Eureka Remediation Tool: Algebra I
Module 1, Topic B

To become mathematically proficient, students **must** access on-grade-level content. This document aims to help teachers who use the Eureka curriculum to target remediation for students needing extra support before and **during** approaching on-grade-level work, creating opportunities for on-time remediation directly connected to the new learning.

About this Topic

Focus Standards:

A1: A-SSE.A.2: Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$, or see $2x^2 + 8x$ as $(2x)(x) + 2x(4)$, thus recognizing it as a polynomial whose terms are products of monomials and the polynomial can be factored as $2x(x + 4)$.*

A1: A-APR.A.1: Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Topic Overview per the Eureka Curriculum

In Lessons 6 and 7 of this topic, students develop a precise understanding of what it means for expressions to be algebraically equivalent. By exploring geometric representations of the distributive, associative, and commutative properties for positive whole numbers and variable expressions assumed to represent positive whole numbers, students confirm their understanding of these properties and expand them to apply to all real numbers. Students use the properties to generate equivalent expressions and formalize that two algebraic expressions are equivalent if we can convert one expression into the other by repeatedly applying the commutative, associative, and distributive properties and the properties of rational exponents to components of the first expression. A goal of this topic is to address a fundamental, underlying question: Why are the commutative, associative, and distributive properties so important in mathematics? The answer to the question is, of course, because these three properties help to generate all equivalent algebraic expressions discussed in Algebra I.

Lessons 6 and 7 also engage students in their first experience using a recursive definition for building algebraic expressions. Recursive definitions are sometimes confused with being circular in nature because the definition of the term uses the very term one is defining. However, a recursive definition or process is not circular because it has what is referred to as a base case. For example, a definition for algebraic expression is presented as follows:

An algebraic expression is either:

1. A numerical symbol or a variable symbol or
2. The result of placing previously generated algebraic expressions into the two blanks of one of the four operators ((__) + (__), (__) - (__), (__) × (__),(__) ÷ (__)) or into the base blank of an exponentiation with an exponent that is a rational number.

Part (1) of this definition serves as a base case, stating that any numerical or variable symbol is in itself an algebraic expression. The recursive portion of the definition is in part (2) where one can use any previously generated algebraic expression to form new ones using the given operands. Recursive definitions are an important part of the study of sequences in Module 3. Giving students this early experience lays a nice foundation for the work to come.

Having a clear understanding of how algebraic expressions are built and what makes them equivalent provides a foundation for the study of polynomials and polynomial expressions.

In Lessons 8 and 9, students learn to relate polynomials to integers written in base x, rather than our traditional base of 10. The analogies between the system of integers and the system of polynomials continue as they learn to add, subtract, and multiply polynomials and to find that the polynomials for a system that is closed under those operations (e.g., a polynomial added to, subtracted from, or multiplied by another polynomial) always produces another polynomial. We use the terms polynomial and polynomial expression in much the same way as we use the terms number and numerical expression. Where we would not call 27(3 + 8) a number, we would call it a numerical expression. Similarly, we reserve the word polynomial for polynomial expressions that are written as a sum of monomials.

Overview

Eureka Remediation Tools include:

1. a diagnostic assessment to help teachers determine the misunderstandings or gaps in mathematical knowledge related to a specific Topic in the Eureka curriculum
2. guidance for teachers to analyze student work on the diagnostic assessment
3. suggested materials for targeted remedial instruction

Note: The use of this guidance is not intended to delay students' engagement with on-grade-level learning. On-grade-level learning should be the focus of instructional time and be treated as an opportunity for students to "finish" learning previous skills and deepen conceptual understanding.

Diagnostic Assessment

The diagnostic assessment is designed to be administered to targeted students prior to beginning instruction on the given Topic. When appropriate, it is broken into parts (Part A, Part B, and so on); each part addresses a different prerequisite standard and contains three problems. If a student correctly answers at least 2 out of the 3 problems, it can be assumed that he/she is ready to engage with the new content of the Topic with little to no support needed prior to engaging with the Topic. The diagnostic assessment is designed in this way so that teachers can determine the "entry point" to remedial instruction and/or opportunities for unfinished learning within the context of the new learning. The entry points and opportunities for unfinished learning will vary between students.

Guidance for Remediation

The Remediation Guidance is designed for teacher use. It is also broken into parts (Part A, Part B, and so on) and correlates to the parts on the diagnostic assessment. Each part contains the following:

1. **The focus standard**: The focus standards are strategically chosen to address prerequisite skills and are purposefully arranged in the order that students typically master the skills and knowledge.
2. **Why this is important for current grade level work**: This section describes how the work of the prerequisite standard relates to the standard(s) addressed in the Topic of instruction.
3. **Using the diagnostic assessment to identify gaps**: This section identifies common errors students make on the diagnostic assessment items.
4. **Remediation Resources for Targeted Instruction**: The resources pinpoint specific Eureka lessons and parts of lessons for teachers to use to address gaps in mathematical knowledge. Using Eureka materials to address remediation ensures alignment to the standards, consistency in approach to learning, and similarities in strategies for solving problems.

Part A: 6.EE.A.3

1. Apply properties of operations to write an equivalent expression.
   $t + t + t + t$

2. Apply the distributive property to write an equivalent expression.
   $4(2 + w)$

3. Apply the distributive property to write an equivalent expression.
   $21b + 28c$

Part B: 6.EE.A.4

4. Are these two expressions equivalent? Explain how you know.
   $r + r + r$ and $3r$

5. Are these two expressions equivalent? Explain how you know.
   $4(m + n)$ and $4m + n$

6. Are these two expressions equivalent? Explain how you know.
   $2(x + y + z)$ and $2x + 2(y + z)$

Part C: 7.EE.A.1

7. Add.
   $a + (-8a)$

8. Subtract.
   $\left(\frac{4}{10} + \frac{3}{10}x\right) - \left(\frac{4}{10} - \frac{7}{10}x\right)$

9. Expand the expression and collect like terms.
   $-g - 4(2g - 6)$

Part D: 8.EE.A.1

10. Apply the properties of integer exponents to generate an equivalent expression.
    $2^4 \cdot 2 \cdot 2^3$

11. Apply the properties of integer exponents to generate an equivalent expression.
    $3^2 \cdot 3^2 \cdot 7^2$

12. Apply the properties of integer exponents to generate an equivalent expression.
    $(5^3)^2$

Solutions:

1. $4t$
2. $8 + w$
3. $7(3b + 4c)$
4. (Sample) Yes, three of the same value added together is equivalent to that value times three. For example, 4 + 4 + 4 = 3(4) = 12.
5. (Sample) No, the first expression has a total of four *n*'s while the second expression only has a single *n*. For example, $4(m + n) = (m + n) + (m + n) + (m + n) + (m + n) = 4m + 4n.$
6. (Sample) Yes, each expression has a total of 2 *x*'s, *y*'s, and *z*'s.
7. $-7a$
8. x
9. $-9g + 24$
10. $2^8 = 256$
11. $3^4 \cdot 7^2 = 3{,}969$
12. $5^5 = 3{,}125$

Part A Focus: 6.EE.A.3: Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3(2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6(4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*

<table>
<tr><td colspan="3">Why this is important for current grade level work:
In middle school students worked with algebraic expressions by applying the properties of operations to add, subtract, and rewrite expressions in either factored or expanded form. In Topic B students will make use of structure to show that two expressions are equivalent. Students will apply the commutative, associative, and distributive properties to change the form of equivalent expressions, preparing them for when they must choose the most appropriate form with which to work. Students extend their work with expressions to include polynomial expressions as they generate, identify, and, beginning in Lesson 8, perform operations (addition, subtraction and multiplication) on polynomial expressions.</td><td rowspan="3">Remediation Resources for Targeted Instruction:

<u>6th Grade, Module 4, Topic D, Lesson(s) 9 – 12</u>

Use the Concept Development portion of each Lesson and a sampling of problems from the Problem Set that focus on conceptual understanding.</td></tr>
<tr><td colspan="3">Using the Diagnostic Assessment to identify gaps:</td></tr>
<tr><td>Problem 1:
A student may write an expression such as 2t + 2t and still be considered ready for the target standard. Look for students who change the exponent when adding as this shows a fundamental misunderstanding of addition that likely extends back beyond algebraic expressions.</td><td>Problem 2:
A student may write an expression such as 4(2) + 4(w) and still be considered ready for the target standard. Look for students who only distribute to the first term and suggest using expansion to see the entire expression should be written four times, not just the first term in the expression.</td><td>Problem 3:
Look for students who struggle to engage with the directions, not seeing factoring as an application of the distributive property. Question such students to distinguish which are struggling with the academic vocabulary compared to those who cannot apply the distributive property to factor an expression.</td></tr>
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Remediation Guidance: Algebra I
Eureka Module 1, Topic B

Part B Focus: 6.EE.A.4: Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*

<table>
<tr><td colspan="2">Why this is important for current grade level work:
The target topic spends the first two lessons extending properties of operations to generating equivalent expressions. With simple numeric expressions, students can easily decide if two expressions are equivalent based solely on arithmetic facts. For example, 2 + 3 and 5 are equivalent because 2 + 3 has a value of 5; however, it is not as easy to determine if algebraic expressions are equivalent. For example, 2a + 3 and 5 are equivalent only when a = 1, otherwise they are not equivalent. While the first two lessons of the topic will continue to build understanding of equivalent expressions, students will greatly benefit by having a base understanding of equivalent expressions prior to engaging with Lesson 6.</td><td rowspan="3">Remediation Resources for Targeted Instruction:

<u>6th Grade, Module 4, Topic C, Lesson(s) 8</u>

Use the Concept Development portion of the Lesson and a sampling of problems from the Problem Set that focus on conceptual understanding.</td></tr>
<tr><td colspan="2">Using the Diagnostic Assessment to identify gaps:</td></tr>
<tr><td>Problem 4:
Look for students justify their choices algebraically or by simply showing a single example where the two expressions are equivalent. Push such students to think more deeply about the problem, ensuring they can truly justify their selection of equivalent or not.</td><td>Problems 5 – 6:
Look for students who are unable to accurately apply (or recognize the application of) the distributive property. While Lesson 6 will continue to develop understanding of the distributive property, the lesson does assume that students have some fundamental understanding of the property prior to engaging with the lesson.</td></tr>
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Part C Focus: 7.EE.A.1: Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients to include multiple grouping symbols (e.g., parentheses, brackets, and braces).

Why this is important for current grade level work:

Lesson 8 teaches students how to add and subtract polynomials. The basis for this is adding and subtracting linear expressions, a subset of polynomials. If students struggle to accurately add and/or subtract the expressions in problems 7 and 8, they will likely need remediation prior to engaging with Lesson 8. Lesson 9 focuses on multiplying polynomials which build from students' ability to apply the distributive property to expand linear expressions. Problem 9 involves expanding and collecting like terms. The most important look-fors here are the accuracy of the answers, ensuring students are not changing exponents when collecting like terms.

Using the Diagnostic Assessment to identify gaps:

Problem 7:
Look for students who change the exponent when adding as this shows a fundamental misunderstanding of addition that likely extends back beyond algebraic expressions. A student who creates an expression in terms of *a* but uses the incorrect coefficient may still be considered ready for the target standard although such a student would likely benefit from remediation on adding/subtracting integers.

Problem 8:
Look for students who struggle to subtract the second expression from the first, not recognizing that the variable term will actually need to be added. Look for students that at least subtract the two constant terms correctly as this shows some understanding of subtracting expressions.

Problem 9:
Look for students only distribute to the first term and/or do not take into account the negative terms when distributing. Each error reveals a different gap in student understanding, with the former being more pertinent to the grade-level content than the latter.

Remediation Resources for Targeted Instruction:

7th Grade, Module 3, Topic A, Lesson(s) 1 - 6

Use the Concept Development portion of each Lesson, based on the diagnosed gaps in understanding, and a sampling of problems from the Problem Set focused on conceptual understanding

Part D Focus: 8.EE.A.1: Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.

Why this is important for current grade level work:	Remediation Resources for Targeted Instruction:
Lesson 9 extends students work with integer exponents to multiplying polynomials. For students to be successful generating equivalent algebraic expressions involving integer exponents, they must fully understand the properties of integer exponents. The problems here focus only on the properties students will use while multiplying polynomials; thus, you do not see any problems involving negative exponents or the quotient of powers. Given that the problems are numerical expressions whereas the grade-level work is with algebraic expressions, if students make a calculation error in evaluating their numerical expressions, check how they handled the exponents before determining where the gap exists. The most important look-fors here are how students handle the exponents, not their ability to multiply factors like 81 and 49 (see problem 11). The problems scaffold in difficulty.	
Using the Diagnostic Assessment to identify gaps: **Problem 10:** Look for students who multiply the bases while adding the exponents (e.g., $\mathbf{8^8}$) as this shows a serious gap in their understanding of exponents. Students who produce an answer of $\mathbf{2^7}$ likely have learned a rule without having the conceptual understanding to support the rule. Expanding the expression to include no exponents greater than 1 will help such students see the eight 2s being multiplied, not seven. **Problem 11:** Look for students who ignore the different bases and respond with a single base, either 3^6 or 7^6, as both show a gap in understanding of exponents. Students who leave the answer as $3^4 \cdot 7^2$ should still be considered ready for the grade-level work as the grade-level work will have variable bases. **Problem 12:** Look for students who think the answer is 5^5 instead of 5^6 as this likely points again to the students learning a rule without the necessary understanding to support the rule. Expanding the expression multiple times to finally have an expression with no exponents greater than 1 will help students see the six 5's, not five.	<u>8th Grade, Module 1, Topic A, Lesson(s) 1 - 3</u> Use the Concept Development portion of each Lesson and a sampling of problems from the Problem Set focused on conceptual understanding and/or procedural skill and fluency.