Government Purchase (ECR)

Overview

Students will create equations for three different scenarios using the same variables defined in the stem of the item. Then students will solve the equations they have written.

Standards

Create equations that describe numbers or relationships.

HSA-CED.A.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*★

Solve equations and inequalities in one variable.

HSA-REI.B.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade-Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSA-CED.A.1	• 7.EE.B.4 • 8.EE.C.7	1. Jaquel bought *j* pairs of jeans at \$24.99 each and *s* shirts at \$14.99 each for a total of \$104.95 before taxes. Write an equation to represent this situation. a. $24.99j + 14.99s = 104.95$ 2. http://www.illustrativemathematics.org/illustrations/581 3. http://www.illustrativemathematics.org/illustrations/582	• http://www.illustrativemathematics.org/illustrations/643 • http://www.illustrativemathematics.org/illustrations/583 • http://www.illustrativemathematics.org/illustrations/999 • http://learnzillion.com/lessonsets/120-create-equations-and-inequalities-in-one-variable-and-use-them-to-solve-problems
HSA-REI.B.3	• 7.EE.B.4a • 8.EE.C.7	1. Solve the following: a. $\frac{3}{4}x + \frac{5}{6} = 5x - \frac{125}{3}$ i. 10 2. Find s if $24.99j + 14.99s = 104.95$ and $j = 3$. a. $s = 29.98$	• http://www.illustrativemathematics.org/illustrations/550 • http://www.illustrativemathematics.org/illustrations/392 • http://learnzillion.com/lessonsets/741-solve-linear-equations-and-inequalities-in-one-variable • http://learnzillion.com/lessonsets/180-solve-linear-equations-and-inequalities-in-one-variable

After the Task

Students may struggle with writing the equation for each situation since each scenario has a different unknown quantity. Encourage students to write an equation with all of the variables first, as if they were creating a formula. Relate this equation to finding the cost of items at a store or tickets at a concert. If students can write the equation $B = zx + wy$ based on the description in the stem, then students can substitute given values for parts a, b, and c to find the solution.

For part c, students must write the value of *z* in terms of w. Students who struggle with this may benefit from listing the values of all the variables prior to writing the equation. Have students write the verbal expression first, then translate the verbal expression into an algebraic expression.

Provide students who struggle with this task additional practice writing and solving equations based on real-world scenarios.

Student Extended Constructed Response

The government buys x fighter planes at z dollars each and y tons of wheat at w dollars each. It spends B dollars. In each of the following, write and solve an equation to find the unknown quantity.

1. Write and solve an equation to find the number of tons of wheat the government can afford to buy if it spends a total of \$100 million, wheat costs \$300 per ton, and it must buy 5 fighter planes at \$15 million each. Show your work and state your answer in the context of the given situation.

2. Write and solve an equation to find the price of fighter planes if the government bought 3 of them, in addition to 10,000 tons of wheat at \$500 per ton, for a total of \$50 million. Show your work and state your answer in the context of the given situation.

3. Write and solve an equation to find the price of a ton of wheat, given that a fighter plane costs 100,000 times as much as a ton of wheat, and that the government bought 20 fighter planes and 15,000 tons of wheat for a total cost of \$90 million. Show your work and state your answer in the context of the given situation.

Task adapted from http://www.illustrativemathematics.org/illustrations/580.

Extended Constructed Response Exemplar Response

The government buys *x* fighter planes at *z* dollars each and *y* tons of wheat at *w* dollars each. It spends *B* dollars. In each of the following, write an equation in terms of the unknown quantity.

1. Write and solve an equation to find the number of tons of wheat the government can afford to buy if it spends a total of $100 million, wheat costs $300 per ton, and it must buy 5 fighter planes at $15 million each. Show your work and state your answer in the context of the given situation.

   Given B = 100,000,000, w = 300, x = 5, and z = 15,000,000; y = ?

$$100{,}000{,}000 = 15{,}000{,}000(5) + 300y$$

$$100{,}000{,}000 = 75{,}000{,}000 + 300y$$

$$25{,}000{,}000 = 300y$$

$$\frac{25{,}000{,}000}{300} = \frac{300y}{300}$$

$$83{,}333\frac{1}{3} = y$$

   The government can buy 83,333 1/3 tons of wheat.

2. Write and solve an equation to find the price of fighter planes if the government bought 3 of them, in addition to 10,000 tons of wheat at $500 per ton, for a total of $50 million. Show your work and state your answer in the context of the given situation.

   Given x = 3, y = 10,000, w = 500, and B = 50,000,000; z = ?

$$50{,}000{,}000 = 3z + 10{,}000(500)$$

$$50{,}000{,}000 = 3z + 5{,}000{,}000$$

$$45{,}000{,}000 = 3z$$

$$\frac{45{,}000{,}000}{3} = \frac{3z}{3}$$

$$15{,}000{,}000 = z$$

   The price of a fighter plane is $15 million.

3. Write and solve an equation to find the price of a ton of wheat, given that a fighter plane costs 100,000 times as much as a ton of wheat, and that the government bought 20 fighter planes and 15,000 tons of wheat for a total cost of $90 million. Show your work and state your answer in the context of the given situation.

Solution: Given z = 100,000w, x = 20, y = 15,000, B = 90,000,000; w = ?

$$90{,}000{,}000 = 20(100{,}000w) + 15{,}000w$$

$$90{,}000{,}000 = 2{,}000{,}000w + 15{,}000w$$

$$90{,}000{,}000 = 2{,}015{,}000w$$

$$\frac{90{,}000{,}000}{2{,}015{,}000} = \frac{2{,}015{,}000w}{2{,}015{,}000}$$

$$44.67 \approx w$$

The price of a ton of wheat is approximately $44.67

Technical Support Center (ECR)

Overview

Students will solve systems of inequalities and equations to determine the number of cubicles to be built and the number of operators to be employed in a technical support center.

Standards

Create equations that describe numbers or relationships.

HSA-CED.A.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

Solve systems of equations.

HSA-REI.C.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Represent and solve equations and inequalities graphically.

HSA-REI.D.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade-Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSA-CED.A.3	• HSA-CED.A.1 • HSA-CED.A.2 • HAS-REI.D.10	1. D'Andre earns some money by helping people with their yards. It takes him an average of 2 hours to weed a flowerbed and an average of 1.5 hours to mow and edge a lawn. He can work no more than 6 hours a day. He charges $60 to weed a flowerbed and $75 to mow and edge a lawn. He needs to cover his expenses of $150 to make a profit. Write a system of inequalities that would help determine how many flowerbeds and lawns he could complete on a given day.	• http://www.illustrativemathematics.org/illustrations/582 • http://www.illustrativemathematics.org/illustrations/1010 • http://www.illustrativemathematics.org/illustrations/1351 • http://www.illustrativemathematics.org/illustrations/1066 • http://learnzillion.com/lessonsets/516-represent-constraints-by-linear-equations-inequalities-and-systems-interpret-solutions-as-viable-and-nonviable

Grade-Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
		a. $2f + 1.5m \leq 6$ $60f + 75m \geq 150$ Where f = # of flowerbeds and m = # of lawns to be mowed and edged 2. http://www.illustrativemathematics.org/illustrations/610 3. http://www.illustrativemathematics.org/illustrations/1351	
HSA-REI.C.6	• 8.EE.C.8 • HSA-CED.A.2	1. Solve the system: $-2x = 6y + 36$ $5x - 18y = 141$ a. (3, 7) 2. http://www.illustrativemathematics.org/illustrations/462	• http://www.illustrativemathematics.org/illustrations/469 • http://www.illustrativemathematics.org/illustrations/472 • http://www.illustrativemathematics.org/illustrations/1003 • http://learnzillion.com/lessonsets/486-solve-systems-of-two-linear-equations-in-two-variables • http://learnzillion.com/lessonsets/247-solve-systems-of-linear-equations-exactly-and-approximately
HSA-REI.D.12	• HSA-REI.C.5 • HSA-REI.D.11	1. http://www.illustrativemathematics.org/illustrations/644 2. http://www.illustrativemathematics.org/illustrations/1205	• http://learnzillion.com/lessonsets/678-graph-the-solutions-to-a-linear-inequality-as-a-halfplane-graph-the-solutions-to-a-system-of-linear-inequalities-as-an-intersection-of-halfplanes • http://learnzillion.com/lessonsets/624-graph-the-solutions-to-a-linear-inequality-as-a-halfplane-and-the-solutions-to-a-system-of-linear-inequalities-as-an-intersection-of-halfplanes

Real-World Preparation: The following questions will prepare students for some of the real-world components of this task:

- **What is a technical support center?** Companies typically have technical support centers to answer questions that customers might have about products they have purchased. In some cases, operators who work in the center could provide support to customers by helping them troubleshoot problems they may be having with the products.
- **What is a cubicle?** A cubicle is a workspace provided to employees in order for them to have some privacy, but it is not an office. Typically, cubicles do not have doors and are created using portable wall dividers.
- **What is an operator?** In the case of this task, an operator would be a person answering phone calls and emails from customers.
- **What are employee benefits?** Employers typically pay for benefits such as health insurance, vacation time, and sick-leave time. The cost of these benefits is used to determine the most accurate amount an employee is paid.
- **What is an open floor plan?** *Open floor plan* is a term used to describe any floor plan that makes use of large, open spaces and minimizes the use of small rooms, such as private offices. This allows for the use of "movable" walls to create different arrangements to fit a company's needs.

After the Task

Students may struggle with writing the inequalities for question one as they may try to incorporate the square footage. Remind students to use only the information they know for sure—total number of cubicles and total number of employees—when creating the system of inequalities.

For question 2, students may forget that the axes represent additional constraints. Have students explain why the solutions are confined to quadrant one and determine the best way to state the constraints that would be graphed on the coordinate system.

Students who understand what each of the boundary lines and the shaded region represent will be able to determine the answer to question 3 with little work. Other students will test many points to find the maximum and minimum values for the operators and cubicles, respectively. Some students may find multiple points rather than the one point that satisfies both conditions. Have students discuss what the boundary lines represent and what their intersection means.

Students may overlook the word "**both"** in the question and provide the answer (0, 10) or 10 large cubicles. Remind students to read the question closely. Since the question requires the use of both types of cubicles, there must be at least one small cubicle. If this is the case, there will only be 19 employees.

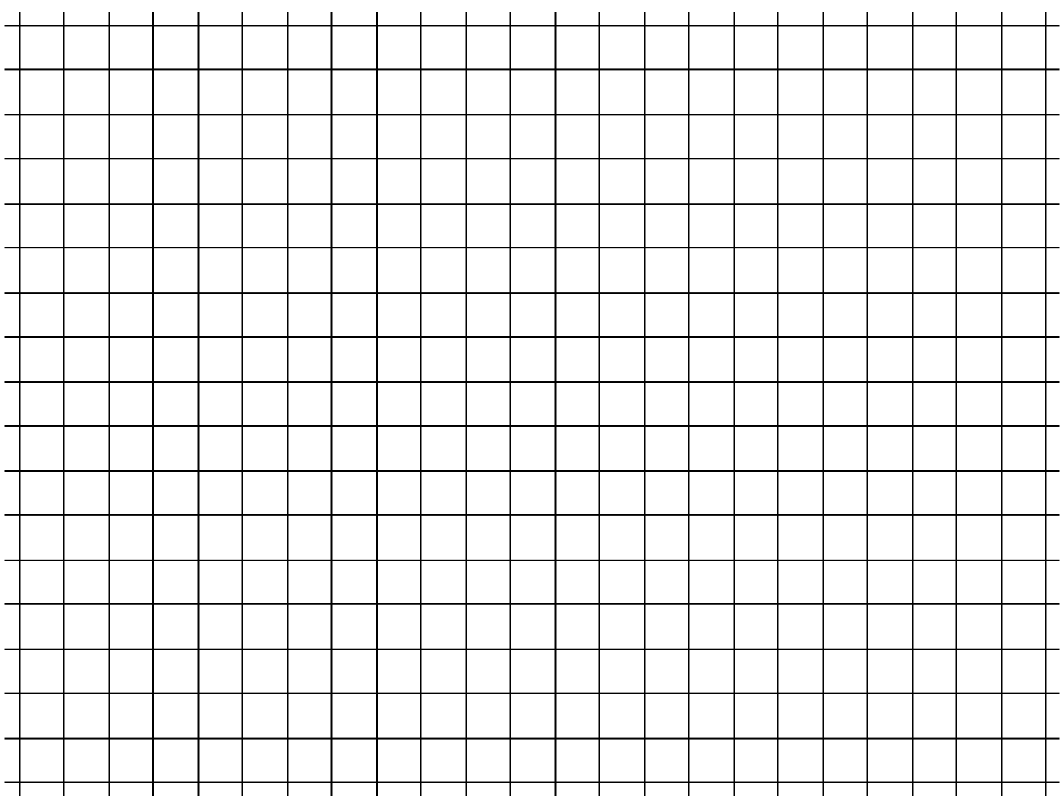
For question 5, students may try to use the inequality $s + 2l \leq 20$ rather than the expression $s + 2l$. Remind students that there is no longer a maximum value placed on the number of operators and therefore students only need to know how many cubicles of each type they will need. If necessary, provide students with additional optimization problems to help them see the use of these expressions.

Student Extended Constructed Response

A new home security company is planning to open a technical support center to answer customer phone calls and emails. The company plans to purchase a large building with an open floor plan to house cubicles where operators will sit while they work. There are two options for cubicles: a small cubicle for one operator that is approximately 36 square feet and a larger cubicle for two operators that is approximately 60 square feet. The company knows it wants a minimum of 15 cubicles in the technical support center; however, because of the rising costs of employee benefits, the company wants to keep the total number of operators at a maximum of 20.

1. Write a system of inequalities to model the requirements for the new technical support center that would help the company determine the number of cubicles to build and the number of operators that could be employed. *Be sure to define any variables used.*

2. Graph the solution set of the system you created. State any additional constraints the model may have. *Be sure to label the graph.*



3. Which combination of small and large cubicles will allow the company to employ the maximum number of operators with the minimum number of cubicles? *Provide evidence to support your response.*

4. The first location the company finds is smaller than it would like. The space will only hold 10 cubicles total. If the company still wants to employ up to 20 operators, is it possible in this space to have the maximum number of cubicles and operators with a combination of **both** small and large cubicles? *Provide evidence to support your response.*

5. The location the company decides to use has an 1,800-square-foot open floor plan. The owner of the company also decides to remove the maximum limit from the number of operators he is willing to employ. He wants six fewer small cubicles than large cubicles. Calculate the maximum number of operators that can be employed in the 1,800-square-foot space using six fewer small cubicles than large cubicles. *Provide evidence to support your response.*

Task adapted with permission from Universal Achievement, LLC.

Extended Constructed Response Exemplar Response

A new home security company is planning to open a technical support center to answer customer phone calls and emails. The company plans to purchase a large building with an open floor plan to house cubicles where operators will sit while they work. There are two options for cubicles: a small cubicle that is approximately 36 square feet that will house one operator and a larger cubicle of approximately 60 square feet that will house two operators. The company knows it wants a minimum of 15 cubicles in the technical support center; however, because of the rising costs of employee benefits, the company wants to keep the total number of operators at a maximum of 20.

1. Write a system of inequalities to model the requirements for the new technical support center that would help the company determine the number of cubicles to build and the number of operators that could be employed. *Be sure to define any variables used.*

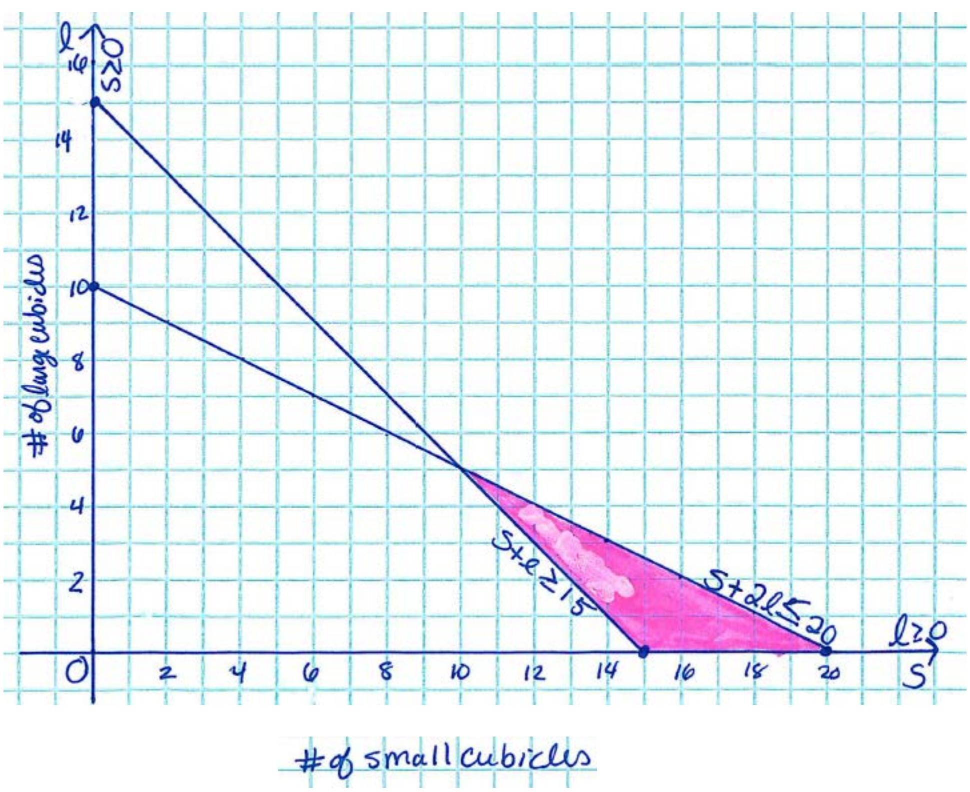
   s = # of small cubicles   *Total number of cubicles:* $s + l \geq 15$

   l = # of large cubicles   *Total number of operators:* $s + 2l \leq 20$

2. Graph the solution set of the system you created. State any additional constraints the model may have.

   Other constraints on the model are that $s \geq 0$ *and* $l \geq 0$ *because there cannot be a negative number of cubicles. The region shaded in pink, including solutions on the boundaries of the region, would represent all of the possible solutions to this system. Also, the solutions can only be whole numbers as there cannot be a fraction of a cubicle.*

   ***Note: Students' responses are dependent upon their response to question 1. Therefore, student work in this section will need to be checked for accuracy based on the inequalities they wrote.*



3. Which combination of small and large cubicles will allow the company to employ the maximum number of operators with the minimum number of cubicles? *Provide evidence to support your response.*

***Note: Student responses are dependent upon the system of inequalities in question 1. If those are incorrect, student approaches to answer this question will need to be assessed based on their previous incorrect responses.*

From the information in the problem, the minimum number of cubicles is 15. The maximum number of operators is 20. The only point in the solution set that satisfies these conditions is (10, 5), which is the point where the boundary lines intersect. This ordered pair represents 10 small cubicles and 5 large cubicles.

$s + l \geq 15$	$s + 2l \leq 20$
$10 + 5 \geq 15$	$10 + 2(5) \leq 20$
$15 \geq 15$	$20 \leq 20$

All other points in the region would be combinations that provide for:

- *exactly 15 cubicles but fewer than 20 operators; or*
- *exactly 20 operators but more than 15 cubicles; or*
- *more than 15 cubicles and fewer than 20 operators.*

4. The first location the company finds is smaller than it would like. The space will only hold 10 cubicles total. If the company still wants to employ up to 20 operators, is it possible in this space to have the maximum number of cubicles and operators with a combination of **both** small and large cubicles? *Provide evidence to support your response.*

s = # of small cubicles *Total of 10 cubicles:* $s + l = 10$

l = # of large cubicles *Total of 20 operators:* $s + 2l = 20$

$-1(s + l) = -1(10) \quad \rightarrow \quad -s - l = -10$

$s + 2l = 20$

By combining these equations: $l = 10$

So there could be 10 large cubicles that would house 20 operators. This means there would be no small cubicles $(s + 10 = 10; s = 0)$*. Therefore, this space will not work because it cannot have the maximum number of cubicles and the maximum number of operators with a combination of **both** large and small cubes.*

5. The location the company decides to use has an 1,800-square-foot open floor plan. The owner of the company also decides to remove the maximum limit from the number of operators he is willing to employ. He wants six fewer small cubicles than large cubicles. Calculate the maximum number of operators that can be employed in the 1,800-square-foot space using six fewer small cubicles than large cubicles. *Provide evidence to support your response.*

s = # of small cubicles

l = # of large cubicles

Total number of cubicles: $s = l - 6$

Total area of cubicles: $36s + 60l = 1800$

$36(l - 6) + 60l = 1800$

$36l - 216 + 60l = 1800$

$96l - 216 = 1800$

$96l = 1800 + 216$

$\frac{96l}{96} = \frac{2016}{96}$

$l = 21$

$s = 21 - 6$

$s = 15$

Number of operators: $s + 2l$

$15 + 2(21) = 57$

So, using all 1,800 square feet, there can be 21 large cubicles and 5 small cubicles. The company would be able to employ 57 operators.

Pool Dimensions (ECR)

Overview

Students will solve quadratic equations to find the dimensions of a pool and a walkway surrounding the pool.

Standards

Create equations that describe numbers or relationships.

HSA-CED.A.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

Solve equations and inequalities in one variable.

HSA-REI.B.4 Solve quadratic equations in one variable.

a. Use the method of completing the square to transform any quadratic equation in *x* into an equation of the form $(x-p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers *a* and *b*.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade-Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSA-CED.A.1	• 7.EE.B.4 • 8.EE.C.7	1. The length of a rectangular field is 5 feet more than its width. If the area of the field is 2500 square feet, write an equation that could be used to find the dimensions of the field. Define any variables used. a. *w* = width of field in feet; length of field is (*w* + 5) feet. An equation to find the dimensions could be $w(w+5) = 2500$ OR $w^2 + 5w = 2500$. **2.** http://www.illustrativemathematics.org/illustrations/581	• http://www.illustrativemathematics.org/illustrations/986 • http://www.illustrativemathematics.org/illustrations/999 • http://learnzillion.com/lessonsets/120-create-equations-and-inequalities-in-one-variable-and-use-them-to-solve-problems
HSA-REI.B.4	• 8.EE.A.2	1. Solve the equation $3(4x+1)(x) = 9$. a. $x = \frac{3}{4}$ or -1	• http://learnzillion.com/lessonsets/24-

Grade-Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
		2. http://www.illustrativemathematics.org/illustrations/618 3. http://www.illustrativemathematics.org/illustrations/586	solve-quadratic-equations

Real-World Preparation: The following discussions will help prepare students for some of the real-world components of this task:

- Discuss with students that the depth of a pool typically slopes down from the shallow end to the deep end. However, for this task, students will ignore the slope as they calculate the dimensions of the pools described.
- Students will need to remember how to find the volume of a rectangular right prism, which is an expectation of 5^{th} and 6^{th} grade students. For those students who may need the additional scaffolding, the volume formula may be provided.

After the Task

Students may struggle with writing the expressions to represent the dimensions of the pool. Provide students with additional practice in translating verbal expressions into algebraic expressions as needed.

When solving the quadratic equations, students may struggle with multiplying polynomials. Provide additional practice with multiplying polynomials and reinforce the distributive property as needed.

When students have the equation written in factored form, for example $70 = (3d - 1)(d)$, they may set each factor equal to the constant. Discuss with students the Zero Product Property and why setting factors equal to other constants will not always work. Provide additional practice with solving quadratic equations by factoring as needed.

For question 3, students may struggle with writing the expressions for the width of the walkway, as well as the expressions for the total width and length of the pool and walkway. Have students draw a diagram of the scenario and label the information they are given. If necessary, provide a diagram for students who may need the additional scaffolding. Discuss with students which portion of the drawing would represent the area they are given. Then have students identify how someone would likely find the area of that region. Additional practice with finding the area of various regions in given diagrams may be necessary.

Student Extended Constructed Response

A hotel is planning to redesign its pool to accommodate a new diving board and waterslide for guests to use. The existing rectangular pool is 28 feet long. The width of the existing pool is one foot less than three times the measure of the pool's depth.

1. The existing pool can hold up to 1,960 cubic feet of water. What are the dimensions of the existing pool? Be sure to define any variables used. Show all work.

2. In order to accommodate the new diving board and waterslide, the hotel management wants the length of the new rectangular pool to be 30 feet. The deep end of the new pool (where the diving board and waterslide will be placed) will be 10 feet deeper than the shallow end. The new pool's width will equal three feet more than double the depth of the shallow half. The new pool will hold 4,950 cubic feet of water. How deep will each half of the pool be? Show your work and be sure to define any variables used.

3. The walkway to be constructed around the new pool will cover an area of 1,054 square feet, excluding the area of the pool. The width of the walkway will be the same all around the pool. There will be new lounge chairs placed on the walkway for guests to use. The width of the walkway is 1.5 feet more than the length of one lounge chair in order to allow space to walk around the pool. What is the length of a single lounge chair? What is the width of the walkway? Show your work and define any variables used.

Task adapted with permission from Universal Achievement, LLC.

Extended Constructed Response Exemplar Response

A hotel is planning to redesign its pool to accommodate a new diving board and waterslide for guests to use. The existing rectangular pool is 28 feet long. The width of the existing pool is one foot less than three times the measure of the pool's depth.

1. The existing pool can hold up to 1,960 cubic feet of water. What are the dimensions of the existing pool? Be sure to define any variables used. Show all work.

   d = the measure of the pool's depth in feet

   Width of the pool = (3d – 1) feet

   Volume of the pool = 1960 cubic feet

   Volume of the pool = length of pool × width of pool × depth of pool

$$1960 = 28(3d - 1)(d)$$

$$1960 = 28(3d^2 - d)$$

$$\frac{1960}{28} = \frac{28(3d^2 - d)}{28}$$

$$70 = 3d^2 - d$$

$$0 = 3d^2 - d - 70$$

$$0 = (3d + 14)(d - 5)$$

$$3d + 14 = 0 \; or \; d - 5 = 0$$

$$d = -\frac{14}{3} \; or \; d = 5$$

   Because d is the depth of the pool in feet, the negative answer is not viable; therefore, the depth of the pool is 5 feet. The width of the pool is 3(5) – 1, which is 14 feet. The length of the pool was given as 28 feet.

   ***Note: This is not the only solution method. Students may also choose to complete the square or use the quadratic formula.*

2. In order to accommodate the new diving board and waterslide, the hotel management wants the length of the new rectangular pool to be 30 feet. The deep end of the new pool (where the diving board and waterslide will be placed) will be 10 feet deeper than the shallow end. The new pool's width will equal three feet more than double the depth of the shallow half. The new pool will hold 4,950 cubic feet of water. How deep will each half of the pool be? Show your work and be sure to define any variables used.

   s = depth of the shallow half of the pool in feet

   s + 10 = depth of the deeper half of the pool in feet

2s + 3 = width of the pool in feet

4950 cubic feet = total volume of the pool

15 feet = the length of each half of the pool (30 feet total ÷ 2)

volume of shallow end + volume of deep end = total volume of the pool

$$15(2s+3)(s)+15(2s+3)(s+10)=4950$$

$$15(2s+3)(s+s+10)=4950$$

$$\frac{15(2s+3)(s+s+10)}{15}=\frac{4950}{15}$$

$$(2s+3)(2s+10)=330$$

$$4s^2+26s+30=330$$

$$4s^2+26s=300$$

$$\frac{4s^2+26s}{4}=\frac{300}{4}$$

$$s^2+\frac{13}{2}s=75$$

$$s^2+\frac{13}{2}s+\frac{169}{16}=75+\frac{169}{16}$$

$$\left(s+\frac{13}{4}\right)^2=\frac{1369}{16}$$

$$s+\frac{13}{4}=\pm\frac{37}{4}$$

$$s=-\frac{13}{4}\pm\frac{37}{4}$$

$$s=\frac{-13+37}{4}\ or\ s=\frac{-13-37}{4}$$

$$s=\frac{24}{4}$$

$$s=6$$

Because s is the depth of the shallow half in feet, the answer cannot be negative. Since the numerator of the second answer (–13 – 37) is negative, that answer is not viable. Therefore, the depth of the shallow half of the pool is 6 feet. The depth of the deeper half of the pool is 16 feet (6 + 10).

***Note: This is not the only solution method. Students may also choose to use the quadratic formula or factor the quadratic equation.*

3. The walkway to be constructed around the new pool will cover an area of 1,054 square feet, excluding the area of the pool. The width of the walkway will be the same all around the pool. There will be new lounge chairs placed on the walkway for guests to use. The width of the walkway is 1.5 feet more than the length of one lounge chair in order to allow space to walk around the pool. What is the length of a single lounge chair? What is the width of the walkway? Show your work and define any variables used.

   l = length of one lounge chair in feet

   l + 1.5 = the width of the walkway in feet

   1054 square feet = total area of the walkway

   Total width of pool plus walkway: 15+2(l + 1.5) or (2l + 18) feet (when simplified)

   Total length of pool plus walkway: 30+2(l + 1.5) or (2l + 33) feet (when simplified)

   Area of the walkway = total area of pool and walkway – area of the pool

$$1054 = (2l + 18)(2l + 33) - (15)(30)$$

$$1054 = 4l^2 + 66l + 36l + 594 - 450$$

$$1054 = 4l^2 + 102l + 144$$

$$0 = 4l^2 + 102l - 910$$

$$l = \frac{-102 \pm \sqrt{102^2 - 4(4)(-910)}}{2(4)}$$

$$l = \frac{-102 \pm \sqrt{24964}}{8}$$

$$l = \frac{-102 \pm 158}{8}$$

$$l = \frac{-102 + 158}{8} \; or \; l = \frac{-102 - 158}{8}$$

$$l = \frac{56}{8}$$

$$l = 7$$

   Because l is the length of one lounge chair in feet, the answer cannot be negative. Since the numerator of the second answer (–102 – 158) is negative, that answer is not viable. Therefore, the length of one lounge chair is 7 feet. The width of the walkway is 8.5 feet.

   ***Note: This is not the only solution method. Students may also choose to complete the square or factor the quadratic equation. Also, students may choose to find the area of four smaller rectangles (around the pool) and four squares (at the corners) and add all of these areas rather than subtracting the pool area from the total area.*

Trophy Boxes (ECR)

Overview

Students will use polynomials to model the volume of boxes used to package trophies.

Standards

Interpret the structure of expressions.

HSA-SSE.A.1 Interpret expressions that represent a quantity in terms of its context. ★

a. Interpret parts of an expression, such as terms, factors, and coefficients.
b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1 + r)^n$ *as the product of P and a factor not depending on P.*

Perform arithmetic operations on polynomials.

HSA-APR.A.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Prior to the Task

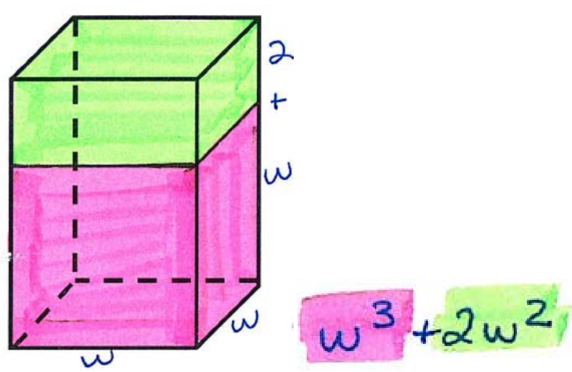
Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade-Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSA-SSE.A.1	• 6.EE.A.2b • 7.EE.A.2	1. The expression $(4x^2 + 6x)$ in.^{2} represents the area of a rectangular region whose width is 2*x* in. Explain the terms of the expression in the context of the area of the region. a. The expression $4x^2$ represents the area of a square region with side lengths of 2*x*. The expression $6x$ represents the area of a rectangular region with a width of 2*x* and a length of 3 in. Together, these regions would be combined to create a rectangular region with an area of $(4x^2 + 6x)$ in.^{2} 2. http://www.illustrativemathematics.org/illustrations/389 3. http://www.illustrativemathematics.org/illustrations/21	• http://www.illustrativemathematics.org/illustrations/1450 • http://learnzillion.com/lessonsets/749-interpret-quadratic-expressions-by-understanding-their-parts • http://learnzillion.com/lessonsets/649-interpret-complicated-expressions-in-context-understanding-the-meaning-of-specific-terms-factors-and-coefficients
HSA-APR.A.1	• 6.EE.A.3 • 6.EE.A.4 • 7.EE.A.1 • 8.EE.A.1	1. Simplify the following: a. $x^3(2x^2 - 3x)$ i. $2x^5 - 3x^4$ b. $(3x + 4)\left(\frac{2}{3}x - 1\right)$ i. $2x^2 - \frac{1}{3}x - 4$	• http://www.illustrativemathematics.org/illustrations/542 • http://www.illustrativemathematics.org/illustrations/461 • http://www.illustrativemathematics.org/illustrations/541

Grade-Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
			• http://www.illustrativemathematics.org/illustrations/395 • http://learnzillion.com/lessonsets/558-understand-that-polynomials-are-closed-under-addition-subtraction-and-multiplication-add-subtract-and-multiply-polynomials • http://learnzillion.com/lessonsets/494-understand-that-polynomials-are-closed-under-addition-subtraction-and-multiplication-perform-these-operations-on-polynomials

After the Task

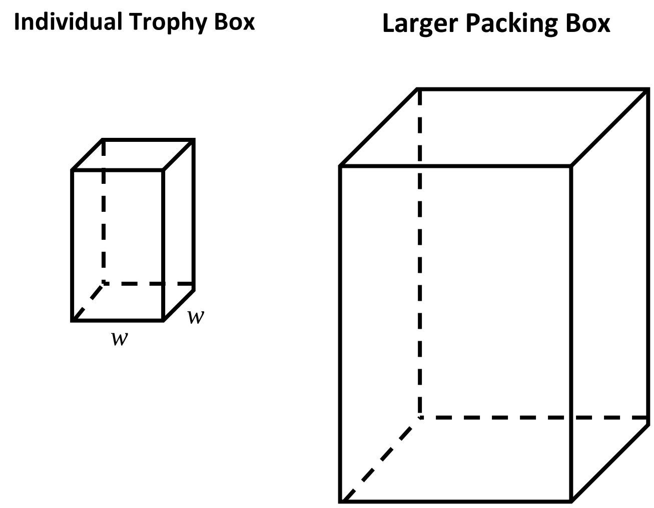
Students will need to remember how to find the volume of a rectangular solid (Grade 6) in order to write the polynomial expressions for the volume of the boxes in the task. When writing a simplified expression for the volume, students might multiply the exponents instead of adding them when multiplying the terms in the polynomials. Provide additional practice as needed with the rules regarding exponents. Students who struggle to interpret the terms of the expression as each relates to the volume of the boxes should be encouraged to draw a diagram and divide the edges of the right rectangular prism into the parts of the expressions they used for length, width, and height. Then they should find the volume of each separate right rectangular prism and relate those expressions to the terms of the expression they found in questions 1 and 2. See the drawing below.



For question 3, students may initially say they can only fit 32 boxes in the packing box since they can only stand two four-inch-tall boxes on top of one another inside of a 10-inch-tall packing box. Have students determine the lengths of the sides of the space that would remain in the box and ask them if they could find a way to fit any additional boxes in the remaining space.

Student Extended Constructed Response

The employees at Trophy Depot are trying to determine how many individual trophy boxes will fit in a larger packing box. Diagrams of the individual trophy box and the larger packing box are shown below. Both the individual trophy box and the larger packing box have square bases and heights that are two inches taller than their widths.



Not drawn to scale

1. Provide a simplified polynomial expression that could be used to model the volume of the individual trophy box. Interpret each term of the simplified expression as it relates to the volume of the individual trophy box.

2. The larger packing box is designed to be as wide as four trophy boxes. Provide a simplified polynomial expression that could be used to model the volume of the larger packing box. Interpret each term of the simplified expression as it relates to the volume of the larger packing box.

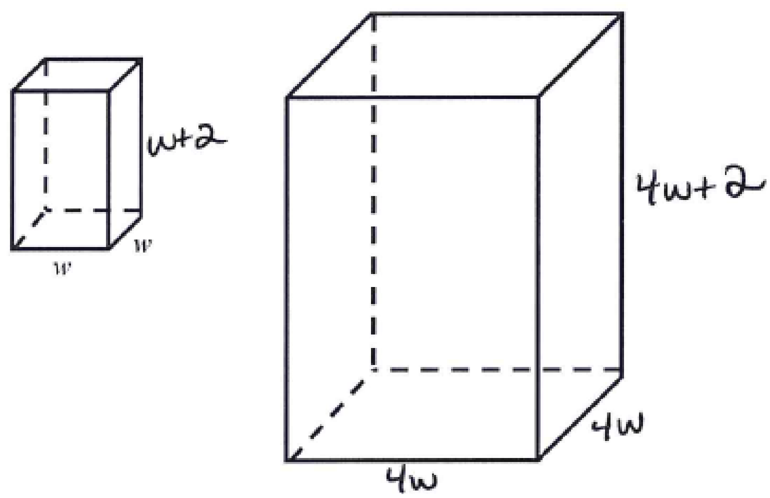
3. If the trophy boxes are three inches wide, what is the greatest number of individual trophy boxes that can fit into one packing box? Explain your reasoning.

Task adapted with permission from Universal Achievement, LLC.

Extended Constructed Response Exemplar Response

The employees at Trophy Depot are trying to determine how many individual trophy boxes will fit in a larger packing box. Diagrams of the individual trophy box and the larger packing box are shown below. Both the individual trophy box and the larger packing box have square bases and heights that are two inches taller than their widths.

Individual Trophy Box **Packing Box**



Not drawn to scale

1. Provide a simplified polynomial expression that could be used to model the volume of the individual trophy box. Interpret each term of the simplified expression as it relates to the volume of the individual trophy box.

   Width = w in.; length = w in.; height = (w + 2) in.

   Volume = area of the base x height

   Volume = $[w^2(w+2)]in^3$

   Volume = $(w^3 + 2w^2)\ in^3$

   Each term of the expression could represent the volume of two smaller right rectangular prisms, which would be stacked on top of each other inside of the trophy box. The term w^3 *represents the volume of a cube with edge lengths of (w) in. The term* $2w^2$ *represents the volume of a right rectangular prism with a base area of* $(w^2)\ in^2$ *and a height of 2 in.*

2. The larger packing box is designed to be as wide as four trophy boxes. Provide a simplified polynomial expression that could be used to model the volume of the larger packing box. Interpret each term of the simplified expression as it relates to the volume of the larger packing box.

   Width = 4w in.; length = 4w in.; height = (4w + 2) in.

   Volume = area of the base x height

   Volume = $[(4w)^2(4w + 2)]in^3$

   Volume = $(64w^3 + 32w^2)\ in^3$

   Each term of the expression could represent the volume of two smaller right rectangular prisms, which would be stacked on top of each other inside of the packing box. The term $64w^3$ *represents the volume of a cube with edge lengths of (4w) in. The term* $32w^2$ *represents the volume of a right rectangular prism with a base area of* $(16w^2)\ in.^2$ *and a height of 2 in.*

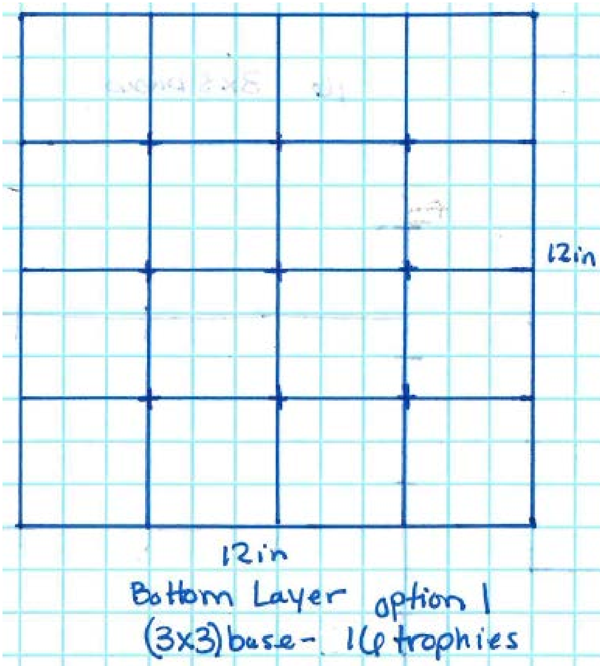
3. If the trophy boxes are three inches wide, what is the greatest number of individual trophy boxes that can fit into one packing box? Explain your reasoning.

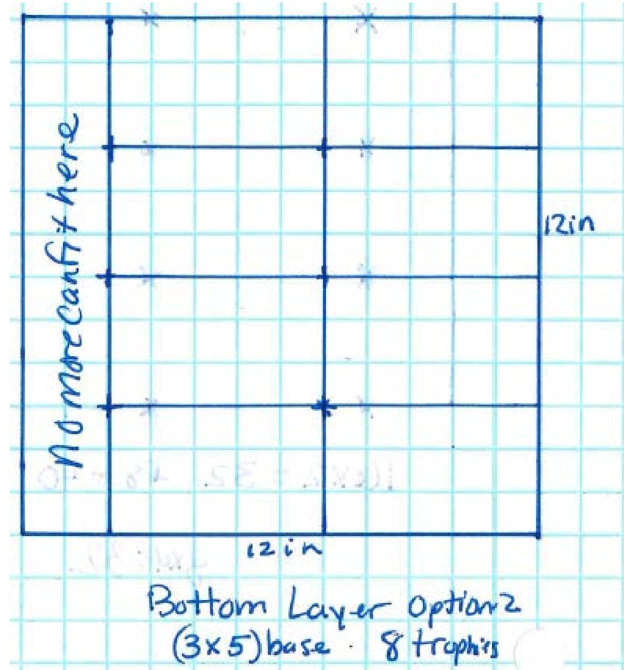
   ***Note: The ability to find the volume of the boxes in order to answer this question does not rely upon the expressions students wrote in the previous parts. If students do use incorrect expressions from questions 1 and 2, their work will need to be checked for correct reasoning and answers based on the incorrect expressions. This is a sample answer; other approaches and explanations are possible.*

$$\frac{Volume\ of\ Packing\ Box}{Volume\ of\ Ind. Trophy\ Box} = \frac{[64(3)^3 + 32(3)^2]in^3}{[(3)^3 + 2(3)^2]in^3} = \frac{2016}{45} = 44.8$$

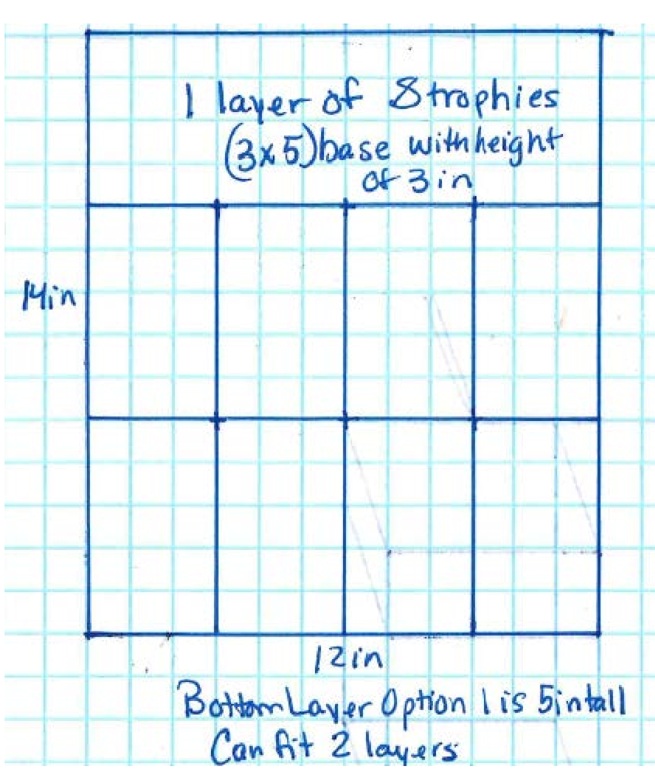
   The volume of the packing box is 44.8 times the volume of one individual trophy box. Therefore, up to 44 individual trophy boxes could fit into the packing box. The length and width of the packing box are each 4(3) = 12 in. The height of the packing box is 4(3) + 2 = 14 in. The individual trophy boxes have a length and width of 3 in and a height of 5 in (3 + 2).

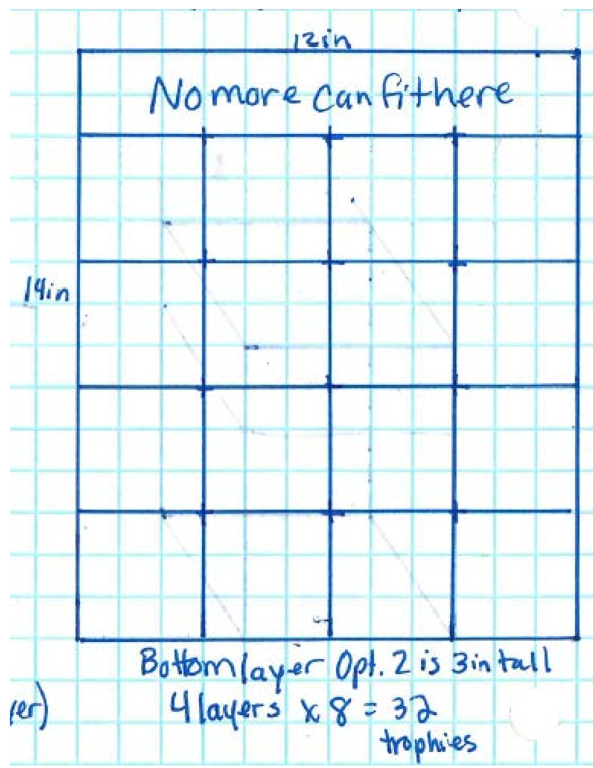
   There are two ways I could arrange the trophies in the box—the individual trophy boxes could be laid horizontally to create a base of 3 in x 5 in for each individual trophy box or they could stand up vertically to create a base of 3 in x 3 in for each individual trophy box. Below are two drawings to show the arrangements. Option 1 would create a bottom layer of 16 trophies, while option 2 would create a bottom layer of 8 trophies.





To find how many layers I can fit into one packing box, I used the height of the packing box. Below are the pictures for each option to determine the number of layers.





Option 1 can fit 2 layers to give 32 trophies (16 x 2). In the space at the top, which is 12 in x 12 in x 4 in, I can fit one layer of trophies lying on their side like Option 2. That would be an additional 8 trophies for a total of 40 trophies that could fit in this box. There would be a space of [(2 in x 12 in x 3 in) + (1 in x 12 in x 12 in)] or 216 cubic inches that would not be able to fit any more trophies.

Option 2 can fit 4 layers of 8 trophies, which would be 32 trophies. However, no additional trophies could fit into the space at the top of the box. Also, no additional trophies would fit into the space on the side. So there would be a space of [(2 in x 12 in x 12 in) +(2 in x 12 in x 12 in)] or 576 cubic inches that would not be able to fit any more trophies.

Therefore, the greatest number of individual trophy boxes that are 3 inches wide that could fit into the larger packing box with a width of 12 inches is 40.

Rate of Change (ECR)

Overview

Students will answer questions concerning average rate of change, maximum height, and function transformations based on a given function.

Standards

Solve equations and inequalities in one variable.

HSA-REI.B.4 Solve quadratic equations in one variable.

b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers *a* and *b*.

Interpret functions that arise in applications in terms of the context.

HSF-IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★

HSF-IF.B.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

Build new functions from existing functions.

HSF-BF.B.3 Identify the effect on the graph of replacing *f*(*x*) by *f*(*x*) + *k*, *k f*(*x*), *f*(*kx*), and *f*(*x* + *k*) for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade-Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSA-REI.B.4b	• 7.EE.A.1 • 8.EE.A.2 • HSA-REI.B.4a	1. Solve the equation $x^2 - 26x + 100 = -20$ for *x*. a. $x = 6$ or $x = 20$	• http://www.illustrativemathematics.org/illustrations/541 • http://www.illustrativemathematics.org/illustrations/1827 • http://learnzillion.com/lessonsets/98-

<table>
<tr><th>Grade-Level Standard</th><th>The Following Standards Will Prepare Them</th><th>Items to Check for Task Readiness</th><th>Sample Remediation Items</th></tr>
<tr><td></td><td></td><td>2. Solve the equation $5x^2 + 10x - 13 = 0$ for x.
a. $x = \frac{-5+3\sqrt{10}}{5}$ or $x = \frac{-5-3\sqrt{10}}{5}$
3. http://www.illustrativemathematics.org/illustrations/375
4. http://www.illustrativemathematics.org/illustrations/586</td><td>solve-quadratic-equations-by-inspection-taking-square-roots-completing-the-square-the-quadratic-formula-and-factoring
• http://learnzillion.com/lessonsets/26-understand-and-choose-methods-to-solve-quadratic-equations</td></tr>
<tr><td>HSF-IF.B.4</td><td>• 8.F.B.5
• HSN-Q.A.1
• HSF-IF.A.1</td><td>1. A shot-put throw can be modeled using the equation $y = -0.0241x^2 + x + 5.5$ where x is distance traveled (in feet) and y is height traveled (in feet). Find the maximum point of this function and explain the meaning of this point.
a. The maximum point is approximately (20.75, 15.87). This means that the maximum height of the shot-put throw was 15.87 feet and that maximum height occurred when the shot put had traveled 20.75 feet.
2. http://www.illustrativemathematics.org/illustrations/637
3. http://www.illustrativemathematics.org/illustrations/639</td><td>• http://www.illustrativemathematics.org/illustrations/628
• http://www.illustrativemathematics.org/illustrations/632
• http://www.illustrativemathematics.org/illustrations/85
• http://www.illustrativemathematics.org/illustrations/588
• http://www.illustrativemathematics.org/illustrations/630
• http://learnzillion.com/lessonsets/477-graph-quadratic-functions-and-show-intercepts-maxima-and-minima
• http://learnzillion.com/lessonsets/470-graph-linear-functions-and-show-intercepts-maxima-and-minima</td></tr>
<tr><td>HSF-IF.B.6</td><td>• 8.F.B.4
• HSF-IF.A.2</td><td>1. The Just for Fun T-shirt company used the function $P(q) = -100 + 0.5q + 0.01q^2$ to determine the profit $P(q)$, in dollars, of selling q T-shirts. Compare the average rate of change between 100 T-shirts and 200 T-shirts to the average rate of change between 200 T-shirts and 300 T-shirts.
a. The average rate of change between 100 and 200 shirts is $3.50 per shirt. The average rate of change between 200 and 300 shirts is $5.50 per shirt. This shows that the amount of profit per shirt made is increasing as more shirts are sold.
2. http://www.illustrativemathematics.or</td><td>• http://www.illustrativemathematics.org/illustrations/120
• http://www.illustrativemathematics.org/illustrations/247
• http://www.illustrativemathematics.org/illustrations/634
• http://www.illustrativemathematics.org/illustrations/626</td></tr>
</table>

Grade-Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
		g/illustrations/577 3. http://www.illustrativemathematics.org/illustrations/1500	
HSF-BF.B.3		1. Given the function $f(x) = 2(x-1)^2 + 3$, write the function $h(x) = f(x) + 2$. a. $h(x) = 2(x-1)^2 + 5$. 2. Given the functions $f(x) = 2(x-1)^2 + 3$ and $h(x) = f(x) - 6$, describe how the graphs of $f(x)$and $h(x)$ are different. a. The *y*-intercept of $f(x)$ is (0, 5). The graph of $h(x)$ will shift the graph of $f(x)$ down 6 units, so the *y*-intercept of $h(x)$ will be (0, –1). 3. http://www.illustrativemathematics.org/illustrations/742 4. http://www.illustrativemathematics.org/illustrations/695	• http://learnzillion.com/lessonsets/766-manipulate-quadratic-functions • http://learnzillion.com/lessonsets/764-manipulate-linear-functions

After the Task

Students who struggle with finding the average rate of change for the given intervals may benefit from additional work with finding the slope of lines given two points. Students need to understand that the average rate of change for non-linear functions is the slope of the line connecting two points on the graph of the given function. Have students graph the quadratic function and identify the points on the graph for the specified intervals. Then draw a line through the two identified points. Students should then be able to find the slope of the line they have drawn. For students who need additional scaffolding, use this technique with quadratic functions (and other nonlinear functions) that have simpler numbers.

Students may also struggle with solving the given equation in order to find when the object hits the ground and find the maximum height. Have students state what information they know about the object hitting the ground in terms of time and height. Additionally, relate the solution and the maximum height to the graph of the function and have students answer the questions from the graph. Connect the graphical solution with the algebraic solution.

Student Extended Constructed Response

An object is launched at an initial velocity of 19.6 meters per second from a 58.8-meter-tall platform. This situation can be modeled by the function $h(t) = -4.9t^2 + 19.6t + 58.8$ where t represents the time, in seconds, that the object is in motion, and $h(t)$ is the height of the object.

1. Find the average rate of change from 0 seconds to 1 second. Find the average rate of change from 1 second to 2 seconds. Explain what each average rate of change means based on the problem. Compare the two average rates of change to explain what is happening with the object.

2. Find the average rate of change from 3 seconds to 5 seconds and explain its meaning.

3. What is the maximum height of the object? When does the object reach its maximum height?

4. When does the object hit the ground? How do you know?

5. What is the average rate of change between 1 second and 3 seconds? Why?

6. How would the function and graph of the function change if the object were launched from a platform that is 63.8 meters tall? Explain your reasoning.

Extended Constructed Response Exemplar Response

An object is launched at an initial velocity of 19.6 meters per second from a 58.8-meter-tall platform. This situation can be modeled by the function $h(t) = -4.9t^2 + 19.6t + 58.8$ where *x* represents the time, in seconds, that the object is in motion, and $h(t)$ is the height of the object.

1. Find the average rate of change from 0 seconds to 1 second. Find the average rate of change from 1 second to 2 seconds. Explain what each average rate of change means based on the problem. Compare the two average rates of change to explain what is happening with the object.

   $\frac{h(1)-h(0)}{1-0} = \frac{73.5-58.8}{1} = 14.7$ *meters per second. This means that the object is moving upward (because it's positive) at an average rate of 14.7 meters per second on this time interval.*

   $\frac{h(2)-h(1)}{2-1} = \frac{78.4-73.5}{1} = 4.9$ *meters per second. This means that the object is moving upward at an average rate of 4.9 meters per second on this time interval.*

   The average rate of change during the second interval (1 second to 2 seconds) is slower than the first interval (0 seconds to 1 second) because as the object is traveling upward it will start to slow down as it reaches its maximum height and begins its descent. It is still traveling upward because the average rate of change is positive for both intervals.

2. Find the average rate of change from 3 seconds to 5 seconds and explain its meaning.

   $\frac{h(5)-h(3)}{5-3} = \frac{34.3-73.5}{2} = -19.6$ *meters per second. This means the object is falling at an average rate of 19.6 meters per second. It is falling because the rate is negative.*

3. What is the maximum height of the object? When does the object reach its maximum height?

   The maximum height of the object is reached at 2 seconds and will be located at 78.4 meters.

4. When does the object hit the ground? How do you know?

   The object will hit the ground at 6 seconds. The object is on the ground when h(t) = 0 since h(t) represents the height of the object. So we solve the quadratic equation when set equal to zero, which means to find the values of t that would make the equation true. There are two answers: 6 and –2. Since t represents time in seconds and time cannot be negative, the answer must be 6 seconds.

5. What is the average rate of change between 1 second and 3 seconds? Why?

   The average rate of change between 1 second and 3 seconds is 0 meters per second because the object reached its maximum height at 2 seconds, which means it traveled the same distance from 1 to 2 seconds as it did from 2 to 3 seconds, only in opposite directions. Also, if a line were drawn such that it passes through the two points (1, 73.5) and (3, 73.5), it would be a horizontal line as the h(t) values are the same. Horizontal lines have a slope of zero, and the average rate of change between two points is the slope of the line that passes through the two points.

6. How would the function and graph change if the object were launched from a platform that is 63.8 meters tall? Explain your reasoning.

   The function would become $h(t) = -4.9t^2 + 19.6t + 63.8$ *since the initial height of the object now changed. This shifts the graph of the function up by 5 units because the platform is 5 meters higher than the original. The maximum height would then be 83.4 meters. The time the object hits the ground would be approximately 6.13 seconds.*