Constructions and Proof (ECR)

Overview

Students will perform basic constructions and prove two lines are parallel.

Standards

Prove geometric theorems.

HSG-CO.C.9 Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly equidistant from the segment's endpoints.*

Make geometric constructions.

HSG-CO.D.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSG-CO.C.9	• 4.MD.C.7 • 7.G.B.5 • 8.G.A.5	1. Explain how you would show that two lines cut by a transversal are parallel. a. To show that two lines, which are cut by a transversal, are parallel, show that (a) corresponding angles are congruent; or (b) alternate interior angles are congruent; or (c) alternate exterior angles are congruent; or (d) consecutive interior angles are supplementary. **2.** http://www.illustrativemathematics.org/illustrations/967	• http://www.illustrativemathematics.org/illustrations/59 • http://www.illustrativemathematics.org/illustrations/56 • http://www.illustrativemathematics.org/illustrations/1501 • http://www.illustrativemathematics.org/illustrations/1503

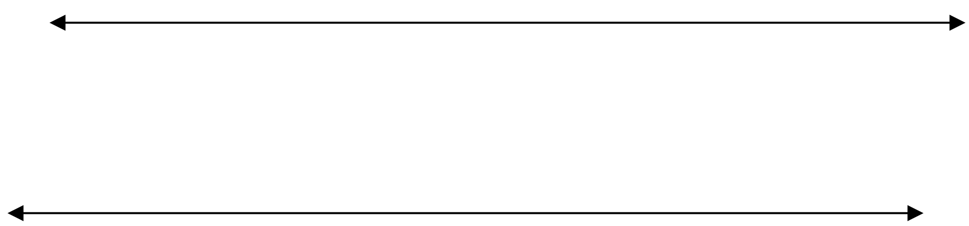
Grade Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSG-CO.D.12	• 4.MD.C.6 • 7.G.A.2	1. http://www.illustrativemathematics.org/illustrations/1320 2. http://www.illustrativemathematics.org/illustrations/966	• http://www.illustrativemathematics.org/illustrations/909

After the Task

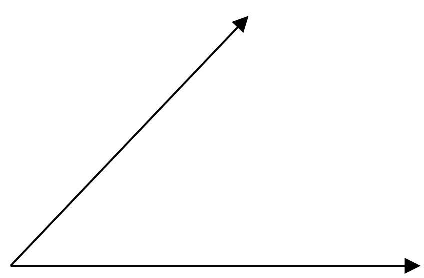
- ***For problem 1***, students are able to draw any transversal they choose and should use a protractor to measure the angles. Students may need assistance with using a protractor or remembering the different ways to show that two lines are parallel.
- ***For problems 2-4***, students should use a compass and a straightedge to complete the constructions. Students who are struggling can visit the websites listed in the exemplar response to see how the constructions are performed and practice them with new segments and angles. Students should be discouraged from using a ruler or protractor to measure segments or angles in order to make the constructions.
- ***For problem 4***, students may fail to see the 90° triangle that is formed. Students should be encouraged to label the figure throughout the construction to mark congruent segments, angles, or the measures of angles they know. Provide students with additional practice with constructions throughout the year.

Student Extended Constructed Response

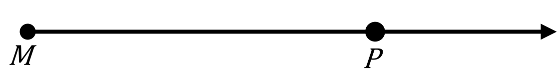
1. Using the following diagram, draw a transversal. Use the transversal and a protractor to explain why these lines are parallel.



2. Copy the angle below. Then, bisect the copied angle.



3. Construct $\angle M$ given $\overrightarrow{MP}$ such that $\angle M$ is a right angle.



4. Given $\overline{BC}$, perform the following constructions and answer the question.

    a. Construct the perpendicular bisector of $\overline{BC}$. Label the midpoint of $\overline{BC}$ as *M*.

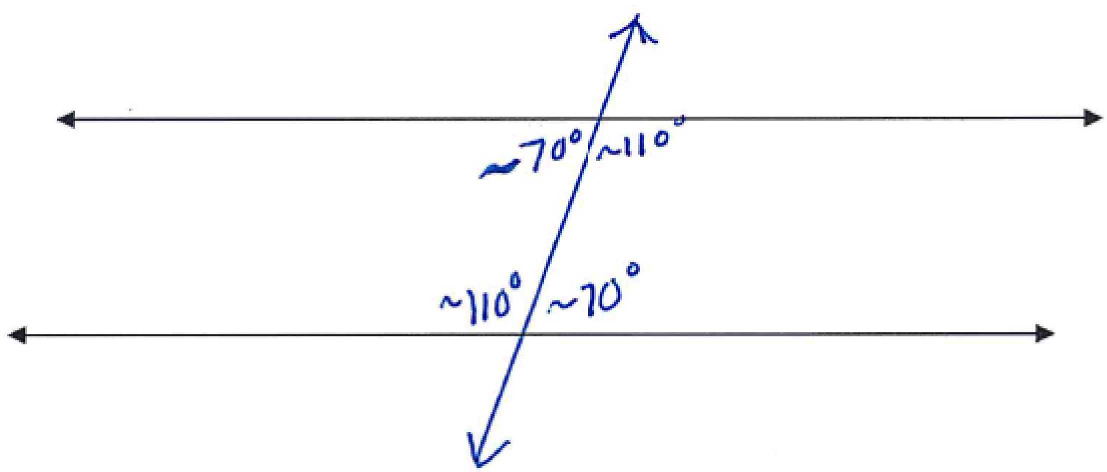
    b. Construct $\overline{MP}$ so that the length of $\overline{MP}$ is equal to the length of $\overline{BM}$ and so that $\overline{MP} \perp \overline{BC}$.

    c. Draw a line connecting *B* and *P*.

    What is the measure of $\angle PBM$? Explain your reasoning.

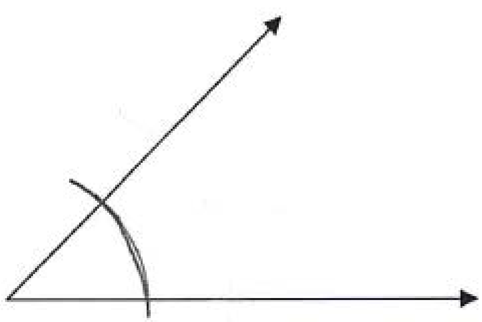
Extended Constructed Response Exemplar Response

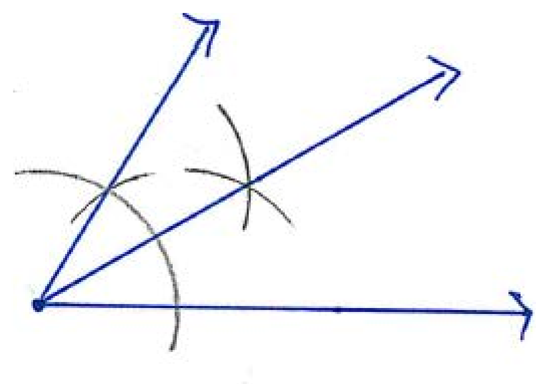
1. Using the following diagram, draw a transversal. Use the transversal and a protractor to explain why these lines are parallel.



After drawing in the transversal, I measured the angles and found the measures of the angles shown. Since the alternate interior angles have the same measure, they are congruent. If two lines are cut by a transversal such that the alternate interior angles are congruent, then the lines are parallel. Thus, the lines above are parallel.

2. Copy the angle below. Then, bisect the copied angle.

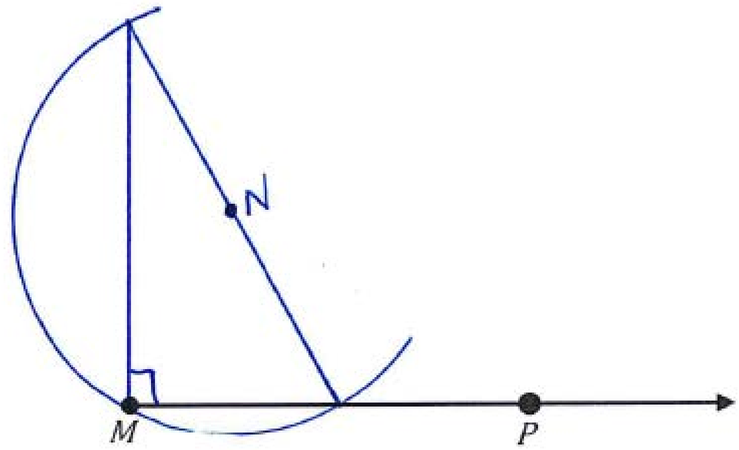




Teacher Note: For the steps to copy an angle, visit http://www.mathopenref.com/printcopyangle.html.

For the steps to bisect an angle, visit http://www.mathopenref.com/printbisectangle.html.

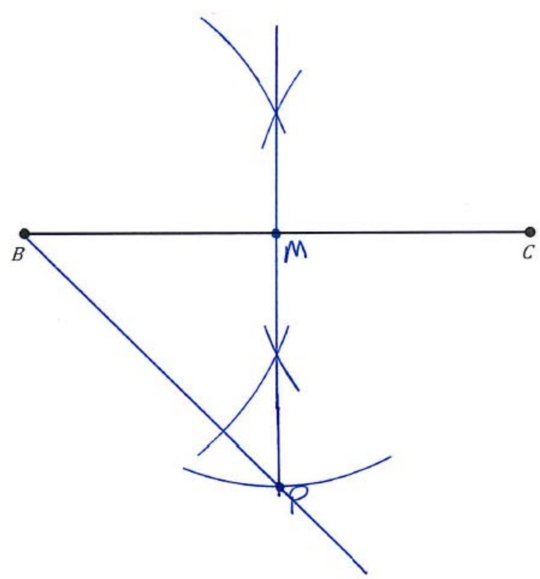
3. Construct $\angle M$ given $\overrightarrow{MP}$ such that $\angle M$ is a right angle.



Teacher Note: For the steps to construct a perpendicular at the endpoint of a ray, visit http://www.mathopenref.com/constperpendray.html.

4. Given $\overline{BC}$, perform the following constructions and answer the question.
   a. Construct the perpendicular bisector of $\overline{BC}$. Label the midpoint of $\overline{BC}$ as *M*.
   b. Construct $\overline{MP}$ so that the length of $\overline{MP}$ is equal to the length of $\overline{BM}$ and so that $\overline{MP} \perp \overline{BC}$.
   c. Draw a line connecting *B* and *P*.

What is the measure of $\angle PBM$? Explain your reasoning.



The measure of $\angle PBM$ is 45°. Triangle BMP is an isosceles right triangle because the length of $\overline{MP}$ is the same as the length of $\overline{BM}$ and the two segments are perpendicular. In an isosceles right triangle, the acute angles are both 45°.

Teacher Note: For the steps to construct a 45-degree angle, visit http://www.mathopenref.com/constangle45.html.

Equations of Circles (ECR)

Overview

Using information about two circles, students will derive the equations of the circles. These equations will be used to prove that the two circles are similar and to prove or disprove that a point lies on a circle.

Standards

Understand and apply theorems about circles.

HSG-C.A.1 Prove that all circles are similar.

Translate between the geometric description and the equation for a conic section.

HSG-GPE.A.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

Use coordinates to prove simple geometric theorems algebraically.

HSG-GPE.B.4 Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).*

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSG-C.A.1	• 7.G.B.4 • HSG-SRT.A.2	1. Draw two circles on your page. Are they similar? a. Yes; all circles are similar. 2. http://www.illustrativemathematics.org/illustrations/1368	• http://www.illustrativemathematics.org/illustrations/34 • http://www.illustrativemathematics.org/illustrations/603
HSG-GPE.A.1	• 8.G.B.8 • HSA-REI.B.4 • HSG-SRT.B.4 • HSG-SRT.C.8	1. What is the equation of a circle with center (-2, 7) and a radius of 4? a. $(x+2)^2+(y-7)^2=16$ 2. http://www.illustrativemathematics.org/illustrations/1425	• http://www.illustrativemathematics.org/illustrations/375 • http://www.illustrativemathematics.org/illustrations/1568 • http://www.illustrativemathematics.org/illustrations/1095 • http://www.illustrativemathematics.org/illustrations/962

Grade Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
			• http://learnzillion.com/lessonsets/763-derive-the-equation-of-a-circle-using-the-pythagorean-theorem • http://learnzillion.com/lessonsets/724-derive-the-equation-of-a-circle-and-complete-the-square-to-find-the-center-and-radius • http://learnzillion.com/lessonsets/28-understand-and-use-equations-for-circles
HSG-GPE.B.4	• 8.G.B.8 • HSG-GPE.B.5	1. Does the point (2, 22) lie on the parabola with equation $y = x^2 + 5x + 8$? a. Yes 2. http://www.illustrativemathematics.org/illustrations/605	• http://www.illustrativemathematics.org/illustrations/1556 • http://www.illustrativemathematics.org/illustrations/1347 • http://www.illustrativemathematics.org/illustrations/1348 • http://learnzillion.com/lessons/286-prove-whether-a-point-is-on-a-circle

After the Task

The following list identifies possible student difficulties and suggestions for helping students by problem number:

- Students may struggle with completing the square. They may need to be reminded to move 56 to the other side of the equation. They may also stop when they encounter fractions. Remind them that equations of circles can have fractions in them.
- Encourage students to draw a picture before attempting to find the equation.
- Students may solve this problem using their equations from part 2 or using the Pythagorean Theorem.
- Students may only say that all circles are similar. Encourage them to explain how they know that all circles are similar.

Student Extended Constructed Response Task

James has two circles, Circle A and Circle B. Use the information given below to answer the questions that follow.

Circle A:

Equation: $100x^2 + 100y^2 - 100x + 240y - 56 = 0$

Circle B:

$center$ $(0, -4)$ with a radius of 3

1. Complete the square in order to find the center and radius of Circle A. Show your work and state the center and radius.

2. Derive the equation of Circle B using the Pythagorean Theorem. Explain how you found your equation.

3. Does the point $(\sqrt{8}, -3)$ lie on Circle B? Support your answer with evidence.

4. Are Circle A and Circle B similar? Prove your answer.

Extended Constructed Response Exemplar Response

James has two circles, Circle A and Circle B. Use the information given below to answer the questions that follow.

Circle A:

Equation: $100x^2 + 100y^2 - 100x + 240y - 56 = 0$

Circle B:

$center$ $(0, -4)$ with a radius of 3

1. Complete the square in order to find the center and radius of Circle A. Show your work and state the center and radius.

$$100x^2 + 100y^2 - 100x + 240y - 56 = 0$$
$$100x^2 - 100x + 100y^2 + 240y = 56$$
$$x^2 - x + y^2 + \frac{12}{5}y = \frac{14}{25}$$
$$\left(x^2 - x + \frac{1}{4}\right) + \left(y^2 + \frac{12}{5}y + \frac{36}{25}\right) = \frac{14}{25} + \frac{1}{4} + \frac{36}{25}$$
$$\left(x - \frac{1}{2}\right)^2 + \left(y + \frac{6}{5}\right)^2 = \frac{9}{4}$$
$$\left(x - \frac{1}{2}\right)^2 + \left(y - \left(-\frac{6}{5}\right)\right)^2 = \left(\frac{3}{2}\right)^2$$

The center of Circle A is $\left(\frac{1}{2}, -\frac{6}{5}\right)$, and the radius of Circle A is $\frac{3}{2}$ units.

2. Derive the equation of Circle B using the Pythagorean Theorem. Explain how you found your equation.

$center$: $(0, -4)$ with a radius of 3

The center of the circle is at (0, -4). Label a point (x, y) on the circle.

From the center, you would move over to x. The length of this leg of the triangle would be $|x - 0|$ or x. You would move from -4 to y, so the length of the second leg would be $|y - -4|$ or $y + 4$.

Connecting this point, (x, y), to the center on the circle, (0, -4), would form the hypotenuse of a right triangle.

Using the Pythagorean Theorem,

$$(x)^2 + (y + 4)^2 = 3^2$$
$$(x)^2 + (y + 4)^2 = 9$$

3. Does the point $(\sqrt{8}, -3)$ lie on Circle B? Support your answer with evidence.

Yes, the point $(\sqrt{8}, -3)$ does lie on Circle B.

$$(x)^2 + (y + 4)^2 = 9$$
$$\left(\sqrt{8}\right)^2 + (-3 + 4)^2 = 9$$
$$\left(\sqrt{8}\right)^2 + (1)^2 = 9$$
$$8 + 1 = 9$$
$$9 = 9$$

Alternate solution:

$center$: $(0, -4)$ *with a radius of 3*

The center of the circle is at (0, -4). Label a point $(\sqrt{8}, -3)$ on the circle.

From the center, you would move over to $\sqrt{8}$. The length of this leg of the triangle would be $\left|\sqrt{8} - 0\right|$ or $\sqrt{8}$. You would move from -4 to -3, so the length of the second leg would be $|-3 - -4|$ or 1.

Connecting this point, $(\sqrt{8}, -3)$, to the center on the circle, (0, -4), would form the hypotenuse of a right triangle.

Using the Pythagorean Theorem,

$$\left(\sqrt{8}\right)^2 + (1)^2 = 3^2$$
$$8 + 1 = 9$$
$$9 = 9$$

Yes, the point $(\sqrt{8}, -3)$ does lie on Circle B.

Note: If a student's answer is correct based on an incorrect equation in part 2, then this part should be marked correct.

4. Are Circle A and Circle B similar? Prove your answer.

Yes, Circle A and Circle B are similar (all circles are similar). A translation and a dilation transform Circle A to Circle B. A translation of $\left(x - \frac{1}{2}, y - \frac{14}{5}\right)$ and a dilation by a scale factor of 2 maps Circle A onto Circle B.

Note: If a student's answer is correct based on an incorrect equation in part 1, then this part should be marked correct.

Trigonometric Ratios (ECR)

Overview

This task requires students to decide if right triangles are similar and then develop a conceptual understanding of trigonometric ratios for acute angles of right triangles using properties of similarity.

Standards

Understand similarity in terms of similarity transformations.

HSG-SRT.A.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

Define trigonometric ratios and solve problems involving right triangles.

HSG-SRT.C.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

HSG-SRT.C.7 Explain and use the relationship between the sine and cosine of complementary angles.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade Level Standards	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSG-SRT.A.2	• 8.G.A.4 • 8.G.B.8 • HSG-SRT.A.1	1. Triangle ABC has coordinates $A(-3,-1), B(-5,-4)$, and $C(-$. Triangle XYZ has coordinates $X(4,5),\ Y(2,2)$ and $Z(5,1)$. Are Triangle ABC and Triangle XYZ similar? Explain your reasoning. a. Yes. Triangle ABC is translated seven units to the right and six units up. A translation is a rigid transformation, so it preserves both congruence and similarity. 2. http://www.illustrativemathematics.org/illustrations/603	• http://www.illustrativemathematics.org/illustrations/602 • http://www.illustrativemathematics.org/illustrations/1556 • http://learnzillion.com/lessonsets/774-decide-if-two-figures-are-similar-and-explain-the-meaning-of-triangle-similarity

Grade Level Standards	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSG-SRT.C.6	• HSG-SRT.A.2	1. When two triangles are similar triangles, what do you know about the ratios of their corresponding side lengths? a. You know that they are equivalent. 2. http://www.illustrativemathematics.org/illustrations/1635	• http://www.illustrativemathematics.org/illustrations/603 • http://learnzillion.com/lessonsets/767-understand-side-ratios-in-right-triangles-are-properties-of-the-angles-define-trigonometric-ratios-for-acute-angles-of-a-right-triangle • http://learnzillion.com/lessonsets/524-relate-side-ratios-to-angles-and-derive-trigonometric-ratios
HSG-SRT.C.7	• 7.G.B.5 • HSG-SRT.C.6	1. In a figure, angle A measures 30°. What would be the measure of an angle complementary to angle A? Explain your reasoning. a. 60° because complementary angels are angles whose sum is 90°. Therefore, 90 - 30 = 60. 2. http://www.illustrativemathematics.org/illustrations/1443	• http://learnzillion.com/lessonsets/768-relate-the-sine-and-cosine-of-complementary-angles • http://learnzillion.com/lessonsets/768-relate-the-sine-and-cosine-of-complementary-angles

After the Task

To complete various portions of this task, students will need to find the length of the sides of the given triangles. Students may struggle with finding the length of each side of the triangles. They might count the length of the legs of the right triangles (horizontal and vertical sides) and use the Pythagorean Theorem or the distance formula to find the hypotenuse of each right triangle, or they may use the Pythagorean Theorem or the distance formula to find the length of all sides.

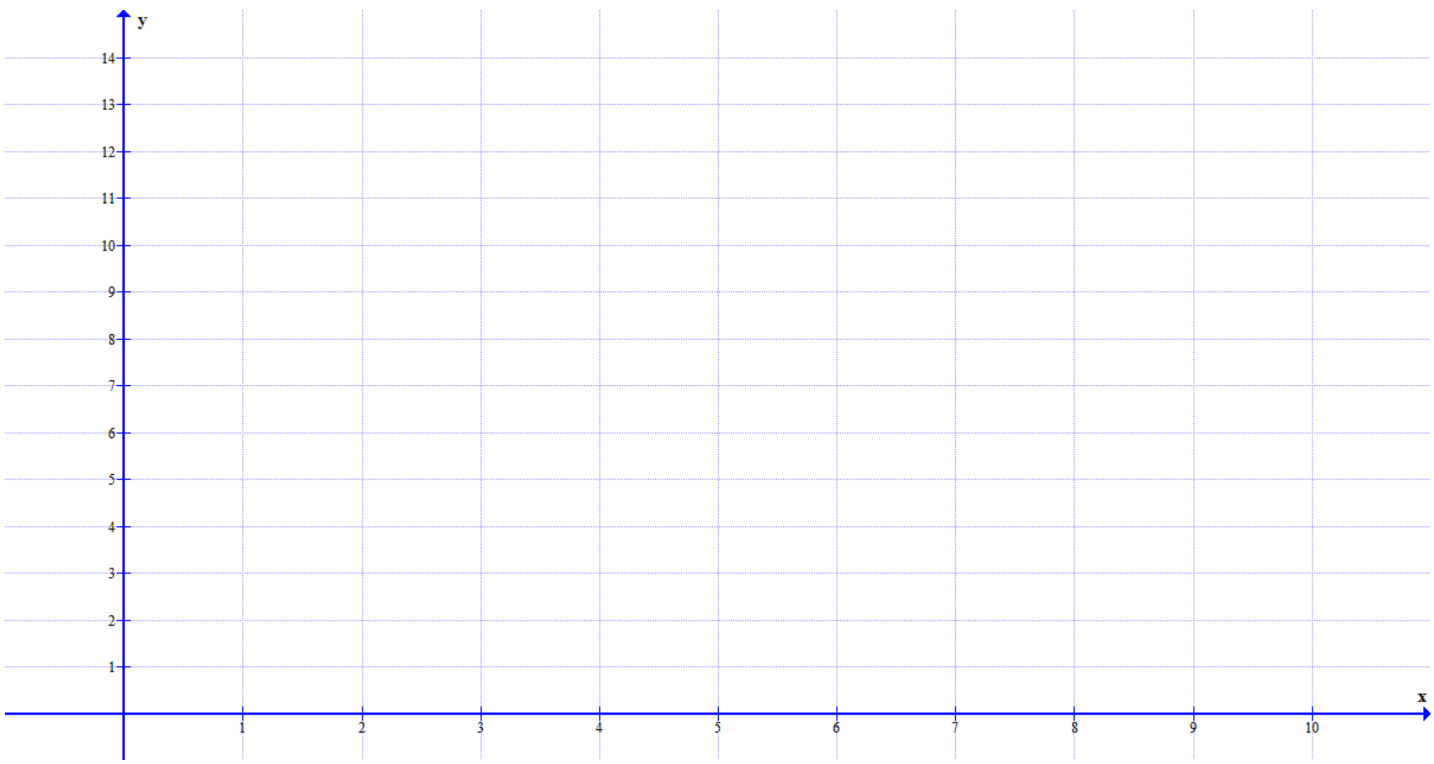
Student Extended Constructed Response

Plot the following triangles on the graph paper below. Label all vertices.

$Triangle\ ABC$: $A(2, 2)$, $B(5, 2)$, and $C(2, 7)$

$Triangle\ XYZ$: $X(4, 4)$, $Y(10, 4)$, and $Z(4, 14)$

$Triangle\ PQR$: $P(1, 1)$, $Q(2.5, 1)$, and $R(1, 3.5)$



Use your graph as needed to answer the following questions.

1. Are Triangle ABC and Triangle XYZ similar? Support your statement with evidence.

2. Are Triangle XYZ and Triangle PQR similar? Support your statement with evidence.

3. Find the sine of $\angle B$ and the sine of $\angle Y$. Write your answers as fractions. What do you notice about the sine values of these two angles? Explain your observation using properties of similarity.

4. Find the cosine of $\angle C$. Write your answer as a fraction. Compare the cosine of $\angle C$ to the sine of $\angle B$. Explain the comparison using properties of similarity or definitions/properties of trigonometric ratios.

5. Using your work from parts 2 and 3, find the cosine of $\angle Z$. Explain your reasoning using properties of similarity or definitions/properties of trigonometric ratios.

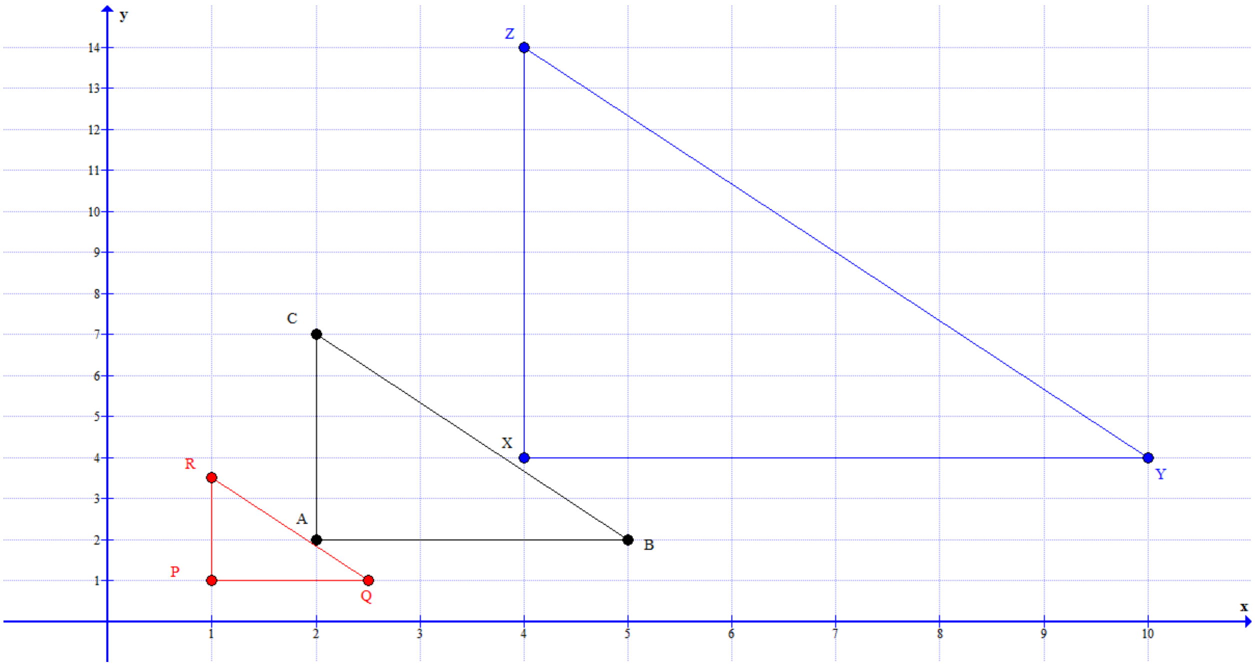
Extended Constructed Response Exemplar Response

Plot the following triangles on the graph paper below. Label all vertices.

$Triangle\ ABC$: $A(2,2)$, $B(5,2)$, and $C(2,7)$

$Triangle\ XYZ$: $X(4,4)$, $Y(10,4)$, and $Z(4,14)$

$Triangle\ PQR$: $P(1,1)$, $Q(2.5,1)$, and $R(1,3.5)$



Use your graph as needed to answer the following questions.

1. Are Triangle ABC and Triangle XYZ similar? Support your statement with evidence.

   Yes, the triangles are similar. One way that I can tell that the triangles are similar is by looking at the ordered pairs. If I make Triangle ABC the pre-image, then Triangle XYZ is a dilation with the origin as center with a scale factor of 2. I know that dilations are a similarity transformation, so Triangles ABC and XYZ are similar.

   Alternate explanation:

   I can tell that the three triangles are similar using side lengths and angle measures.

$$\frac{AB}{XY} = \frac{AC}{XZ}$$

$$\frac{3}{6} = \frac{5}{10}$$

$$\frac{1}{2} = \frac{1}{2}$$

Angle A and Angle X are both 90°, so they are congruent. Triangle ABC is similar to Triangle XYZ by SAS Similarity.

2. Are Triangle XYZ and Triangle PQR similar? Support your statement with evidence.

   Yes, the triangles are similar. One way that I can tell that the triangles are similar is by looking at the ordered pairs. If I make Triangle XYZ the pre-image, Triangle PQR is a dilation with the origin as center with a scale factor of $\frac{1}{4}$*. I know that dilations are a similarity transformation, so Triangles XYZ and PQR are similar.*

   Alternate explanation:

$$\frac{XY}{PQ} = \frac{XZ}{PR}$$

$$\frac{6}{1.5} = \frac{10}{2.5}$$

$$4 = 4$$

   Angle X and Angle P are both 90°, so they are congruent. Triangle XYZ is similar to Triangle PQR by SAS Similarity.

3. Find the sine of $\angle B$ and the sine of $\angle Y$. Write your answers as fractions. What do you notice about the sine values of these two angles? Explain your observation using properties of similarity.

$$\sin B = \frac{5}{\sqrt{34}}$$

$$\sin Y = \frac{10}{\sqrt{136}}$$

$$= \frac{10}{2\sqrt{34}}$$

$$= \frac{5}{\sqrt{34}}$$

   The sine values of these two angles are the same. This occurs because $\angle B$ *and* $\angle Y$ *are corresponding angles in similar triangles. Therefore, the ratios* $\frac{AC}{BC}$ and $\frac{XZ}{YZ}$ *are equal. This is verified because when these expressions are evaluated using the side lengths, they both equal* $\frac{5}{\sqrt{34}}$.

4. Find the cosine of $\angle C$. Write your answer as a fraction. Compare the cosine of $\angle C$ to the sine of $\angle B$. Explain the comparison using properties of similarity or definitions/properties of trigonometric ratios.

   $\cos C = \frac{5}{\sqrt{34}}$

   The cosine of $\angle C$ *is the same as the sine of* $\angle B$*. The sine and cosine of complementary angles are the same because in a right triangle, the opposite side of one complementary angle is the adjacent side of the other complementary angle.*

   $\sin\theta = \frac{opposite}{hypotenuse}$ *and* $\cos\theta = \frac{adjacent}{hypotenuse}$

Since the opposite and adjacent sides are the same sides for complementary angles, then $\sin\theta = \cos(90 - \theta)$*.*

5. Using your work from parts 2 and 3, find the cosine of $\angle Z$. Explain your reasoning using properties of similarity or definitions/properties of trigonometric ratios.

   The cosine of $\angle Z$ *will be* $\frac{5}{\sqrt{34}}$ *because the cosine of* $\angle C$ *is* $\frac{5}{\sqrt{34}}$*. These two angles are corresponding angles in similar triangles, so they will have the same cosine values.*

   $$\cos\theta = \frac{adjacent}{hypotenuse}$$

   Since these two angles are corresponding angles of similar triangles, then we know that the corresponding sides of the two triangles are proportional, so $\frac{XZ}{YZ} = \frac{AC}{BC}$, so *cosine of* $\angle Z$= *cosine of* $\angle C$.

Similar Triangles (ECR)

Overview

This task requires that students determine the similarity of figures using given information.

Standards

Understand similarity in terms of similarity transformations.

HSG-SRT.A.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade Level Standards	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSG-SRT.A.2	• 8.G.A.4 • HSG-SRT.A.1	1. Explain what you know about the corresponding pairs of angles and sides in similar triangles. a. In similar triangles, all pairs of corresponding angles are congruent, and all pairs of corresponding sides are proportional.	• http://www.illustrativemathematics.org/illustrations/602 • http://learnzillion.com/lessonsets/774-decide-if-two-figures-are-similar-and-explain-the-meaning-of-triangle-similarity

During the Task

In this problem, students are given a figure in which two triangles appear to be similar, but their similarity cannot be proven without further information. Asking students to provide a sequence of similarity transformations that maps one triangle to the other focuses them on the work of standard HSG-SRT.A.2, using the definition of similarity in terms of similarity transformations. Teachers will need to remind students to show that their sequences of similarity transformations have the intended effect; that is, that all parts of one triangle get mapped to the corresponding parts of the other triangle.

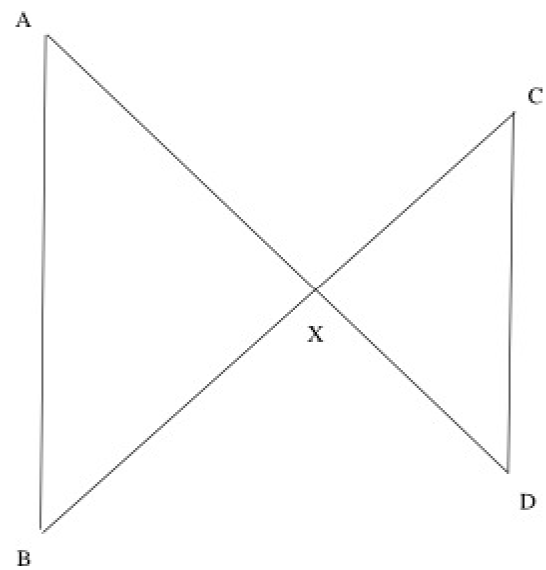
As noted in the solution, almost any attempt to draw a counterexample works. But on this part, a teacher may have students observe that they do not seem to have enough information to prove similarity, rather than showing that it is impossible to prove similarity.

After the Task

After establishing similarity through the use of transformations, ask students to prove or disprove that the triangles are similar for each problem using properties of parallel lines and the definition of similarity. This shifts the focus of this task from G-SRT.2 to G-SRT.5.

Student Extended Constructed Response

In the picture given below, $\overline{AD}$ and $\overline{BC}$ intersect at *X*. $\overline{AB}$ and $\overline{CD}$ are drawn, forming two triangles, *AXB* and *CXD*.



In each part (a-d) below, some additional assumptions about the picture are given. In each problem, determine whether the given assumptions are enough to prove that the two triangles are similar; and if so, what the correct correspondence of vertices is. If the two triangles must be similar, prove this result by describing a sequence of similarity transformations that maps one triangle to the other. If not, explain why not.

a. The lengths *AX* and *XD* satisfy the equation 2*AX* = 3*XD*.

b. The lengths *AX, BX, CX*, and *DX* satisfy the equation $\frac{AX}{BX} = \frac{DX}{CX}$.

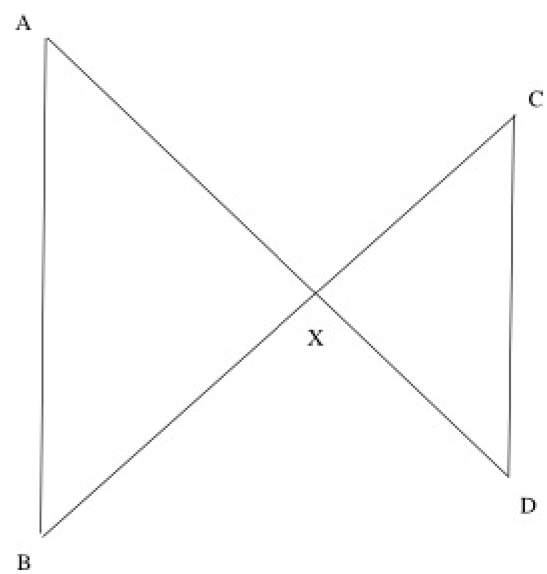
c. $\overline{AB}$ and $\overline{CD}$ are parallel.

d. $\angle XAB \cong \angle XCD$.

Task adapted from http://www.illustrativemathematics.org/illustrations/603.

Extended Constructed Response Exemplar Response

In the picture given below, $\overline{AD}$ and $\overline{BC}$ intersect at *X*. $\overline{AB}$ and $\overline{CD}$ are drawn, forming two triangles, *AXB* and *CXD*.



In each part (a-d) below, some additional assumptions about the picture are given. In each problem, determine whether the given assumptions are enough to prove that the two triangles are similar; and if so, what the correct correspondence of vertices is. If the two triangles must be similar, prove this result by describing a sequence of similarity transformations that maps one triangle to the other. If not, explain why not.

a. The lengths *AX* and *XD* satisfy the equation 2*AX* = 3*XD*.

   I am given that 2AX = 3XD. This is not enough information to prove similarity. To see that in a simple way, I can draw an arbitrary triangle ⊿AXB. Extend AX and choose a point D on the extended line so that 2AX = 3XD. Extend BX and choose a point C on the extended line so that 2BX = XC. Now triangles AXB and CXD satisfy the given conditions but are not similar.

b. The lengths *AX, BX, CX*, and *DX* satisfy the equation $\frac{AX}{BX} = \frac{DX}{CX}$.

   I am given that $\frac{AX}{BX} = \frac{DX}{CX}$. *Rearranging this proportion gives* $\frac{AX}{DX} = \frac{BX}{CX}$. *Let* $k = \frac{AX}{DX}$. *Suppose I rotate the triangle DXC 180 degrees about point X: Since AD is a straight line, DX and AX align upon rotation of 180 degrees, as do CX and BX, and so* $\angle DXC$ *and* $\angle AXB$ *coincide after this rotation. Then dilate the Triangle DXC by a factor of k about the center X. This dilation moves the point D to A, since k(DX) = AX, and moves C to B, since k(CX) = BX. Then, since the dilation fixes X, and dilations take line segments to line segments, I see that the Triangle DXC is mapped to Triangle AXB. So the original triangle DXC is similar to Triangle AXB.*

c. $\overline{AB}$ and $\overline{CD}$ are parallel.

   Again, rotate triangle DXC so that $\angle DXC$ *coincides with* $\angle AXB$. *Then the image of the side CD under this rotation is parallel to the original side CD, so the new side CD is still parallel to side AB. Now, apply a dilation about point X that moves the vertex C to point B. This dilation moves the line CD to a line through B parallel to the previous line CD. I already know that line AB is parallel to line CD, so the dilation must move the line CD onto the line AB. Since the dilation moves D to a point on the ray XA and on the line AB, D must move to A. Therefore, the rotation and dilation map the Triangle DXC to the Triangle AXB. Thus, triangle DXC is similar to Triangle AXB.*

d. $\angle XAB \cong \angle XCD$.

   Suppose I draw the bisector of $\angle AXC$, *and reflect the triangle CXD across this angle bisector. This maps the segment XC onto the segment XA; and since reflections preserve angles, it also maps segment XD onto segment XB. Since* $\angle XAB \cong \angle XCD$, *I also know that the image of side CD is parallel to side AB. Therefore, if I apply a dilation about the point X that takes the new point C to A, then the new line CD will be mapped onto the line AB, by the same reasoning used in (c). Therefore, the new point D is mapped to B, and thus, the Triangle XCD is mapped to Triangle XAB. So Triangle XCD is similar to Triangle XAB.*

Solving Right Triangles (ECR)

Overview

This task requires students to solve real-world problems using trigonometric ratios.

Standards

HSG-SRT.C.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade Level Standards	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSG-SRT.C.8	• 8.G.B.7 • HSG-SRT.B.4 • HSG-SRT.C.6 • HSG-SRT.C.7	1. The angle of elevation from a point on the ground to the top of a building is 50^{o}. The point on the ground is 6 feet from the bottom of the building. How tall is the building? a. Approximately 7.15 feet	• http://www.illustrativemathematics.org/illustrations/655 • http://www.illustrativemathematics.org/illustrations/1635 • http://www.illustrativemathematics.org/illustrations/1443 • http://learnzillion.com/lessonsets/769-solve-problems-using-trigonometric-ratios-and-the-pythagorean-theorem • http://learnzillion.com/lessonsets/663-solve-right-triangles-using-trigonometric-ratios-and-the-pythagorean-theorem

Real-World Preparation: The following questions will prepare students for some of the real-world components of this task:

- **What is a clinometer?** A clinometer is an instrument used to find the measure of the angle of elevation in order to find the height of objects such as trees or buildings. (http://www.merriam-webster.com/dictionary/clinometer)

- **Why would someone buy a house and have trees cut down?** Trees may be removed for many reasons. Sometimes trees have a disease or get struck by lightning, and branches fall to the ground, causing damage to people or property. Another reason trees may be removed is because they are too close to a house, other structure, or power lines. If a tree is too close to a house, the house may become un-level.

- **What is stump grinding?** Stump grinding is the use of a machine to chip up the remainder of a tree stump. The machine chips the base of the tree into small pieces of wood until the tree stump is below ground level.

After the Task

Common errors/misunderstandings by problem number:

- **Problem 1**: Some students may forget to add in 6 feet for the height of the clinometer from the ground. Also, students may forget to put their calculator in degree mode.
- **Problem 2**: Students may give the incorrect answer of $1,854 because they do not notice that the total to remove the trees is over $1,200.
- **Problem 4**: Remind students to draw a picture. Also, students may forget to put their calculator in degree mode.

Student Extended Constructed Response

Bob just bought a new house, and he has three trees that he needs to have cut down. He has saved $1,000 to get the trees removed, but he is not sure if he has enough money. He contacts Jason's Tree Service, a local tree removal service, to get some preliminary information. The information below was provided by the company.

Jason's Tree Service Pricing

	Small Trees **25 feet or shorter**	**Medium Trees** **25-50 feet**	**Large Trees** **50-90 feet**
Tree Removal	$8 per foot	$9 per foot	$10 per foot
Stump Grinding	$75	$150	$300

When calculating the height of the trees, round up to the next whole number of feet.

If the cost to remove all trees is more than $1,200, we will throw in the stump grinding for free.

1. Bob needs to estimate the height of each tree to determine if he has enough money to have them removed. Bob uses a clinometer to measure the angle of elevation to the top of the tree. **He holds the clinometer 6 feet from the ground**. Complete the chart below to determine the height of each tree. Show all work.

	Angle of Elevation	**Bob's Distance from the Base of the Tree (feet)**	**Height of the Tree (feet)**
Tree 1	74°	4	
Tree 2	80°	13	
Tree 3	45°	35	

2. Based on the information from Jason's Tree Service, how much should Bob be charged for the removal and stump grinding of the three trees? Show all work and explain your reasoning.

3. Does Bob have enough money to get the 3 trees removed from his property? Show all work and explain your reasoning.

4. Bob has a tree in his backyard that he is going to chop down himself. In order to keep the tree from falling toward him, he decides to tie a rope to the top of the tree and stake it to the ground. If there is 25 feet of rope between the stake and the top of the tree, and the tree is 15 feet tall, what angle does the rope make with the ground? Show all work.

Extended Constructed Response Exemplar Response

Bob just bought a new house, and he has three trees that he needs to have cut down. He has saved $1,000 to get the trees removed, but he is not sure if he has enough money. He contacts Jason's Tree Service, a local tree removal service, to get some preliminary information. The information below was provided by the company.

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1. Bob needs to estimate the height of each tree to determine if he has enough money to have them removed. Bob uses a clinometer to measure the angle of elevation to the top of the tree. **He holds the clinometer 6 feet from the ground**. Complete the chart below to determine the height of each tree. Show all work.

	Angle of Elevation	Bob's Distance from the Base of the Tree (feet)	Height of the Tree (rounded to the nearest foot)
Tree 1	74°	4	***20***
Tree 2	80°	13	***80***
Tree 3	45°	35	***41***

Height of Tree 1:

$$\tan 74° = \frac{x}{4}$$
$$4 \times tan74° = x$$
$$13.95 \approx x$$
$$\approx 14\, feet$$
$$14\, ft + 6\, ft = 20\, feet$$

Height of Tree 2:

$$\tan 80° = \frac{x}{13}$$
$$13 \times tan\, 80° = x$$
$$73.73 \approx x$$
$$\approx 74\, feet$$
$$74\, ft + 6\, ft = 80\, feet$$

Height of Tree 3:

$$\tan 45° = \frac{x}{35}$$
$$35 \times tan\, 45° = x$$
$$35 = x$$
$$\approx 35\, feet$$
$$35 ft + 6\, ft = 41\, feet$$

Alternate Solution for Height of Tree 3:

This is an isosceles triangle (45°, 45°, 90°), so the second leg would be congruent to the first. As a result, that side is 35 feet, and the total height of the tree is 35 + 6 = 41 feet.

2. Based on the information from Jason's Tree Service, how much should Bob be charged for the removal and stump grinding of the three trees? Show all work and explain your reasoning.

Tree 1:

$$20\, feet \times \$8 = \$160$$
$$\$160 + 75 = \$235$$

Tree 2:

$$80\, feet \times \$10 = \$800$$
$$\$800 + 300 = \$1100$$

Tree 3:

$$41\, feet \times \$9 = \$369$$
$$\$369 + 150 = \$519$$

If Bob had to pay for all three trees to be removed and the stumps ground, it would be as follows:
$\$235 + \$1100 + \$519 = \$1854$.

However, the cost for the three trees to be removed is as follows:
$\$160 + \$800 + \$369 = \$1329$. *This total is more than $1,200, so the stump grinding will be free.*

The total estimate is $1,329 to have all 3 trees removed from Bob's property and the stumps ground.

3. Does Bob have enough money to get the 3 trees removed from his property? Show all work and explain your reasoning.

Bob does not have enough money to have the tree removed. $\$1329 - \$1000 = \$329$ *Bob needs to save an additional $329 to be able to afford to have the three trees removed from his property.*

Note: If a student's answer is correct based on an incorrect equation in part 2, then this part should be marked correct.

4. Bob has a tree in his backyard that he is going to chop down himself. In order to keep the tree from falling toward him, he decides to tie a rope to the top of the tree and stake it to the ground. If there is 25 feet of rope between the stake and the top of the tree, and the tree is 15 feet tall, what angle does the rope make with the ground?

$$sin\,\alpha = \frac{15}{25}$$
$$\mathrm{m}\angle\alpha = \sin^{-1}\frac{15}{25}$$
$$m\angle\alpha \approx 36.87°$$