1.NBT.C.4)	Subtract multiples of 10 using concrete or visual models properties, and relationships. (1.NBT.C.6)		
	Understand two-digit numbers are composed of tens and ones. (1.NBT.B.2)			
K	Compose/decompose numbers from 11-19 using objects or drawings(K.NBT.A.1)			
	ADD w/in 100	SUBTRACT w/in 100	ADD w/in 1000	SUBTRACT w/in 1000

Addition/Subtraction of Whole Numbers - *Strategies other than the standard algorithm*

Grade	Addition	Subtraction
2	• Front-end addition (breaking apart by place value, writing each partial sum, then adding them) • Adding on successively, if efficient (within 1,000) • Record newly composed units above or below the next place (within 1,000) • Informal use of commutative and associative properties	• (De)composing by place value (can use concrete strategies) • Converting to an unknown addend problem • Counting on or counting down
3	• Maintaining or building addition fluency within 1,000 using strategies from Grade 2	• Maintaining or building subtraction fluency within 1,000 using strategies from Grade 2
4	*Standard algorithm*	*Standard algorithm*

SUMMARY OF CORE ACTIONS

Core Action 1

Ensure the work of the lesson reflects the Shifts required by the CCSS for Mathematics.

Indicators

A. The lesson focuses on the depth of grade-level cluster(s), grade-level content standard(s) or part(s) thereof.

B. The lesson intentionally relates new concepts to students' prior skills and knowledge.

C. The lesson intentionally targets the aspect(s) of rigor (conceptual understanding, procedural skill and fluency, application) called for by the standard(s) being addressed.

Core Action 2

Employ instructional practices that allow all students to master the content of the lesson.

Indicators

A. The teacher makes the mathematics of the lesson explicit by using explanations, representations, and/or examples.

B. The teacher provides opportunities for students to work with and practice grade-level problems and exercises.

C. The teacher strengthens all students' understanding of the content by sharing a variety of students' representations and solution methods.

D. The teacher deliberately checks for understanding throughout the lesson and adapts the lesson according to student understanding.

E. The teacher summarizes the mathematics with references to student work and discussion in order to reinforce the focus of the lesson.

Core Action 3

Provide all students with opportunities to exhibit mathematical practices in connection with the content of the lesson.

Indicators

A. The teacher poses high-quality questions and problems that prompt students to share their developing thinking about the content of the lesson.

   Students share their developing thinking about the content of the lesson.

B. The teacher encourages reasoning and problem solving by posing challenging problems that offer opportunities for productive struggle.

   Students persevere in solving problems in the face of initial difficulty.

C. The teacher establishes a classroom culture in which students explain their thinking.

   Students elaborate with a second sentence (spontaneously or prompted by the teacher or another student) to explain their thinking and connect it to their first sentence.

D. The teacher creates the conditions for student conversations where students are encouraged to talk about each other's thinking.

   Students talk about and ask questions about each other's thinking, in order to clarify or improve their own mathematical understanding.

E. The teacher connects and develops students' informal language to precise mathematical language appropriate to their grade.

   Students use precise mathematical language in their explanations and discussions.

F. The teacher establishes a classroom culture in which students choose and use appropriate tools when solving a problem.

   Students use appropriate tools strategically when solving a problem.

G. The teacher asks students to explain and justify work and provides feedback that helps students revise initial work.

   Student work includes revisions, especially revised explanations and justifications.

Eureka Remediation Tool: Grade 5
Module 1, Topic A

To become mathematically proficient, students **must** access on-grade-level content. This document aims to help teachers who use the Eureka curriculum to target remediation for students needing extra support before and **during** approaching on-grade-level work, creating opportunities for on-time remediation directly connected to the new learning.

About this Topic

Focus Standards:

5.NBT.A.1: Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.
5.NBT.A.2: Explain and apply patterns in the number of zeros of the product when multiplying a number by powers of 10. Explain and apply patterns in the values of the digits in the product or the quotient, when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10. *For example,* $10^0 = 1$, $10^1 = 10$ *... and* $2.1 \times 10^2 = 210$.
5.MD.A.1: Convert among different-sized standard measurement units within a given measurement system, and use these conversions in solving multi-step, real-world problems (e.g., convert 5 cm to 0.05 m; 9 ft to 108 in).

Topic Overview per the Eureka Curriculum

Topic A begins with a conceptual exploration of the multiplicative patterns of the base ten system. This exploration extends the place value work done with multi-digit whole numbers in Grade 4 to larger multi-digit whole numbers and decimals. Students use place value disks and a place value chart to build the place value chart from millions to thousandths. They compose and decompose units crossing the decimal with a view toward extending their knowledge of the *10 times as large* and *1/10 as large* relationships among whole number places to that of adjacent decimal places. This concrete experience is linked to the effects on the product when multiplying any number by a power of ten. For example, students notice that multiplying 0.4 by 1,000 shifts the position of the digits to the left three places, changing the digits' relationships to the decimal point and producing a product with a value that is $10 \times 10 \times 10$ as large (400.0) (**5.NBT.2**). Students explain these changes in value and shifts in position in terms of place value. Additionally, students learn a new and more efficient way to represent place value units using exponents (e.g., 1 thousand = $1{,}000 = 10^3$) (**5.NBT.2**). Conversions among metric units such as kilometers, meters, and centimeters give students an opportunity to apply these extended place value relationships and exponents in a meaningful context by exploring word problems in the last lesson of Topic A (**5.MD.1**).

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Overview

Eureka Remediation Tools include:

1. a diagnostic assessment to help teachers determine the misunderstandings or gaps in mathematical knowledge related to a specific Topic in the Eureka curriculum
2. guidance for teachers to analyze student work on the diagnostic assessment
3. suggested materials for targeted remedial instruction

Note: The use of this guidance is not intended to delay students' engagement with on-grade-level learning. On-grade-level learning should be the focus of instructional time and be treated as an opportunity for students to "finish" learning previous skills and deepen conceptual understanding.

Diagnostic Assessment

The diagnostic assessment is designed to be administered to targeted students prior to beginning instruction on the given Topic. It is broken into parts (Part A, Part B, and so on); each part addresses a different prerequisite standard and contains three problems. If a student correctly answers at least 2 out of the 3 problems, it can be assumed that he/she is ready to engage with the new content of the Topic with little to no support needed prior to engaging with the Topic. The diagnostic assessment is designed in this way so that teachers can determine the "entry point" to remedial instruction and/or opportunities for unfinished learning within the context of the new learning. The entry points and opportunities for unfinished learning will vary between students.

Guidance for Remediation

The Remediation Guidance is designed for teacher use. It is also broken into parts (Part A, Part B, and so on) and correlates to the parts on the diagnostic assessment. Each part contains the following:

1. **The focus standard**: The focus standards are strategically chosen to address prerequisite skills and are purposefully arranged in the order that students typically master the skills and knowledge.
2. **Why this is important for current grade level work**: This section describes how the work of the prerequisite standard relates to the standard(s) addressed in the Topic of instruction.
3. **Using the diagnostic assessment to identify gaps**: This section identifies common errors students make on the diagnostic assessment items.
4. **Remediation Resources for Targeted Instruction**: The resources pinpoint specific Eureka lessons and parts of lessons for teachers to use to address gaps in mathematical knowledge. Using Eureka materials to address remediation ensures alignment to the standards, consistency in approach to learning, and similarities in strategies for solving problems.

Remediation Guidance: Grade 5
Eureka Module 1, Topic A

Part A: 4.NBT.A.1:

1. Write a number where the value of the 4 is ten times the value of the 4 in the number 62,347.

2. Write a number where the value of the 2 is ten times the value of the 2 in the number 62,347.

3. Write a number where the value of the 6 is ten times the value of the 6 in the number 62,347.

Part B: 4.NBT.A.2:

4. Write the following number in expanded form: 12,497

5. Write the following number in expanded form: 64,025

6. Write the following number in standard form: (4 x 100,000) + (9 x 1,000) + (6 x 100) + (7 x 10) + (5 x 1)

Part C: 4.NF.C.6:

7. What is the decimal form of the fraction 3/10?

8. What is the decimal form of the fraction 8/100?

9. What is the fraction form of 0.90?

Part D: 4.MD.A.1:

10. Complete the following table.

1 kilometer	=	________ meters
1 meter	=	________ centimeters
1 kilogram	=	________ grams
1 liter	=	________ milliliters

11. Complete the following table.

2 kilometers	=	________ meters
3 meters	=	________ centimeters

12. Complete the following table.

2,000 grams	=	________ kilograms
5,000 milliliters	=	________ liters

Solutions:

1. 400 (sample)
2. 20,000 (sample)
3. 600,000 (sample
4. (1 x 10,000) + (2 x 1,000) + (4 x 100) + (9 x 10) + (7 x 1)
5. (6 x 10,000) + (4 x 1,000) + (2 x 10) + (5 x 1)
6. 409,671
7. 0.3
8. 0.08
9.
10. 1,000 (meters)
    100 (centimeters)
    1,000 (grams)
    1,000 (milliliters)
11. 2,000 (meters)
    300 (centimeters)
12. 2 (kilograms)
    5 (liters)

Diagnostic Assessment: Grade 5
Eureka Module 1, Topic A

Remediation Guidance: Grade 5
Eureka Module 1, Topic A

Part A Focus: 4.NBT.A.1 Recognize that in a multi-digit whole number less than or equal to 1,000,000, a digit in one place represents ten times what it represents in the place to its right. *For example, (1) recognize that 700 ÷ 70 = 10; (2) in the number 7,246, the 2 represents 200, but in the number 7,426 the 2 represents 20, recognizing that 200 is ten times as large as 20, by applying concepts of place value and division.*

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Why this is important for current grade level work:
This standard calls for students to understand the value of a digit in one place represents ten times what it represents in the place to its right. In Grade 5 students extend their understanding to explore the relationship between a digit in one place and what it would represent in the place to its left, extending the relationship to include decimal place values. The most important look-fors here are the accuracy of the student's answer. The questions scaffold in difficulty.

Using the Diagnostic Assessment to identify gaps:

Problem 1:
Students may misinterpret the item and identify the place value of the 4 in the given number. This does not necessarily point to a gap in understanding of place value.

Problem 2:
Students should understand the relationship between places, and use the patterns of multiplying by ten to write a number that has the digit 2 in the ten-thousands place. Look for students who need to draw a place-value chart. This highlights an opportunity to move students to a more efficient, abstract understanding of place value and the relationships between digits.

Problem 3:
Look for students who do not add the hundred-thousands place to his/her answer. This might show a lack of understanding of place value and the concept of ten times greater.

Remediation Resources for Targeted Instruction:

<u>4th Grade, Module 1, Topic A, Lesson(s) 2 - 3</u>

Use the Concept Development portion of each Lesson and a sampling of problems from the Problem Set focused on conceptual understanding.

Remediation Guidance: Grade 5
Eureka Module 1, Topic A

Part B Focus: 4.NBT.A.2 Read and write multi-digit whole numbers less than or equal to 1,000,000 using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

5

<table>
<tr>
<td colspan="3">Why this is important for current grade level work:
Since students will be representing place value units using exponents in Grade 5, they must understand the relationship between the standard and expanded form of numbers. Place value work extends in Grade 5 to larger multi-digit whole numbers and decimals. Look for students who do not use multiplication in their expanded form. While this does not necessarily show a gap in understanding, it does show a place where the students' understanding may not readily connect to the on grade-level work. The problems scaffold in difficulty.</td>
<td rowspan="3">Remediation Resources for Targeted Instruction:

<u>4th Grade, Module 1, Topic A, Lesson(s) 3 - 4</u>

Use the Concept Development portion of this Lesson and a sampling of problems from the Problem Set focused on conceptual understanding.</td>
</tr>
<tr>
<td colspan="3">Using the Diagnostic Assessment to identify gaps:</td>
</tr>
<tr>
<td>Problem 4:
Students may recognize the pattern for writing numbers in expanded form and write the answer correctly. This shows an understanding of the pattern of expanded form.</td>
<td>Problem 5:
This problem has a zero in the hundreds place. Look for students who incorrectly represent the hundreds place. This shows a gap in their fundamental understanding of expanded form.</td>
<td>Problem 6:
Look for students who write the answer as 49,675, when the correct answer is 409,675. This shows a gap in their fundamental understanding of place value.</td>
</tr>
</table>

Remediation Guidance: Grade 5
Eureka Module 1, Topic A

Part C Focus: 4.NF.C.6 Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram; represent 62/100 of a dollar as $0.62.*

<table>
<tr>
<td colspan="3">Why this is important for current grade level work:
When multiplying and dividing by powers of 10, students must have a conceptual understanding of place value and the connection between fractions and decimals. These questions ask students to represent a fraction as a decimal and to represent a decimal as a fraction. These items ensure students have the necessary understanding of decimal notation for fractions to connect to the target standard.</td>
<td rowspan="3">Remediation Resources for Targeted Instruction:

<u>4th Grade, Module 6, Topic A, Lesson(s) 1 - 3</u>

Use the Concept Development portion of each Lesson and a sampling of problems from the Problem Set focused on conceptual understanding.</td>
</tr>
<tr>
<td colspan="3">Using the Diagnostic Assessment to identify gaps:</td>
</tr>
<tr>
<td>Problem 7:
Look for students who misrepresent 3/10 as a decimal. This shows a gap in conceptual understanding of place value.</td>
<td>Problem 8:
Look for students who misrepresent the fraction as a decimal, not understanding the hundredths place. This shows a gap in their understanding of decimal notation for fractions.</td>
<td>Problem 9:
Students may represent .90 as 9/10 or 90/100. Students representing the decimal correctly as a fraction should be considered ready for the target standard.</td>
</tr>
</table>

Remediation Guidance: Grade 5
Eureka Module 1, Topic A

Part D Focus: 4.MD.A.1 Know relative sizes of measurement units within one system of units including ft, in; km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. (Conversions are limited to one-step conversions.) *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs* (*1, 12*), (*2, 24*), (*3, 36*), *...*

Why this is important for current grade level work:
In Lesson 4 students begin to connect their work with place value to their work with metric conversions. This connection will help finish the learning for some students, while deepening the understanding of others through a direct application of place value understanding. Accurate answers to these problems show basic understanding of metric measurement and conversions. The problems scaffold in difficulty.

Using the Diagnostic Assessment to identify gaps:

Problem 10:
Some students may not remember the correct conversions for all items in the table. This does not mean they do not understand the concept of converting metric measurements.

Problem 11:
Students whose answers represent accurate conversions understand the concept of expressing measurement of a larger unit in terms of a smaller unit, i.e., kilometers to meters.

Problem 12:
While this item is beyond the explicit expectations of the target Grade 4 standard, it will help you identify which students are ready to engage with the new concepts of Grade 5.

Remediation Resources for Targeted Instruction:

4th Grade, Module 2, Topic A, Lesson(s) 1 - 3

Use the Concept Development portion of each Lesson and a sampling of problems from the Problem Set focused on conceptual understanding and/or procedural skill and fluency.

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STUDENT
ACHIEVEMENT
PARTNERS

ANet
ACHIEVEMENT NETWORK
Learning. Together.

Recommendations for Targeted Math Support and Interventions

Use this resource while addressing gaps in student understanding in both universal instruction and math interventions. Gain knowledge of common pitfalls schools fall into, and adjust approaches accordingly based on the recommendations below.

Common Misstep	Recommendation
Blindly adhering to a pacing guide/calendar	**Use formative data** to gauge student understanding and inform pacing
Halting instruction for a broad review	**Provide just in time** support within each unit or during intervention
Trying to address every gap a student has	**Prioritize most essential prerequisite** skills and understanding for upcoming content
Trying to build from the ground up or going back too far in the learning progression	**Trace the learning progression, diagnose, and go back just enough** to provide access to grade level material
Re-teaching students using previously failed methods and strategies	**Provide a new experience** for students to re-engage, where appropriate
Disconnecting intervention from content students are learning in math class	**Connect learning experiences** in intervention and universal instruction
Choosing content for intervention based solely on students' weakest areas	**Focus on major work** clusters from current or previous grades as it relates to upcoming content
Teaching all standards in intervention in a step-by-step, procedural way	**Consider the aspect of rigor** called for in the standards when designing and choosing tasks, activities, or learning experiences
Over-reliance on computer programs in intervention	**Facilitate rich learning experiences** for students to complete unfinished learning from previous or current grade

Student C

Part A: 4.NBT.A.1:

1. Write a number where the value of the 4 is ten times the value of the 4 in the number 62,347.

   $4 \times 10 = 40$

2. Write a number where the value of the 2 is ten times the value of the 2 in the number 62,347.

   $2 \times 10 = 20$

3. Write a number where the value of the 6 is ten times the value of the 6 in the number 62,347.

   $6 \times 10 = 60$

Part B: 4.NBT.A.2:

4. Write the following number in expanded form: 12,497

   10000 + 2000 + 400 + 90 + 7

5. Write the following number in expanded form: 64,025

   60000 + 4000 + 0 + 20 + 5

6. Write the following number in standard form: (4 x 100,000) + (9 x 1,000) + (6 x 100) + (7 x 10) + (5 x 1)

   49,675

Part C: 4.NF.C.6:

7. What is the decimal form of the fraction 3/10?

   .310

8. What is the decimal form of the fraction 8/100?

   .8100

9. What is the fraction form of 0.90?

   $\frac{.90}{100}$

Name ______________________________ Date ______________

1. Solve.

   a. $32.1 \times 10 =$ ______________

   b. $3632.1 \div 10 =$ ______________

2. Solve.

   a. $455 \times 1{,}000 =$ ______________

   b. $455 \div 1{,}000 =$ ______________

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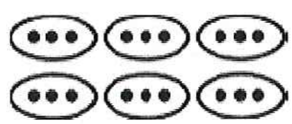
Name ______________________ Date __________ Class __________

6th Grade A3 Pre-assessment

Directions: This quarter, we will be studying expressions and equations which builds on math skills from previous grades. This pre-assessment will help us understand what you know and understand about some of those math topics. I will use this information when I plan lessons this quarter so make sure you do your best to answer all of the questions and show your work.

1. What is another way of expressing 8 x 12?
   A. (8 x 10) + (8 x 2)
   B. (8 x 1) + (8 x 2)
   C. (8 x 10) + 2
   D. 8 + (10 x 2)

2. A student started making a drawing to show that 6 x 3 = 3 x 6. However, she only finished the first part of her drawing. Fill in the other part to complete the picture. Write a sentence explaining why you drew what you did.



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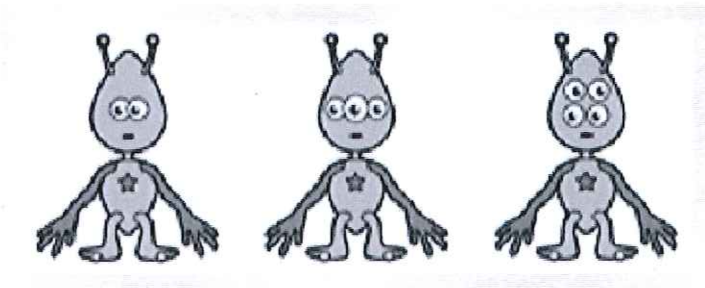
6 x 3 3 x 6

3. Eric is playing a video game. At a certain point in the game, he has 31,500 points. Then the following events happen, in order. Write an expression for the number of points Eric has at the end of the game.

- He earns 2450 additional points.
- He loses 3310 points.
- The game ends, and his score doubles.

4. The two-eyed space creatures, three-eyed space creatures, and four-eyed space creatures are having a contest to create a group with 24 total eyes.



(a) How many two-eyed space creatures are needed to make a group with 24 total eyes?

(b) How many three-eyed space creatures are needed to make a group with 24 total eyes?

(c) Somebody told the five-eyed space creatures that they could not join the contest. Explain why five-eyed space creatures cannot make a group with 24 eyes.

5. Two rules for creating number patterns are given below. Each rule begins with a number called the *input* and creates a number called the *output*.

Rule 1: Multiply the input by 2. Then add 3 to the result to get the output.

Rule 2: Multiply the input by 3. Then add 1 to the result to get the output.

Which input and output table works for **both** rules? How do you know?

Table A

Input	**Output**
2	7

Table B

Input	**Output**
3	10

Table C

Input	**Output**
4	11

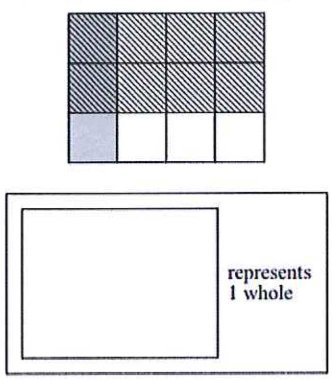
Table D

Input	**Output**
5	13

6. Solve. $\frac{3}{4} + \frac{4}{5} - \frac{7}{10} = ?$

7. Which number is equal to 10^4?
   A. 100
   B. 1,000
   C. 10,000
   D. 100,000

8. The model below can be used to find the product of $\frac{2}{3} \times \frac{1}{4}$.



What is the product of $\frac{2}{3} \times \frac{1}{4}$?

   A. $\frac{2}{9}$
   B. $\frac{3}{9}$
   C. $\frac{2}{12}$
   D. $\frac{9}{12}$

Answer Key & Standards Alignment
6th Grade A3 Pre-assessment

Item	Answer	Standard	Source
1	A	3.OA.B.5	NY State Assessment
2	Drawing showing three groups of six dots each	3.OA.B.5	ANet Open Response

3	(31,500 + 2,450 - 3,310) x 2 or equivalent expression	5.OA.A.2	Illustrative Mathematics
4	a) 12 b) 8 c) 5 is not a factor of 24, 24 is not a multiple of 5	4.OA.B.4	Smarter Balanced Assessment Consortium (SBAC)
5	A	5.OA.B.3	PARCC
6	$\frac{17}{20}$	5.NF.A.1	PARCC
7	C	5.NBT.A.2	SBAC
8	C	5.NF.B.4	Massachusetts State Assessment (MCAS)

Problem	1	2	3	4a	4b	4c	5	6	7	8
Standard	3.OA.B.5	3.OA.B.5	5.OA.A.2	4.OA.B.4	4.OA.B.4	4.OA.B.4	5.OA.B.3	5.NF.A.1	5.NBT.A.2	5.NF.B.4
Content	distributive	commutative	write expressions	factors	factors	factors	rules for input/output	add/sub fractions	powers of 10	mult fractions
Correct Answer	A	3 groups of 6 dots	2(31,500+2,450-3,310) or equivalent	12	8	5 not a factor of 24	A	17/20	C	C
Student 1	A	3 groups of 6 dots	added 2,450 and 3,310	24	8	5 does not go into 24	B and D	no answer	C	D
Student 2	A	3 groups of 6 dots	found score, no expression	2 groups of 12	3 groups of 11	won't add up to 24	D	6/5.	C	B
Student 3	A	3 groups of 6 dots	31,500+2,450-3,310	12	8	it will be a odd number than a even	A	9/20.	D	C
Student 4	A	3 groups of 6 dots	no expression, found correct game score	12	8	his number don't equal 24	D	1/9.	C	D
Student 5	A	3 groups of 6 dots	10, 400	11 (mistake in work)	8	they go past 24 the go up to 25	A	14/19	C	B
Student 6	A	3 groups of 6 dots	no expression, found correct game score	12	8	5 can not go into 24	A	17/20	C	C
Student 7	A	3 groups of 6 dots	rewrote info from the problem	12	8	5 is odd, 24 is even, he will get to 25	A - work for rule 1 only	8/6.	A	C
Student 8	A	3 groups of 6 dots	put -3310, 2450, 4900 on number line	12	8	5 doesn't go into 25	checked A, C, D	17/20	C	C
Student 9	A	3 groups of 6 dots	computed 2450 - 3310 + 2450	12	8	5 doesn't land on 24 when you count it	A	8/24.	B	C
Student 10	A	3 groups of 6 dots	no answer	12 x 2	8 x 3	5 cannot go into 24	identified one rule per table	0/5.	C	C
Student 11	A	3 groups of 6 dots	no expression, found incorrect game score	11	8	because it's gone be over 24	Identified one rule for A and B	1/10.	C	D
Student 12	A	3 groups of 6 dots	2450 - 3310 x 2 = 2280 (no order of operations)	12	8	they don't go in evenly	A - "I did the work"	0	D	C
Student 13	A	wrote "6 x 6"	no answer	no answer	no answer	no answer	B	0/1.	A	C
Student 14	C	3 groups of 6 dots	no expression, found incorrect game score	11 more eyes	7 more eyes	24 doesn't go into 5; 5, 10, 15, 20, 25	"A and B have the input"	6/10.	D	C
Student 15	A	3 groups of 6 dots	no expression, found correct game score	12	8	24 is not a factor of 5	Wrote "no" w/ explanation	31/20.	C	C
Student 16	A	3 groups of 6 dots	no expression, found incorrect game score	12	8	5, 10, 15, 20, 25. It skips 24.	A (no explanation)	14/19	C	D
Student 17	B	drew 20 squares	computed 3310 - 2450	no answer	"the three eyed"	no answer	"A and B cause it shows"	no answer	B	A
Student 18	A	described 6x3 model	computed 3450 + 3310	11	8	because you can't multiply 5 to get 24	A - "it has the 2 in the input"	no answer	C	D
Student 19	D	draw bars with 3 and 6 parts	3310 - 2450 x 2	12	8	"goes from 20 - 25 not 24"	C and D (work for rule 1 shown)	1	C	C
Student 20	A	3 groups of 6 dots	incorrectly computed 2450 - 3310 and doubled	12	8	"won't add up to 24...going to go over"	D	1	C	C
Student 21	C	3 groups of 6 dots	computed 2450 + 3310 + 5760 (sum of 2 previous)	8	8	"it would be 29 not 24" (model shown)	A	2 and 1/9	C	C
Student 22	A	3 groups of 6 dots	computed 31500 + 2450 - 3310	12	8	24 is not a multiple of 5	A	17/20	D	C
Student 23	D	described 6x3 model	40,100	13	8	15 more	C (11 x 11 + 5 = 4...)	13/19	C	C
FULL CREDIT	78%	78%	0% (FULL)	52% (FULL)	78% (FULL)	9% (FULL)	26% (FULL)	13% (FULL)	65%	65%
PARTIAL CREDIT			22% (PARTIAL)	13% (PARTIAL)	4% (PARTIAL)	57% (PARTIAL)	30% (PARTIAL)	13% (PARTIAL)		
NO CREDIT	22%	22%	78% (NO CREDIT)	35% (NO CREDIT)	18% (NO CREDIT)	35% (NO CREDIT)	43% (NO CREDIT)	74% (NO CREDIT)	35%	35%

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ANet
ACHIEVEMENT NETWORK
Learning. Together.

Using pre-assessment data to adapt instruction (example)
EngageNY, 6th Grade, Module 4

Focus standards: All 6.EE standards

Foundational standards: Foundational standards listed in Module Overview and/or vertical progressions chart:

- properties of operation (1.OA.B.3, 3.OA.B.5)
- factors and multiples (4.OA.B.4, 6.NS.B.4)
- writing and interpreting numerical expressions (5.OA.A.2)
- analyzing patterns and relationships (5.OA.B.3)
- understanding place value (5.NBT.A.2)
- understanding and using ratio reasoning to solve problems (6.RP.A.3)
- angles and angle measure (4.MD.C.5, 4.MD.C.6, 4.MD.C.7) -- *(in Module Overview only)*
- graphing on the coordinate plane (5.G.A.1, 5.G.A.2) -- *(in Module Overview only)*
- operations with fractions (5.NF.A.1, 5.NF.B.4) -- *(in vertical progressions chart only)*

Action Plan: Whole class (additional lessons)

Topic/Standards	Data and Action
writing and interpreting numerical expressions (5.OA.A.2)	**Data:** 0% answered correctly, 22% partial credit **Action:** Prior to lessons 5 and 6, engage students in tasks related to: 1. writing numerical expressions: Words to Expressions 1 2. order of operations: Using Operations and Parenthesis 3. meaning of equal sign: Valid Equalities?
Add/subtract fractions (5.NF.A.1)	**Data:** 13% were able to add and subtract fractions; a large majority of the class did not recognize the need to find common denominators or lacked the skill to write equivalent fractions in order to add and subtract the fractions **Action:** Prior to lesson 23, spend 2-3 class periods building understanding+skill with fractions: • Lesson: Understanding equivalent fractions (Learn Zillion) • Video: Adding mixed fractions: regrouping • Tasks: Finding Common Denominators to Add, Finding Common Denominators to Subtract, Mixed Numbers with Unlike Denominators)
analyzing patterns and relationships (5.OA.B.3)	**Data:** 26% full credit, 30% partial credit **Action:** Prior to lesson 31, repurpose problem 5 from the pre-assessment as an instructional task. Allow students to work in groups on the problem and then share/discuss answers as a whole class. Students who received full credit can work on the Sidewalk Patterns task or an extension.

Action Plan: Whole class (spiral or reinforce)

ACHIEVEMENT NETWORK
Learning. Together.

Topic/Standards	Data and Action
properties of operation (1.OA.B.3, 3.OA.B.5)	**Data:** 70% answered both questions correctly, but most student explanations were weak **Action:** Strengthen explanations and precision of language related to commutative and distributive properties throughout the module.
factors and multiples (4.OA.B.4, 6.NS.B.4)	**Data:** approximately 50% of students correctly identified the missing factors parts a and b; 66% of students show partial understanding in part c, but explanations lacked precision **Action:** Strengthen precision of language in student explanations related to factors and multiples throughout the module.
understanding place value (5.NBT.A.2)	**Data:** 65% correct **Action:** Include numerical expressions that include multiplying numbers by powers of 10; reinforce the meaning and use of exponents

Action Plan: Small group intervention

Topic/Standards	Students	Action
properties of operation (1.OA.B.3, 3.OA.B.5)	Student 13, 14, 18, 21 (50%) Students 17, 19, 23 (0%)	1. Re-engage students with problems 1 and 2 from the pre-assessment related to properties of operations 2. If necessary, engage students in tasks to reinforce the commutative and distributive properties
factors and multiples (4.OA.B.4, 6.NS.B.4)	Students 11, 21, 23 (33%) Students 13, 14, 17 (0%-17%)	1. Watch video to build a conceptual understanding of the relationship between factors and multiples 2. Engage in related tasks: Factors and Common Factors and Multiples and Common Multiples, 3. Use the fluency exercise from Lesson 11 to practice finding the GCF
Multiply fractions (5.NF.B.4)	Students 1, 2, 4, 5, 11, 16, 17, 18 (answered 1 question incorrectly)	1. Watch Multiplying unit fractions and whole numbers video 2. Engage with tasks aligned to 5.NF.B.4: Sugar in six cans of soda and To multiply or not multiply

Name ______________________________   Date ______________

1. Solve the addition problems below using the standard algorithm.

a.
$$\begin{array}{r} 6,311 \\ +\ 268 \\ \hline \end{array}$$

b.
$$\begin{array}{r} 6,311 \\ +\ 1,268 \\ \hline \end{array}$$

c.
$$\begin{array}{r} 6,314 \\ +\ 1,268 \\ \hline \end{array}$$

d.
$$\begin{array}{r} 6,314 \\ +\ 2,493 \\ \hline \end{array}$$

e.
$$\begin{array}{r} 8,314 \\ +\ 2,493 \\ \hline \end{array}$$

f.
$$\begin{array}{r} 12,378 \\ +\ 5,463 \\ \hline \end{array}$$

g.
$$\begin{array}{r} 52,098 \\ +\ 6,048 \\ \hline \end{array}$$

h.
$$\begin{array}{r} 34,698 \\ +\ 71,840 \\ \hline \end{array}$$

i.
$$\begin{array}{r} 544,811 \\ +\ 356,445 \\ \hline \end{array}$$

j. 527 + 275 + 752

k. 38,193 + 6,376 + 241,457

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Draw a tape diagram to represent each problem. Use numbers to solve, and write your answer as a statement.

2. In September, Liberty Elementary School collected 32,537 cans for a fundraiser. In October, they collected 207,492 cans. How many cans were collected during September and October?

3. A baseball stadium sold some burgers. 2,806 were cheeseburgers. 1,679 burgers didn't have cheese. How many burgers did they sell in all?

4. On Saturday night, 23,748 people attended the concert. On Sunday, 7,570 more people attended the concert than on Saturday. How many people attended the concert on Sunday?

Name ______________________ Date ____________

1. Solve the addition problems below using the standard algorithm.

a. $\begin{array}{r} 23,607 \\ +\ 2,307 \\ \hline \end{array}$

b. $\begin{array}{r} 3,948 \\ +\ \ \ 278 \\ \hline \end{array}$

c. 5,983 + 2,097

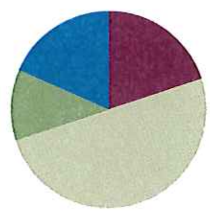
2. The office supply closet had 25,473 large paperclips, 13,648 medium paperclips, and 15,306 small paperclips. How many paperclips were in the closet?

Lesson 11

Objective: Use place value understanding to fluently add multi-digit whole numbers using the standard addition algorithm, and apply the algorithm to solve word problems using tape diagrams.

Suggested Lesson Structure

Fluency Practice	(12 minutes)
Application Problem	(7 minutes)
Concept Development	(30 minutes)
Student Debrief	(11 minutes)
Total Time	**(60 minutes)**



Fluency Practice (12 minutes)

- Round to Different Place Values **4.NBT.3** (5 minutes)
- Multiply by 10 **3.NBT.3** (4 minutes)
- Add Common Units **3.NBT.2** (3 minutes)

Round to Different Place Values (5 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews rounding skills that are building toward mastery.

T: (Write 3,941.) Say the number. We are going to round this number to the nearest thousand.
T: How many thousands are in 3,941?
S: 3 thousands.
T: (Label the lower endpoint of a vertical number line with 3,000.) And 1 more thousand is...?
S: 4 thousands.
T: (Mark the upper endpoint with 4,000.) Draw the same number line.
S: (Draw number line.)
T: What is halfway between 3,000 and 4,000?
S: 3,500.
T: Label 3,500 on your number line as I do the same. Now, label 3,941 on your number line.
S: (Label 3,500 and 3,941.)
T: Is 3,941 nearer to 3,000 or 4,000?

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T: (Write 3,941 ≈ ____.) Write your answer on your personal white board.

S: (Write 3,941 ≈ 4,000.)

Repeat the process for 3,941 rounded to the nearest hundred; 74,621 rounded to the nearest ten thousand and nearest thousand; and 681,904 rounded to the nearest hundred thousand, nearest ten thousand, and nearest thousand.

Multiply by 10 (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity deepens student understanding of base ten units.

T: (Write 10 × ____ = 100.) Say the multiplication sentence.

S: 10 × 10 = 100.

T: (Write 10 × 1 ten = ____.) On your personal white boards, fill in the blank.

S: (Write 10 × 1 ten = 10 tens.)

T: (Write 10 tens = ____ hundred.) On your personal white boards, fill in the blank.

T: (Write ____ ten × ____ ten = 1 hundred.) On your boards, fill in the blanks.

S: (Write 1 ten × 1 ten = 1 hundred.)

Repeat the process for the following possible sequence: 1 ten × 60 = ____, 1 ten × 30 = ____ hundreds, 1 ten × ____ = 900, and 7 tens × 1 ten = ____ hundreds.

Note: Watch for students who say 3 tens × 4 tens is 12 tens rather than 12 hundreds.

Add Common Units (3 minutes)

Materials: (S) Personal white board

Note: This mental math fluency activity prepares students for understanding the importance of the algorithm.

T: (Project 303.) Say the number in unit form.

S: 3 hundreds 3 ones.

T: (Write 303 + 202 = ____.) Say the addition sentence, and answer in unit form.

S: 3 hundreds 3 ones + 2 hundreds 2 ones = 5 hundreds 5 ones.

T: Write the addition sentence on your personal white boards.

S: (Write 303 + 202 = 505.)

Repeat the process and sequence for 505 + 404; 5,005 + 5,004; 7,007 + 4,004; and 8,008 + 5,005.

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Application Problem (7 minutes)

Meredith kept track of the calories she consumed for three weeks. The first week, she consumed 12,490 calories, the second week 14,295 calories, and the third week 11,116 calories. About how many calories did Meredith consume altogether? Which of these estimates will produce a more accurate answer: rounding to the nearest thousand or rounding to the nearest ten thousand? Explain.

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

For the Application Problem, students working below grade level may need further guidance in putting together three addends. Help them to break it down by putting two addends together and then adding the third addend to the total. Use manipulatives to demonstrate.

$$\overbrace{\boxed{12{,}490}\boxed{14{,}295}\boxed{11{,}116}}^{C}$$

ten thousand → 10,000 + 10,000 + 10,000 = 30,000

thousand → 12,000 + 14,000 + 11,000 = 37,000

My 2 estimates are so far apart! But rounding to a smaller unit will always make the estimate closer to the actual answer. So Meredith consumed about 37,000 calories.

Note: This problem reviews rounding from Lesson 10 and can be used as an extension after the Student Debrief to support the objective of this lesson.

Concept Development (30 minutes)

Materials: (T) Millions place value chart (Template) (S) Personal white board, millions place value chart (Template)

Note: Using the template provided within this lesson in upcoming lessons provides students with space to draw a tape diagram and record an addition or a subtraction problem below the place value chart. Alternatively, the unlabeled millions place value chart template from Lesson 2 could be used along with paper and pencil.

Problem 1: Add, renaming once, using place value disks in a place value chart.

T: (Project vertically: 3,134 + 2,493.) Say this problem with me.

S: Three thousand, one hundred thirty-four plus two thousand, four hundred ninety-three.

T: Draw a tape diagram to represent this problem. What are the two parts that make up the whole?

S: 3,134 and 2,493.

T: Record that in the tape diagram.

T: What is the unknown?

S: In this case, the unknown is the whole.

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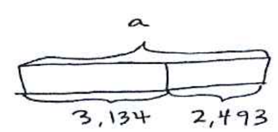
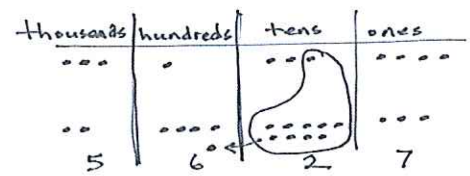




$$\begin{array}{r} 3,134 \\ +\ 2,493 \\ \hline 5,627 \end{array}$$

MP.1

T: Show the whole above the tape diagram using a bracket and label the unknown quantity with an *a*. When a letter represents an unknown number, we call that letter a **variable.**

T: (Draw place value disks on the place value chart to represent the first part, 3,134.) Now, it is your turn. When you are done, add 2,493 by drawing more disks on your place value chart.

T: (Point to the problem.) 4 ones plus 3 ones equals?

S: 7 ones. (Count 7 ones in the chart, and record 7 ones in the problem.)

T: (Point to the problem.) 3 tens plus 9 tens equals?

S: 12 tens. (Count 12 tens in the chart.)

T: We can bundle 10 tens as 1 hundred. (Circle 10 tens disks, draw an arrow to the hundreds place, and draw the 1 hundred disk to show the regrouping.)

T: We can represent this in writing. (Write 12 tens as 1 hundred, crossing the line, and 2 tens in the tens column so that you are writing 12 and not 2 and 1 as separate numbers. Refer to the visual above.)

T: (Point to the problem.) 1 hundred plus 4 hundreds plus 1 hundred equals?

S: 6 hundreds. (Count 6 hundreds in the chart, and record 6 hundreds in the problem.)

T: (Point to the problem.) 3 thousands plus 2 thousands equals?

S: 5 thousands. (Count 5 thousands in the chart, and record 5 thousands in the problem.)

T: Say the equation with me: 3,134 plus 2,493 equals 5,627. Label the whole in the tape diagram, above the bracket, with $a = 5,627$.

Problem 2: Add, renaming in multiple units, using the standard algorithm and the place value chart.

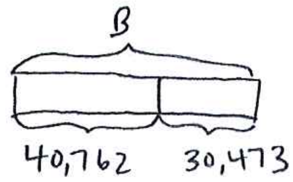
T: (Project vertically: 40,762 + 30,473.) With your partner, draw a tape diagram to model this problem, labeling the two known parts and the unknown whole, using the variable *B* to represent the whole. (Circulate and assist students.)

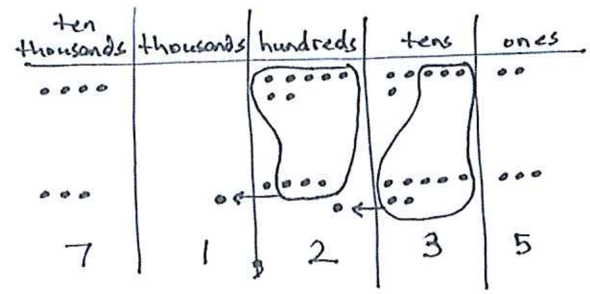
T: With your partner, write the problem, and draw disks for the first addend in your chart. Then, draw disks for the second addend.

T: (Point to the problem.) 2 ones plus 3 ones equals?

S: 5 ones. (Count the disks to confirm 5 ones, and write 5 in the ones column.)

T: 6 tens plus 7 tens equals?





$$\begin{array}{r} 40{,}762 \\ +\ 30{,}473 \\ \hline 71{,}235 \end{array}$$

S: 13 tens. → We can group 10 tens to make 1 hundred. → We do not write two digits in one column. We can change 10 tens for 1 hundred leaving us with 3 tens.

T: (Regroup the disks.) Watch me as I record the larger unit using the addition problem. (First, record the 1 on the line in the hundreds place, and then record the 3 in the tens so that you are writing 13, not 3 then 1.)

T: 7 hundreds plus 4 hundreds plus 1 hundred equals 12 hundreds. Discuss with your partner how to record this. (Continue adding, regrouping, and recording across other units.)

T: Say the equation with me. 40,762 plus 30,473 equals 71,235. Label the whole in the tape diagram with 71,235, and write B = 71,235.

Problem 3: Add, renaming multiple units using the standard algorithm.

$$\begin{array}{r} 207{,}426 \\ +\ 128{,}744 \\ \hline 336{,}170 \end{array}$$

T: (Project: 207,426 + 128,744.) Draw a tape diagram to model this problem. Record the numbers on your personal white board.

T: With your partner, add units right to left, regrouping when necessary using the standard algorithm.

S: 207,426 + 128,744 = 336,170.

Problem 4: Solve a one-step word problem using the standard algorithm modeled with a tape diagram.

The Lane family took a road trip. During the first week, they drove 907 miles. The second week they drove the same amount as the first week plus an additional 297 miles. How many miles did they drive during the second week?

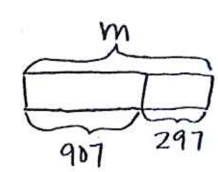
T: What information do we know?

S: We know they drove 907 miles the first week.
We also know they drove 297 miles more during the second week than the first week.

T: What is the unknown information?

S: We do not know the total miles they drove in the second week.

T: Draw a tape diagram to represent the amount of miles in the first week, 907 miles. Since the Lane family drove an additional 297 miles in the second week, extend the bar for 297 more miles. What does the tape diagram represent?



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S: The number of miles they drove in the second week.

T: Use a bracket and label the unknown with the variable *m* for miles.

T: How do we solve for *m*?

S: $907 + 297 = m$.

T: (Check student work to see they are recording the regrouping of 10 of a smaller unit for 1 larger unit.)

T: Solve. What is *m*?

S: $m = 1{,}204$. (Write $m = 1{,}204$.)

T: Write a statement that tells your answer.

S: (Write: The Lane family drove 1,204 miles during the second week.)

NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

English language learners benefit from further explanation of the word problem. Have a conversation around the following: "What do we do if we do not understand a word in the problem? What thinking can we use to figure out the answer anyway?" In this case, students do not need to know what a road trip is in order to solve. Discuss, "How is the tape diagram helpful to us?" It may be helpful to use the RDW approach: Read important information. Draw a picture. Write an equation to solve. Write the answer as a statement.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

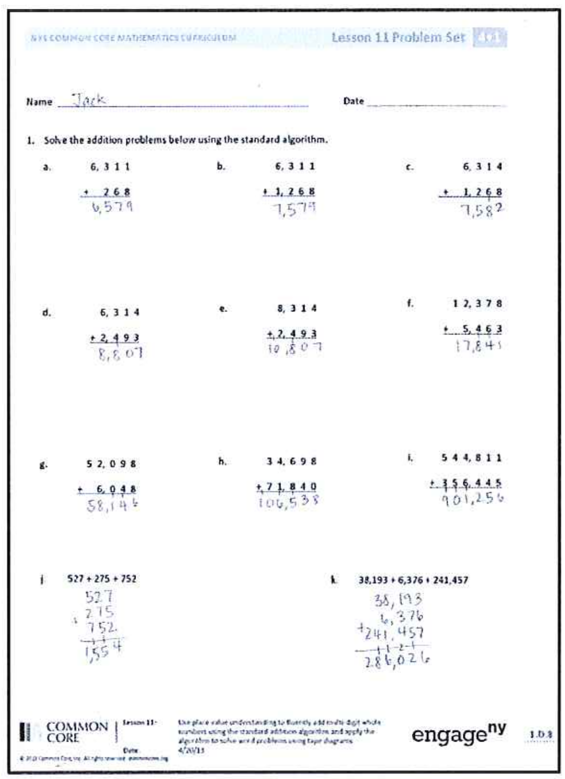
Student Debrief (11 minutes)

Lesson Objective: Use place value understanding to fluently add multi-digit whole numbers using the standard addition algorithm, and apply the algorithm to solve word problems using tape diagrams.

Invite students to review their solutions for the Problem Set and the totality of the lesson experience. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Student Debrief. Guide students in a conversation to debrief the Problem Set.

Any combination of the questions below may be used to lead the discussion.

- When we are writing a sentence to express our answer, what part of the original problem helps us to tell our answer using the correct words and context?



NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 11 Problem Set 4•1

Name Jack Date

1. Solve the addition problems below using the standard algorithm.

a. 6,311 + 268 = 6,579

b. 6,311 + 1,268 = 7,579

c. 6,314 + 1,268 = 7,582

d. 6,314 + 2,493 = 8,807

e. 8,314 + 2,493 = 10,807

f. 12,378 + 5,463 = 17,841

g. 52,098 + 6,048 = 58,146

h. 34,698 + 71,840 = 106,538

i. 544,811 + 356,445 = 901,256

j. 527 + 275 + 752 = 1,554

k. 38,193 + 6,376 + 241,457 = 286,026

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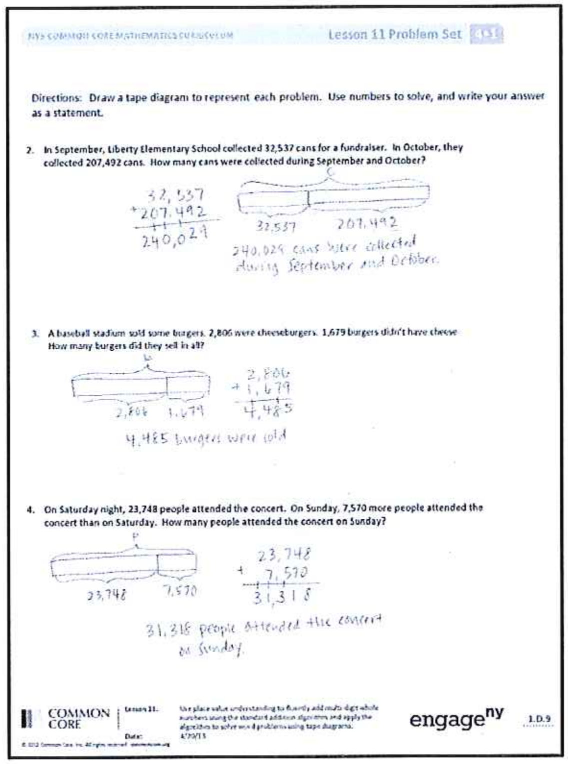
- What purpose does a tape diagram have? How does it support your work?
- What does a **variable**, like the letter *C* in Problem 2, help us do when drawing a tape diagram?
- I see different types of tape diagrams drawn for Problem 3. Some drew one bar with two parts. Some drew one bar for each addend and put the bracket for the whole on the right side of both bars. Will these diagrams result in different answers? Explain.
- In Problem 1, what did you notice was similar and different about the addends and the sums for Parts (a), (b), and (c)?
- If you have 2 addends, can you ever have enough ones to make 2 tens or enough tens to make 2 hundreds or enough hundreds to make 2 thousands? Try it out with your partner. What if you have 3 addends?
- In Problem 1(j), each addend used the numbers 2, 5, and 7 once. I do not see those digits in the sum. Why?


NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 11 Problem Set

Directions: Draw a tape diagram to represent each problem. Use numbers to solve, and write your answer as a statement.

2. In September, Liberty Elementary School collected 32,537 cans for a fundraiser. In October, they collected 207,492 cans. How many cans were collected during September and October?

32,537 + 207,492 = 240,029

240,029 cans were collected during September and October.

3. A baseball stadium sold some burgers. 2,806 were cheeseburgers. 1,679 burgers didn't have cheese. How many burgers did they sell in all?

2,806 + 1,679 = 4,485

4,485 burgers were sold

4. On Saturday night, 23,748 people attended the concert. On Sunday, 7,570 more people attended the concert than on Saturday. How many people attended the concert on Sunday?

23,748 + 7,570 = 31,318

31,318 people attended the concert on Sunday.

COMMON CORE | Lesson 11: Use place value understanding to fluently add multi-digit whole numbers using the standard addition algorithm and apply the algorithm to solve word problems using tape diagrams. engage^ny 1.D.9

- How is recording the regrouped number in the next column when using the standard algorithm related to bundling disks?
- Have students revisit the Application Problem and solve for the actual amount of calories consumed. Which unit, when rounding, provided an estimate closer to the actual value?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

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millions	hundred thousands	ten thousands	thousands	hundreds	tens	ones

millions place value chart

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*Green text: existing supports
*Blue text: additional supports

Lesson 11

Objective: Use place value understanding to fluently add multi-digit whole numbers using the standard addition algorithm, and apply the algorithm to solve word problems using tape diagrams.

STANDARDS ALIGNMENT:
- 4.NBT.B.4: Add and subtract whole numbers with the standard algorithm.
- 4.OA.A.3: Solve multi-step word problems involving whole numbers.

Suggested Lesson Structure

Fluency Practice	(12 minutes)
Application Problem	(7 minutes)
Concept Development	(30 minutes)
Student Debrief	(11 minutes)
Total Time	**(60 minutes)**

ANTICIPATED AREAS OF STRUGGLE:
- Place value understanding; ability to compose/decompose numbers
- Representing numbers with place value disks and bundle
- Fluency with adding multi-digit numbers

Fluency Practice (12 minutes)

- Round to Different Place Values **4.NBT.3** (5 minutes)
- Multiply by 10 **3.NBT.3** (4 minutes)
- Add Common Units **3.NBT.2** (3 minutes)

Round to Different Place Values (5 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews rounding skills that are building towards mastery.

T: (Write 3,941.) Say the number. We are going to round this number to the nearest thousand.
T: How many thousands are in 3,941?
S: 3 thousands.
T: (Label the lower endpoint of a vertical number line with 3,000.) And 1 more thousand is?
S: 4 thousands.
T: (Mark the upper endpoint with 4,000.) Draw the same number line.
S: (Draw number line.)
T: What's halfway between 3,000 and 4,000?
S: 3,500.
T: Label 3,500 on your number line as I do the same. Now, label 3,941 on your number line.
S: (Label 3,5000 and 3,941.)
T: Is 3,941 nearer to 3,000 or 4,000?

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T: (Write 3,941 ≈ ____.) Write your answer on your personal white board.

S: (Write 3,941 ≈ 4,000.)

Repeat process for 3,941 rounded to the nearest hundred, 74,621 rounded to the nearest ten thousand and nearest thousand, and 681,904 rounded to the nearest hundred thousand, nearest ten thousand, and nearest thousand.

Multiply by 10 (4 minutes)

May decide to omit this fluency activity to allow more time for Concept Development

Materials: (S) Personal white board

Note: This fluency activity deepens student understanding of base ten units.

T: (Write 10 × ____ = 100.) Say the multiplication sentence.

S: 10 × 10 = 100.

T: (Write 10 × 1 ten = ____.) On your personal white boards, fill in the blank.

S: (Write 10 × 1 ten = 10 tens.)

T: (Write 10 tens = ____ hundred.) On your personal white boards, fill in the blank.

T: (Write ____ ten × ____ ten = 1 hundred.) On your boards, fill in the blanks.

S: (Write 1 ten × 1 ten = 1 hundred.)

Repeat process for the following possible sequence: 1 ten × 60 = ____, 1 ten × 30 = ____ hundreds, 1 ten × ____ = 900, and 7 tens × 1 ten = ____ hundreds.

Note: Watch for students who say 3 tens × 4 tens is 12 tens rather than 12 hundreds.

Add Common Units (3 minutes)

Use concrete manipulative such as Base Ten Blocks to represent numbers

Materials: (S) Personal white board

Note: This mental math fluency activity prepares students for understanding the importance of the algorithm.

T: (Project 303.) Say the number in unit form.

S: 3 hundreds 3 ones.

T: (Write 303 + 202 = ____.) Say the addition sentence and answer in unit form.

S: 3 hundreds 3 ones + 2 hundreds 2 ones = 5 hundreds 5 ones.

Opportunity to reinforce place value understanding

T: Write the addition sentence on your personal white boards.

S: (Write 303 + 202 = 505.)

Repeat process and sequence for 505 + 404; 5,005 + 5,004; 7,007 + 4,004; and 8,008 + 5,005.

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Break into two parts (addition then estimates)
OR
Omit application problem to make more time for the Concept Development.

Application Problem (7 minutes)

Meredith kept track of the calories she consumed for 3 weeks. The first week, she consumed 12,490 calories, the second week 14,295 calories, and the third week 11,116 calories. About how many calories did Meredith consume altogether? Which of these estimates will produce a more accurate answer: rounding to the nearest thousand or rounding to the nearest ten thousand? Explain.

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

For the Application Problem, students working below grade level may need further guidance in putting together three addends. Help them to break it down by putting two addends together and then adding the third addend to the total. Use manipulatives to demonstrate.

$$\overbrace{\boxed{12{,}490}\boxed{14{,}295}\boxed{11{,}116}}^{C}$$

ten thousand → 10,000 + 10,000 + 10,000 = 30,000

thousand → 12,000 + 14,000 + 11,000 = 37,000

My 2 estimates are so far apart! But rounding to a smaller unit will always make the estimate closer to the actual answer. So Meredith consumed about 37,000 calories.

Note: This problem reviews rounding from Lesson 10 and can be used as an extension after the Debrief to support the objective of this lesson.

Begin with a problem with fewer digits such as 253 + 126 or 257 + 164

Concept Development (30 minutes)

Materials: (T) Millions place value chart (Template) (S) Personal white board, millions place value chart (Template)

Note: Using the template provided within this lesson in upcoming lessons provides students with space to draw a tape diagram and record an addition or a subtraction problem below the place value chart. Alternatively, the unlabeled millions place value chart template from Lesson 2 could be used along with paper and pencil.

Problem 1: Add, renaming once, using place value disks in a place value chart.

T: (Project vertically: 3,134 + 2,493.) Say this problem with me.

S: Three thousand, one hundred thirty-four plus two thousand, four hundred ninety-three.

T: Draw a tape diagram to represent this problem. What are the two parts that make up the whole?

S: 3,134 and 2,493.

T: Record that in the tape diagram.

T: What is the unknown?

S: In this case, the unknown is the whole.

May decide to omit the tape diagram to focus on the other two representations

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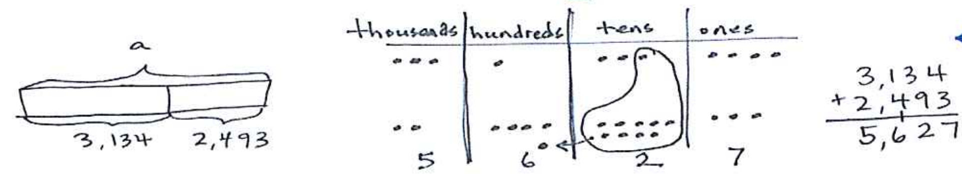
Place value chart



Students who are already familiar with the standard algorithm may begin there and use the place value chart to explain regrouping.

MP.1

T: Show the whole above the tape diagram using a bracket and label the unknown quantity with an a. When a letter represents an unknown number, we call that letter a **variable**.

T: (Draw place value disks on the place value chart to represent the first part, 3,134.) Now, it's your turn. When you are done, add 2,493 by drawing more disks on your place value chart.

T: (Point to the problem.) 4 ones plus 3 ones equals?

S: 7 ones. (Count 7 ones in the chart, and record 7 ones in the problem.)

T: (Point to the problem.) 3 tens plus 9 tens equals?

S: 12 tens. (Count 12 tens in the chart.)

T: We can bundle 10 tens as 1 hundred. (Circle 10 ten disks, draw an arrow to the hundreds place, and draw the 1 hundred disk to show the regrouping.)

T: We can represent this in writing. (Write 12 tens as 1 hundred, crossing the line, and 2 tens in the tens column so that you are writing 12 and not 2 and 1 as separate numbers. Refer to the visual above.)

T: (Point to the problem.) 1 hundred plus 4 hundreds plus 1 hundred equals?

S: 6 hundreds. (Count 6 hundreds in the chart, and record 6 hundreds in the problem.)

T: (Point to the problem.) 3 thousands plus 2 thousands equals?

S: 5 thousands. (Count 5 thousands in the chart, and record 5 thousands in the problem.)

T: Say the equation with me: 3,134 plus 2,493 equals 5,627. Label the whole in the tape diagram, above the bracket, with $a = 5,627$.

Problem 2: Add, renaming in multiple units, using the standard algorithm and the place value chart.

Begin by having students estimate the sum. This will build number sense and help them determine if their answer is reasonable.

T: (Project vertically: 40,762 + 30,473.) With your partner, draw a tape diagram to model this problem, labeling the two known parts and the unknown whole, using the variable B to represent the whole. (Circulate and assist students.)

T: With your partner, write the problem and draw disks for the first addend in your chart. Then, draw disks for the second addend.

T: (Point to the problem.) 2 ones plus 3 ones equals?

S: 5 ones. (Count the disks to confirm 5 ones, and write 5 in the ones column.)

T: 6 tens plus 7 tens equals?

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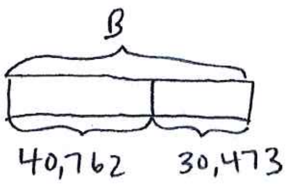
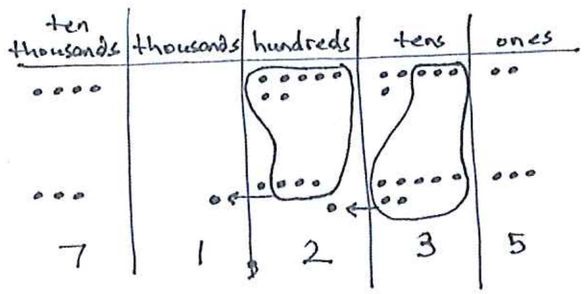




$$\begin{array}{r} 40{,}762 \\ +\ 30{,}473 \\ \hline 71{,}235 \end{array}$$

S: 13 tens. → We can group 10 tens to make 1 hundred. → We don't write two digits in one column. We can change 10 tens for 1 hundred leaving us with 3 tens.

T: (Regroup the disks.) Watch me as I record the larger unit using the addition problem. (First, record the 1 on the line in the hundreds place, and then record the 3 in the tens so that you are writing 13, not 3 then 1.)

T: 7 hundreds plus 4 hundreds plus 1 hundred equals 12 hundreds. Discuss with your partner how to record this. (Continue adding, regrouping, and recording across other units.)

T: Say the equation with me. 40,762 plus 30,473 equals 71,235. Label the whole in the tape diagram with 71,235, and write B = 71,235.

Problem 3: Add, renaming multiple units using the standard algorithm.

T: (Project: 207,426 + 128,744.) Draw a tape diagram to model this problem. Record the numbers on your personal white board.

T: With your partner, add units right to left, regrouping when necessary using the standard algorithm.

S: 207,426 + 128,744 = 336,170.

$$\begin{array}{r} 207{,}426 \\ +\ 128{,}744 \\ \hline 336{,}170 \end{array}$$

Problem 4: Solve a one-step word problem using the standard algorithm modeled with a tape diagram.

The Lane family took a road trip. During the first week, they drove 907 miles. The second week they drove the same amount as the first week plus an additional 297 miles. How many miles did they drive during the second week?

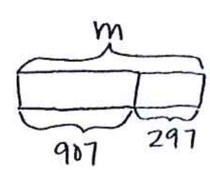
T: What information do we know?

S: We know they drove 907 miles the first week.
We also know they drove 297 miles more during the second week than the first week.

T: What is the unknown information?

S: We don't know the total miles they drove in the second week.

T: Draw a tape diagram to represent the amount of miles in the first week, 907 miles. Since the Lane family drove an additional 297 miles in the second week, extend the bar for 297 more miles. What does the tape diagram represent?



This tip could support all students to understand the word problem.

S: The number of miles they drove in the second week.

T: Use a bracket and label the unknown with the variable *m* for miles.

T: How do we solve for *m*?

S: $907 + 297 = m$.

T: (Check student work to see they are recording the regrouping of 10 of a smaller unit for 1 larger unit.)

T: Solve. What is *m*?

S: $m = 1{,}204$. (Write $m = 1{,}204$.)

T: Write a statement that tells your answer.

S: (Write: The Lane family drove 1,204 miles during the second week.)

NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

English language learners benefit from further explanation of the word problem. Have a conversation around the following: "What do we do if we don't understand a word in the problem? What thinking can we use to figure out the answer anyway?" In this case, students do not need to know what a road trip is in order to solve. Discuss, "How is the tape diagram helpful to us?" It may be helpful to use the RDW approach: Read important information. Draw a picture. Write an equation to solve. Write the answer as a statement.

Problem Set (10 minutes)

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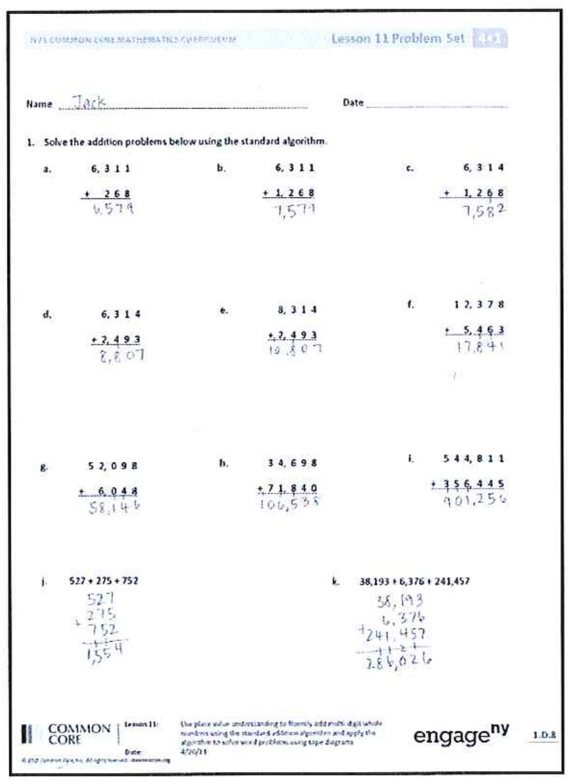

NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 11 Problem Set 4•1

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You may choose to use any combination of the questions below to lead the discussion.

- When we are writing a sentence to express our answer, what part of the original problem helps us to tell our answer using the correct words and context?

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- What purpose does a tape diagram have? How does it support your work?
- What does a **variable,** like the letter *C* in Problem 2, help us do when drawing a tape diagram?
- I see different types of tape diagrams drawn for Problem 3. Some drew one bar with two parts. Some drew one bar for each addend and put the bracket for the whole on the right side of both bars. Will these diagrams result in different answers? Explain.
- In Problem 1, what did you notice was similar and different about the addends and the sums for Parts (a), (b), and (c)?
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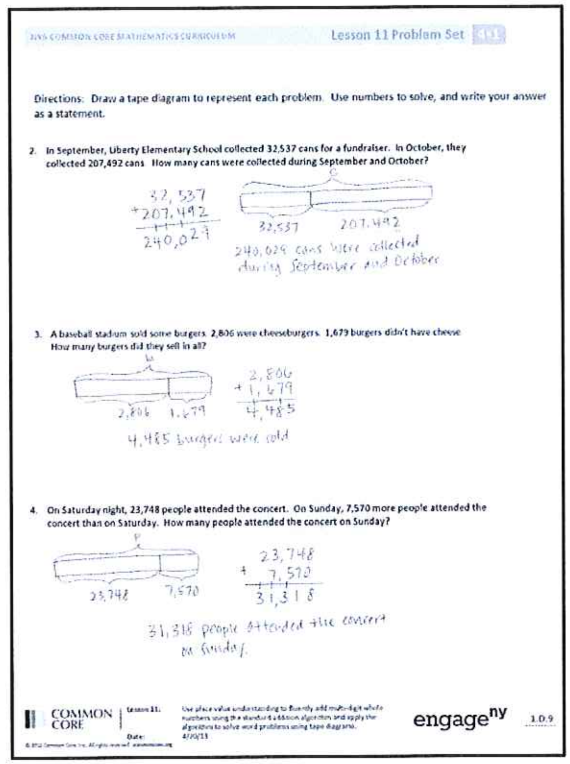
- In Problem 1(j), each addend used the numbers 2, 5, and 7 once. I don't see those digits in the sum. Why?
- How is recording the regrouped number in the next column when using the standard algorithm related to bundling disks?
- Have students revisit the Application Problem and solve for the actual amount of calories consumed. Which unit, when rounding, provided an estimate closer to the actual value?

Prioritize questions in the student debrief based on emphasis in the lesson.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32,537 + 207,492 = 240,029

240,029 cans were collected during September and October.

3. A baseball stadium sold some burgers. 2,806 were cheeseburgers. 1,679 burgers didn't have cheese. How many burgers did they sell in all?

2,806 + 1,679 = 4,485

4,485 burgers were sold.

4. On Saturday night, 23,748 people attended the concert. On Sunday, 7,570 more people attended the concert than on Saturday. How many people attended the concert on Sunday?

23,748 + 7,570 = 31,318

31,318 people attended the concert on Sunday.

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students' understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.

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millions	hundred thousands	ten thousands	thousands	hundreds	tens	ones

millions place value chart

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Math Communication Structures

	Additional information	Your Notes
Who?	*Who is involved in math instruction for the grade level?* May include: other grade level teachers (if self-contained), SPED teacher, interventionists, paras, etc.	
What?	What information does each party need to know? May include: pacing calendar updates, prior grade level skills that need to be reinforced or the actual math content and strategies.	
When?	When will you meet in person? How often? What communication will happen electronically?	
Special Meetings	At least quarterly, the group should review math data together, discuss the effectiveness of intervention and brainstorm actions to take based on data. This can also be a time to discuss how communication is going between all parties and make adjustments as needed. What data will be used? When will this meeting occur?	