Algebra I Guide to Rigor in Mathematics 2.0

In order to provide a quality mathematical education for students, instruction must be rigorous, focused, and coherent. This document provides explanations and a standards-based alignment to assist teachers in providing the first of those: a rigorous education. While this document will help teachers identify the explicit component(s) of rigor called for by each of the Louisiana Student Standards for Mathematics (LSSM), it is up to the teacher to ensure his/her instruction aligns to the expectations of the standards, allowing for the proper development of rigor in the classroom.

This rigor document is considered a "living" document as we believe that teachers and other educators will find ways to improve the document as they use it. Please send feedback to LouisianaStandards@la.gov so that we may use your input when updating this guide.

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Definitions of the Components of Rigor

Rigorous teaching in mathematics does not simply mean increasing the difficulty or complexity of practice problems. Incorporating rigor into classroom instruction and student learning means exploring at a greater depth, the standards and ideas with which students are grappling. There are **three** components of rigor that will be expanded upon in this document, and each is equally important to student mastery: **Conceptual Understanding, Procedural Skill and Fluency,** and **Application**.

- **Conceptual Understanding** refers to understanding mathematical concepts, operations, and relations. It is more than knowing isolated facts and methods. Students should be able to make sense of why a mathematical idea is important and the kinds of contexts in which it is useful. It also allows students to connect prior knowledge to new ideas and concepts.
- **Procedural Skill and Fluency** is the ability to apply procedures accurately, efficiently, and flexibly. It requires speed and accuracy in calculation while giving students opportunities to practice basic skills. Students' ability to solve more complex application tasks is dependent on procedural skill and fluency.
- **Application** provides valuable context for learning and the opportunity to solve problems in a relevant and a meaningful way. It is through real-world application that students learn to select an efficient method to find a solution, determine whether the solution makes sense by reasoning, and develop critical thinking skills.

A Special Note on Procedural Skill and Fluency

While speed is definitely a component of fluency, it is not necessarily speed in producing an answer; rather, fluency can be observed by watching the speed with which a student engages with a particular problem. Furthermore, fluency does not require the most efficient strategy. The standards specify grade-level appropriate strategies or types of strategies with which students should demonstrate fluency (e.g., 1.OA.C.6 allows for students to use counting on, making ten, creating equivalent but easier or known sums, etc.). It should also be noted that teachers should expect some procedures to take longer than others (e.g., fluency with the standard algorithm for division, 6.NS.B.2, as compared to fluently adding and subtracting within 10, 1.OA.C.6).

Standards identified as targeting procedural skill and fluency do not all have an expectation of automaticity and/or rote recall. Only two standards, 2.OA.B.2 and 3.OA.C.7, have explicit expectations of students knowing facts from memory. Other standards targeting procedural skill and fluency do not require students to reach automaticity. For example, in 4.G.A.2, students do not need to reach automaticity in classifying two-dimensional figures.

Recognizing the Components of Rigor

In the LSSM each standard is aligned to one or more components of rigor, meaning that each standard aims to promote student growth in conceptual understanding, procedural skill and fluency, and/or application. Key words and phrases in the standards indicate which component(s) of rigor the standard is targeting: conceptual understanding standards often use terms like *understand*, *recognize*, or *interpret*; procedural skill and fluency standards tend to use words like *fluently*, *find*, or *solve*; and application standards typically use phrases like *word problems* or *real-world problems*. Key words and phrases are underlined in each standard to help clarify the identified component(s) of rigor for each standard.

Focus in the Standards

Not all content in a given grade is emphasized equally in the standards. Some clusters require greater emphasis than others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. More time in these areas is also necessary for students to meet the Louisiana Standards for Mathematical Practice. To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade. Students should spend the large majority of their time on the major work of the grade ( ). Supporting work ( ) and, where appropriate, additional work ( ) can engage students in the major work of the grade.

Algebra I

LSSM – Algebra I		Explicit Component(s) of Rigor		
Code	Standard	Conceptual Understanding	Procedural Skill and Fluency	Application
A1: N-RN.B.3	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	✔		
A1: N-Q.A.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. *	✔		
A1: N-Q.A.2	Define appropriate quantities for the purpose of descriptive modeling. *	✔		
A1: N-Q.A.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. *	✔		
A1: A-SSE.A.1	Interpret expressions that represent a quantity in terms of its context. *	✔		
A1: A-SSE.A.1a	Interpret parts of an expression, such as terms, factors, and coefficients. *	✔		
A1: A-SSE.A.1b	Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.* *	✔		
A1: A-SSE.A.2	Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, or see $2x^2 + 8x$ as $(2x)(x) + 2x(4)$, thus recognizing it as a polynomial whose terms are products of monomials and the polynomial can be factored as $2x(x+4)$.*	✔	✔	
A1: A-SSE.B.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. *	✔	✔	
A1: A-SSE.B.3a	Factor a quadratic expression to reveal the zeros of the function it defines. *	✔	✔	
A1: A-SSE.B.3b	Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. *	✔	✔	
A1: A-SSE.B.3c	Use the properties of exponents to transform expressions for exponential functions emphasizing integer exponents. *For example, the growth of bacteria can be modeled by either $f(t) = 3^{(t+2)}$ or $g(t) = 9(3^t)$ because the expression $3^{(t+2)}$ can be rewritten as $(3^t)(3^2) = 9(3^t)$.* *		✔	
A1: A-APR.A.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	✔	✔	
A1: A-APR.B.3	Identify zeros of quadratic functions, and use the zeros to sketch a graph of the function defined by the polynomial.	✔	✔	

A1: A-CED.A.1	Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear, quadratic, and exponential functions.* *	✔	✔	✔
A1: A-CED.A.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. *	✔	✔	
A1: A-CED.A.3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* *	✔		✔
A1: A-CED.A.4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law $V = IR$ to highlight resistance R.* *		✔	
A1: A-REI.A.1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	✔		
A1: A-REI.B.3	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.		✔	
A1: A-REI.B.4	Solve quadratic equations in one variable.		✔	
A1: A-REI.B.4a	Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.		✔	
A1: A-REI.B.4b	Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as "no real solution. *	✔	✔	
A1: A-REI.C.5	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.	✔		
A1: A-REI.C.6	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.		✔	
A1: A-REI.D.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).	✔		
A1: A-REI.D.11	Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, piecewise linear (to include absolute value), and exponential functions. *	✔	✔	
A1: A-REI.D.12	Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.		✔	

A1: F-IF.A.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f*(*x*).	✔		
A1: F-IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	✔	✔	
A1: F-IF.A.3	Recognize that sequences are functions whose domain is a subset of the integers. Relate arithmetic sequences to linear functions and geometric sequences to exponential functions.	✔		
A1: F-IF.B.4	For linear, piecewise linear (to include absolute value), quadratic, and exponential functions that model a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.* *	✔		
A1: F-IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* *	✔		
A1: F-IF.B.6	Calculate and interpret the average rate of change of a linear, quadratic, piecewise linear (to include absolute value), and exponential function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. *	✔	✔	
A1: F-IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. *	✔	✔	
A1: F-IF.C.7a	Graph linear and quadratic functions and show intercepts, maxima, and minima. *	✔	✔	
A1: F-IF.C.7b	Graph piecewise linear (to include absolute value) and exponential functions. *		✔	
A1: F-IF.C.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.	✔	✔	
A1: F-IF.C.8a	Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.	✔	✔	
A1: F-IF.C.9	Compare properties of two functions (linear, quadratic, piecewise linear [to include absolute value] or exponential) each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, determine which has the larger maximum.*	✔	✔	
A1: F-BF.A.1	Write a linear, quadratic, or exponential function that describes a relationship between two quantities. *	✔	✔	
A1: F-BF.A.1a	Determine an explicit expression, a recursive process, or steps for calculation from a context. *	✔	✔	

A1: F-BF.B.3	Identify the effect on the graph of replacing *f*(*x*) by *f*(*x*) + *k*, *k f*(*x*), *f*(*kx*), and *f*(*x* + *k*) for specific values of *k* (both positive and negative). Without technology, find the value of *k* given the graphs of linear and quadratic functions. With technology, experiment with cases and illustrate an explanation of the effects on the graph that include cases where *f(x)* is a linear, quadratic, piecewise linear (to include absolute value) or exponential function.	✔	✔	
A1: F-LE.A.1	Distinguish between situations that can be modeled with linear functions and with exponential functions. *	✔		
A1: F-LE.A.1a	Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. *	✔		
A1: F-LE.A.1b	Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. *	✔		
A1: F-LE.A.1c	Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. *	✔		
A1: F-LE.A.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). *	✔	✔	
A1: F-LE.A.3	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. *	✔		
A1: F-LE.B.5	Interpret the parameters in a linear or exponential function in terms of a context. *	✔		
A1: S-ID.A.2	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. *	✔	✔	
A1: S-ID.A.3	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). *	✔		
A1: S-ID.B.5	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. *	✔	✔	
A1: S-ID.B.6	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. *	✔	✔	
A1: S-ID.B.6a	Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and quadratic models. *	✔	✔	✔
A1: S-ID.B.6b	Informally assess the fit of a function by plotting and analyzing residuals. *	✔	✔	
A1: S-ID.B.6c	Fit a linear function for a scatter plot that suggests a linear association. *		✔	
A1: S-ID.C.7	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. *	✔		

A1: S-ID.C.8	Compute (using technology) and interpret the correlation coefficient of a linear fit.*	✔	✔	
A1: S-ID.C.9	Distinguish between correlation and causation.*	✔		

*Modeling standard