MATHEMATICS CONTENT MODULES

Module 4: Extending Operations with Rational Numbers

Session 0: Setting the Stage for Module 4

Grades 6-9

The goal of the initiative

Credit: Chart Baker/Jason Reed/Ryan McVay/Photodisc/ Thinkstock

Improve the quality of instruction in math classrooms by developing deep mathematical content knowledge of teachers and connecting that content knowledge to the practical implementation of a quality curriculum.

Math Content Module 4, Session 0
Grades 6-9

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Group norms

- Understand that those who work, learn.
- Look for solutions, not blame.
- Focus on systems, not people.
- Recognize that everyone has expertise.
- Be honest.
- Challenge ideas.
- Share talk time.
- Mistakes are expected, respected, and inspected.

Module 4 goals

Participants will

- Experience math tasks from EngageNY and other Tier 1 resources related to the focus content of operations with rational numbers.
- Explore coherence between the LSSM related to operations with rational numbers and identify implications for teaching and learning.
- Continue to explore effective practices for orchestrating mathematical discourse, with a focus on monitoring student responses during a task.
- Use tools for purposeful planning to develop a common understanding of grade-level/course standards to align teaching and learning using EngageNY and other Tier 1 resources.

Math Content Module 4, Session 0
Grades 6-9

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MATHEMATICS CONTENT MODULES

Module 4: Extending Operations with Rational Numbers

Session 1: Deepening Mathematical Content Knowledge and Exploring Coherence in the LSSM

Engaging in the mathematics: Extending operations with rational numbers

Egyptian Fractions II
Grades 6-9

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Operations with rational numbers

To be successful in solving problems such as those in the Egyptian Fractions task, what do students need to know and be able to do?

Operations with rational numbers

- Identify the standards at your grade level that address operations with rational numbers (addition, subtraction, multiplication, division).
- Make a poster summarizing the work at your grade level and be prepared to present to the whole group.

Operations with rational numbers across the grades

M4_P-02 Operations with Rational Numbers Across the Grade Levels

Fill in the chart summarizing the key understandings at each grade level.

Key Understandings could include:

- Standards
- Consolidated summary of content in standards
- If a skill or concept is introduced, extended, or finalized

	Key Understandings
Upper High School	
Algebra I	
8th grade	
7th grade	
6th grade	
Upper Elementary	

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Reflection

Record your individual reflection on the following questions on a sticky note.

- What did you learn about the vertical progression of operations with rational numbers across the grades?
- What actions will you take in your classroom instruction based on what you learned?

Math Content Module 4, Session 1
Grades 6-9

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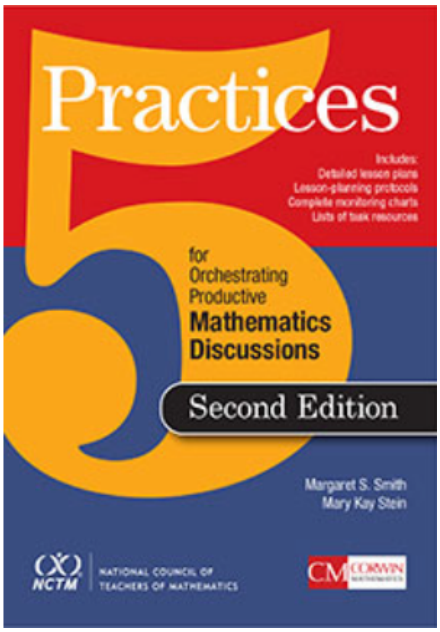
MATHEMATICS CONTENT MODULES

Module 4: Extending Operations with Rational Numbers

Session 2: Instructional Strategies to Improve Curriculum Implementation

Grades 6-9

5 Practices for Orchestrating Productive Mathematics Discussions



— Smith, M. S., & Stein, M. K. (2011). *5 practices for orchestrating productive mathematics discussions.* Reston, VA: National Council of Teachers of Mathematics. Available at http://www.nctm.org/Store/Products/5--Practices-for-Orchestrating-Productive-Mathematics-Discussions

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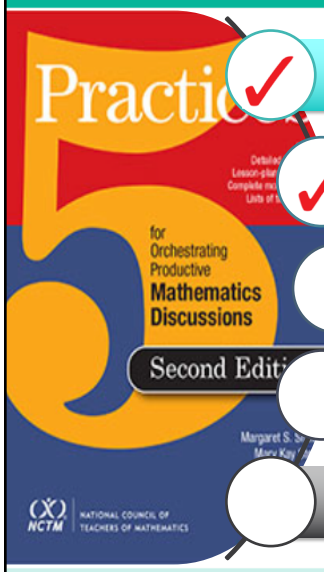
5 practices for orchestrating productive mathematics discussions



- *Anticipating* student responses prior to the lesson
- *Monitoring* students' work on and engagement with the tasks
- *Selecting* particular students to present their mathematical work
- *Sequencing* students' responses in a specific order for discourse
- *Connecting* different students' responses and connecting the responses to key mathematical ideas

Orchestrating productive mathematics discussions

Anticipating student responses prior to the lesson

What should you consider?	How is this supported?
• The strategies that students might use to approach or solve a challenging mathematical task • How to respond to what students produce • Which strategies are most useful in addressing the mathematics to be learned	• *Solving the problem in as many ways as possible* • Solving the problem with other teachers • Drawing on relevant research when possible • Document students responses year to year

— Adapted from Smith, M. S., & Stein, M. K. (2011). *5 Practices for Orchestrating Productive Mathematics Discussions.* Reston, VA: National Council of Teacher of Mathematics.

Orchestrating productive mathematics discussions

Monitoring students' work on, and engagement with, the task

What does this involve?	How is this supported?
• Circulating while students work, watching and listening • Recording interpretations, strategies, and points of confusion • Asking probing questions to get students back "on track" or to advance their understanding	• *Anticipating* student responses beforehand • Using a recording tool • Observing students' actual responses during independent work

— Adapted from Smith, M. S., & Stein, M. K. (2011). *5 Practices for Orchestrating Productive Mathematics Discussions.* Reston, VA: National Council of Teacher of Mathematics.

Math Content Module 4, Session 2
Grades 6-9

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Posing purposeful questions

Working with those at your table:

- Sort your questions into different groups.
- Use a sticky note to label each group of questions and define how you sorted the questions into each group.
- Make note of distinctions between the categories you created.

Math Content Module 4, Session 2
Grades 6-9

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Question types

Question Type	Description
Gathering information	Students recall facts, definitions, or procedures.
Probing thinking	Students explain, elaborate, or clarify their thinking, including articulating the steps in solution methods or the completion of a task.
Making the mathematics visible	Students discuss mathematical structures and make connections among mathematical ideas and relationships.
Encouraging reflection and justification	Students reveal deeper understanding of their reasoning and actions, including making an argument for the validity of their work.

Math Content Module 4, Session 2
Grades 6-9

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Revisit the Egyptian Fractions task and work with your grade-level teams to:

1) Discuss how you anticipate students will respond.

2) Plan possible probing questions to help students.

LDOE: Math Content
Grades 6-9, Module 4

Anticipating and Monitoring Student Responses to
Task: Egyptian Fractions Task

Students will….
- *Rewrite simple rational expressions in different forms.*
- *Use examples and counterexamples to communicate thinking using appropriate vocabulary, symbols, and/or units.*

Column 1: Anticipate student responses to the task. Consider: • Strategies that will be most useful in addressing the learning goals. • Strategies that students are likely to use.	**Column 2:** Plan purposeful questions to respond to likely student work.
Anticipated strategy:	Purposeful questions to respond:
Anticipated strategy:	Purposeful questions to respond:

— Adapted from Smith, M. S., & Stein, M. K. (2011). *5 practices for orchestrating productive mathematics discussions*. Reston, VA: National Council of Teachers of Mathematics.

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Monitoring student responses

Reflection

- How does productive student discourse support a classroom culture that reflects the key shifts?

MATHEMATICS CONTENT MODULES

Module 4: Extending Operations with Rational Numbers

Session 3: Key Shifts in Action Through an EngageNY Lesson

Grades 6-9

EngageNY module overview

- Overview
- Focus Standards
- Foundational Standards

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Evidence of student engagement in the key shifts

Use the handout to:

- Check off student behaviors/actions that you observed.
- Record any observations you made that serve as evidence.

Handout P - 06
Looking for Evidence of Student Engagement in the Key Shifts

Focus	Evidence
☐ The learning goal(s) of the lesson supports grade level standard(s).	
Coherence	
☐ The lesson intentionally relates new concepts to students' prior skills and knowledge. ☐ Students set the foundation for future learning. ☐ Students access prior learning from major work in the grade in order to support new learning.	
Rigor	
Conceptual Understanding ☐ Students access concepts and ideas from a variety of perspectives. ☐ Students explain mathematical ideas behind a particular concept in a variety of ways. ☐ Students use examples and counterexamples to make and support conjectures applied to one problem to multiple situations. ☐ Students create and use a variety of models to analyze relationships. ☐ Students make use of patterns and structure to compose and decompose numbers, shapes, expressions, and equations. **Procedural Skills and Fluency** ☐ Students select tools (e.g. physical objects, manipulatives, drawings, diagrams, algorithms, or strategies) that are relevant and useful for the task or problem. ☐ Students communicate thinking using appropriate vocabulary, symbols and/or units in precise and accurate ways. ☐ Students look for patterns, generalizations, and shortcuts. ☐ Students are flexible in their use of procedures and skills to solve problems. **Application** ☐ Students decontextualize and contextualize quantities in problem situations. ☐ Students plan and choose a solution pathway when applying their mathematical knowledge to different situations.	

Note: To help educators look for evidence of grade-level-appropriate student engagement in mathematical tasks, these narrative descriptions are adapted from Illustrative Mathematics (2014, February 12). *Standards for Mathematical Practice: Commentary and Elaborations for K-5 and 6-8*. Tucson, AZ. Available at http://commoncoretools.me/2014/02/12/k-5-elaborations-of-the-practice-standards

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Math Content Module 4, Session 3
Grades 6-9

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Learning expectations

The student will:

- Use visual fraction models to make connections between multiplication and division of fractions.
- Solve word problems involving division of fractions by fractions with different denominators
- Use examples and counterexamples to communicate thinking using appropriate vocabulary, symbols, and/or units.

Engaging in the math

You have three equivalent fractions. One of the fractions is 10/12. What might the other two fractions be?

Use the sentence stem to defend your response.

The other two equivalent fractions are _____ and ____.
I know these are equivalent because...

Engaging in the math

Molly has 1 ⅜ cups of strawberries. She needs 3/8 cup of strawberries to make one batch of muffins.

How many batches of muffins can Molly make?

Draw a visual model to support your answer.

Engaging in the math

Molly's friend, Xavier, also has 1 ⅜ cups of strawberries. He needs 3/4 cup of strawberries to make a batch of tarts.

How many batches of tarts can Xavier make?

Draw a visual model to support your answer.

Talking about the math

Molly **Xavier**

What is the same?

What is different?

What did you learn from the problem that you could use in solving other problems?

Math Content Module 4, Session 3
Grades 6-9

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Engaging in the math

$$\frac{3}{4} \div \frac{2}{3}$$

What is the question asking?

What type of real-life situation might this problem represent?

How could we model this problem?

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Wrap-up

1) When dividing fractions, how do you know if a quotient is going to be greater than 1 or between 0 and 1?

2) When you are asked to divide two fractions with different denominators, what is one possible way to solve?

Evidence of student engagement in the key shifts

Use the handout to:

- Check off student behaviors/actions that you observed.
- Record any observations you made that serve as evidence.

Handout P - 06
Looking for Evidence of Student Engagement in the Key Shifts

Focus	Evidence
☐ The learning goal(s) of the lesson supports grade level standard(s).	
Coherence	
☐ The lesson intentionally relates new concepts to students' prior skills and knowledge. ☐ Students set the foundation for future learning. ☐ Students access prior learning from major work in the grade in order to support new learning.	
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Conceptual Understanding ☐ Students access concepts and ideas from a variety of perspectives. ☐ Students explain mathematical ideas behind a particular concept in a variety of ways. ☐ Students use examples and counterexamples to make and support conjectures applied to one problem to multiple situations. ☐ Students create and use a variety of models to analyze relationships. ☐ Students make use of patterns and structure to compose and decompose numbers, shapes, expressions, and equations. **Procedural Skills and Fluency** ☐ Students select tools (e.g. physical objects, manipulatives, drawings, diagrams, algorithms, or strategies) that are relevant and useful for the task or problem. ☐ Students communicate thinking using appropriate vocabulary, symbols and/or units in precise and accurate ways. ☐ Students look for patterns, generalizations, and shortcuts. ☐ Students are flexible in their use of procedures and skills to solve problems. **Application** ☐ Students decontextualize and contextualize quantities in problem situations. ☐ Students plan and choose a solution pathway when applying their mathematical knowledge to different situations.	

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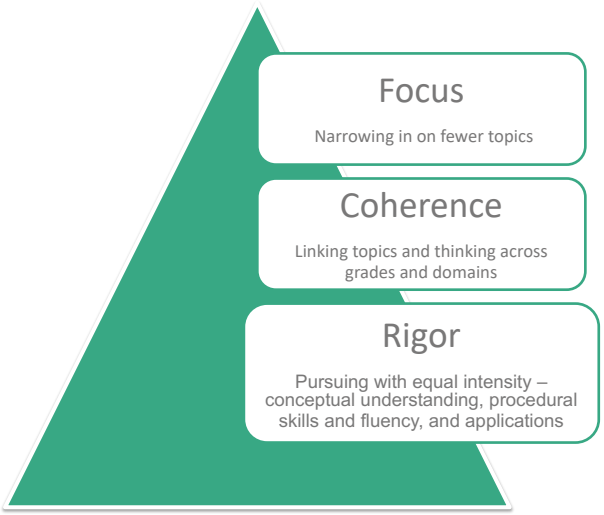
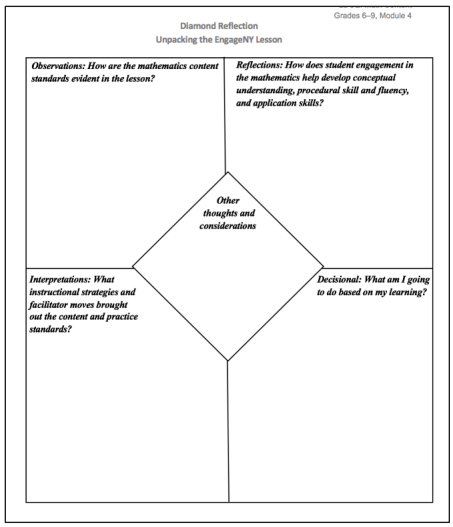
Unpacking the EngageNY lesson

6.NS.A.1

Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*

Evidence of the key shifts





Session reflection

Points to Ponder

- How will unpacking EngageNY lessons for the content standards, SMPs, and key shifts affect your planning and instruction?
- As a result of your learning, identify one *action* you will take to effectively plan for instruction.

MATHEMATICS CONTENT MODULES

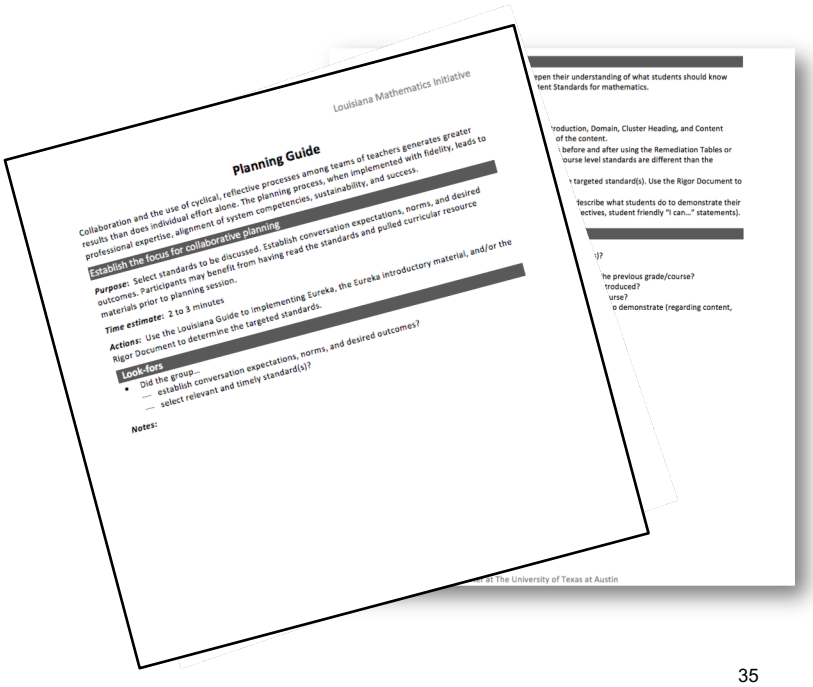
Module 4: Extending Operations with Rational Numbers

Session 4: Purposeful Planning of the EngageNY Curriculum

Grades 6-9

Tool that promotes effective collaborative planning:

The Planning Guide


Louisiana Mathematics Initiative

Planning Guide

Collaboration and the use of cyclical, reflective processes among teams of teachers generates greater results than does individual effort alone. The planning process, when implemented with fidelity, leads to professional expertise, alignment of system competencies, sustainability, and success.

Establish the focus for collaborative planning

Purpose: Select standards to be discussed. Establish conversation expectations, norms, and desired outcomes. Participants may benefit from having read the standards and pulled curricular resource materials prior to planning session.

Time estimate: 2 to 3 minutes

Actions: Use the Louisiana Guide to Implementing Eureka, the Eureka introductory material, and/or the Rigor Document to determine the targeted standards.

Look-fors

- Did the group…
  - establish conversation expectations, norms, and desired outcomes?
  - select relevant and timely standard(s)?

Notes:

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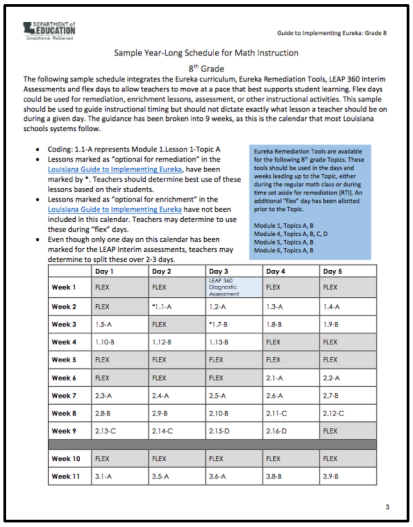
Collaborative planning conversation

Establish the focus standards for collaborative planning:

- Use the *Louisiana Guide to Implementing Eureka* to identify the module and lessons for an upcoming unit.
- Identify the standard(s) related to the upcoming module lessons.


Guide to Implementing Eureka: Grade 8

Sample Year-Long Schedule for Math Instruction

8th Grade

The following sample schedule integrates the Eureka curriculum, Eureka Remediation Tools, LEAP 360 Interim Assessments and flex days to allow teachers to move at a pace that best supports student learning. Flex days could be used for remediation, enrichment lessons, assessment, or other instructional activities. This sample should be used to guide instructional timing but should not dictate exactly what lesson a teacher should be on during a given day. The guidance has been broken into 9 weeks, as this is the calendar that most Louisiana schools systems follow.

- Coding: 1.1-A represents Module 1.Lesson 1-Topic A
- Lessons marked as "optional for remediation" in the Louisiana Guide to Implementing Eureka, have been marked by *. Teachers should determine best use of these lessons based on their students.
- Lessons marked as "optional for enrichment" in the Louisiana Guide to Implementing Eureka have not been included in this calendar. Teachers may determine to use these during "flex" days.
- Even though only one day on this calendar has been marked for the LEAP Interim assessments, teachers may determine to split these over 2-3 days.

Eureka Remediation Tools are available for the following 8th grade Topics. These tools should be used in the days and weeks leading up to the Topic, either during the regular math class or during time set aside for remediation (RTI). An additional "flex" day has been allotted prior to the Topic.

Module 1, Topics A, B
Module 4, Topics A, B, C, D
Module 5, Topics A, B
Module 6, Topics A, B

	Day 1	Day 2	Day 3	Day 4	Day 5
Week 1	FLEX	FLEX	LEAP 360 Diagnostic Assessment	FLEX	FLEX
Week 2	FLEX	*1.1-A	1.2-A	1.3-A	1.4-A
Week 3	1.5-A	FLEX	*1.7-B	1.8-B	1.9-B
Week 4	1.10-B	1.12-B	1.13-B	FLEX	FLEX
Week 5	FLEX	FLEX	FLEX	FLEX	FLEX
Week 6	FLEX	FLEX	FLEX	2.1-A	2.2-A
Week 7	2.3-A	2.4-A	2.5-A	2.6-A	2.7-B
Week 8	2.8-B	2.9-B	2.10-B	2.11-C	2.12-C
Week 9	2.13-C	2.14-C	2.15-D	2.16-D	FLEX
Week 10	FLEX	FLEX	FLEX	FLEX	FLEX
Week 11	3.1-A	3.5-A	3.6-A	3.8-B	3.9-B

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Math Content Module 4, Session 4
Grades 6-9

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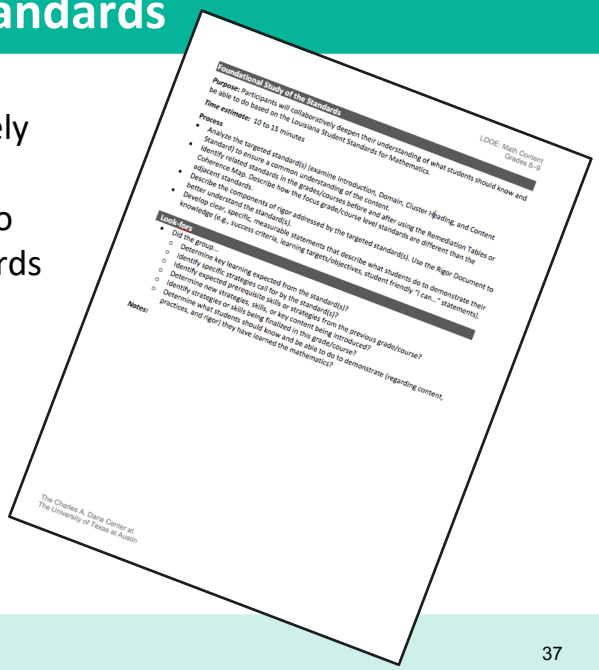
Foundational study of the standards

Purpose: Participants will collaboratively deepen their understanding of what students should know and be able to do based on the Louisiana Student Standards for Mathematics.

Time estimate: 10 to 15 minutes



Math Content Module 4, Session 4
Grades 6-9

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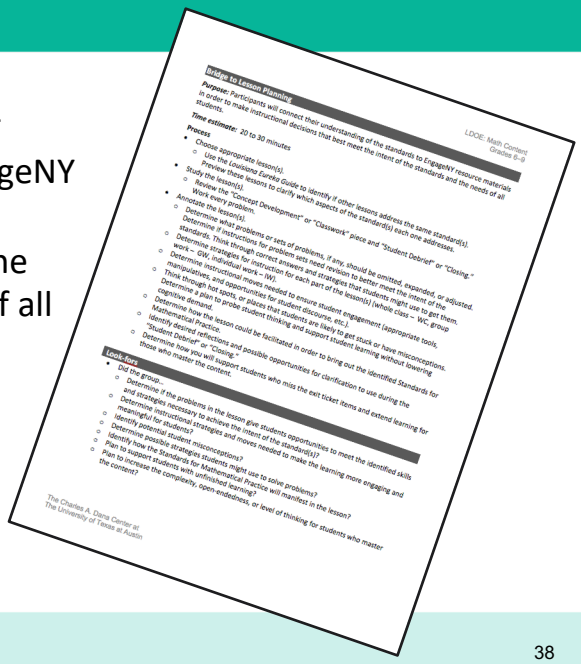
Bridge to lesson planning

Purpose: Participants will connect their understanding of the standards to EngageNY resource materials so they can make instructional decisions that best meet the intent of the standards and the needs of all students.

Time estimate: 20 to 30 minutes



Math Content Module 4, Session 4
Grades 6-9

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Collaborative planning

Work with your grade level group…

- Study your grade level's focus standards using the *Foundational study of the standards*. (15 min)
- Connect your standards conversation to an EngageNY lesson and discuss instructional decisions regarding that lesson using the *Bridge to lesson planning*. (30 min)

Grade 6: 6.NS.A.1, 6.NS.B.2-3
Grade 7: 7.NS.A.1-3
Grade 8: 8.EE.C.7b, 8.NS.A.1-2
Alg I: A1: N-RN.B.3, A1: A-REI.B.4, A1: A-APR.A.1

Math Content Module 4, Session 4
Grades 6-9

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Reflecting on purposeful planning

Points to Ponder

How does purposeful planning help you attend to the components of rigor?

What specific instructional strategies did you focus on?

What changes did you make to the EngageNY lesson(s)? Why?

How did your conversation help determine a plan to fill gaps in learning?

Math Content Module 4, Session 4
Grades 6-9

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MATHEMATICS CONTENT MODULES

Module 4: Extending Operations with Rational Numbers

Module 4 Closing Slides

Grades 6-9

References

Key resources available via the Louisiana Department of Education web page — K-12 Math Planning Resources, https://www.louisianabelieves.com/resources/library/k-12-math-year-long-planning — include the following:
K–12 Louisiana Student Standards for Mathematics (LSSM)
Teachers Companion Documents
Implement The Eureka Curriculum: Louisiana Eureka Guides.
Help Students Who Struggle: Remediation Guides.
The Louisiana Student Standards for Mathematics: A Guide to Rigor in Mathematics 2.0, at https://www.louisianabelieves.com/docs/default-source/year-long-planning/k-12-lssm-alignment-to-rigor.pdf This document (name in link: "K-12 LSSM Alignment to Rigor") as well as the individual rigor documents for each grade level—e.g., "Kindergarten LSSM Alignment to Rigor," "Grade 1 LSSM Alignment to Rigor," and so on) can be downloaded via this web page: https://www.louisianabelieves.com/resources/library/k-12-math-year-long-planning

Other resources referenced include:
EngageNY, from the New York State Department of Education
Various curriculum resources, available via https://www.engageny.org
Pre-Kindergarten-Grade 5 Mathematics: Curriculum Map and Guiding Documents. EngageNY. Available via https://www.engageny.org/resource/pre-kindergarten-grade-5-mathematics-curriculum-map-and-guiding-documents
These curriculum resources are shared under the EngageNY terms of use (https://www.engageny.org/terms-of-use) and the Creative Commons license—Attribution-NonCommercial-ShareAlike 3.0 Unported (CC BY-NC-SA 3.0)—mentioned therein: https://creativecommons.org/licenses/by-nc-sa/3.0

Smith, M. S., & Stein, M. K. (2011). *5 practices for orchestrating productive mathematics discussions.* Reston, VA: National Council of Teachers of Mathematics. Available at http://www.nctm.org/Store/Products/5--Practices-for-Orchestrating-Productive-Mathematics-Discussions.

What to expect in Modules 5–6

- **Deepening Mathematical Content Knowledge** for effective instruction
- **Exploring Coherence** in the LSSM
- **Key Shifts in Action** through an EngageNY lesson
- **Promoting Rigor** in the classroom
- **Instructional Strategies** for implementing the EngageNY curriculum
- The role of a **Productive Classroom Culture** in the EngageNY curriculum
- **Purposeful Planning** of the EngageNY curriculum

Math Content Module 4, Closing Slides
Grades 6-9

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Module evaluation

Your feedback and comments are important to us. Please complete the module evaluation found at:

http://www.utdanacenter.org/ldoe/surveys

Math Content Module 4, Closing Slides
Grades 6-9

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