

Louisiana Teacher Leader Summit

Summer 2016 – Professional Development

What's the Problem?: The Relationship Between
Addition and Subtraction
Grades K–2

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Math learning experience

Participants will...

- Deepen understanding of addition and subtraction problem types as described by the Louisiana Student Standards.
- Use hands-on methods to explore computation methods that support the understanding of the relationship between addition and subtraction.

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Introduction to the Charles A. Dana Center

WHAT IS THE DANA CENTER?

At the Dana Center, we help prepare students to succeed in a complex and changing world. The center advances effective math and science education for all students, from kindergarten to college. We are known for our innovative resources that promote student engagement, motivation, and persistence in support of academic achievement. We work alongside teachers, administrators, and policymakers at the local, state, and national levels to create educational systems that put students on a path to success.

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Making sense of mathematics

“Children who are successful at mathematics are those who believe that mathematics makes sense.”

—Lauren Resnick

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Exploring the relationship between addition and subtraction

Domain – Operations and Algebraic Thinking (OA)

Cluster Heading:

Grade K

- Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

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Exploring the relationship between addition and subtraction

Domain - Operations and Algebraic Thinking (OA)

Cluster Heading:

Grade 1

- Represent and solve problems involving addition and subtraction.
- Understand and apply properties of operations and the relationship between addition and subtraction.
- Add and subtract within 20.
- Work with addition and subtraction equations.

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Exploring the relationship between addition and subtraction

Domain – Operations and Algebraic Thinking (OA)

Cluster Heading:

Grade 2

- Represent and solve problems involving addition and subtraction.
- Add and subtract within 20.

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Problem types

DEPARTMENT OF EDUCATION
Louisiana Believes

Mathematics Standards: Glossary

TABLE 1. Common addition and subtraction situations.⁴

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunions sat on the grass. Three more bunions hopped there. How many bunions are on the grass now? $2+3=?$	Two bunions were sitting on the grass. Some more bunions hopped there. Then there were five bunions. How many bunions hopped over to the first two? $2+?=5$	Some bunions were sitting on the grass. Three more bunions hopped there. Then there were five bunions. How many bunions were on the grass before? $?+3=5$
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? $5-2=?$	Five apples were on the table. I ate two apples. Then there were three apples. How many apples did I eat? $5-?=3$	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $?-2=3$
Put Together/ Take Apart	Total Unknown Three red apples and two green apples are on the table. How many apples are on the table? $3+2=?$	Added Unknown Five apples are on the table. Three are red and the rest are green. How many apples are green? $3+?=5$, $5-3=?$	Both Addends Unknown Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $?=+?$, $5=+?$, $5=?+$
Compare	Difference Unknown ("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2+?=5$, $5-2=?$	Bigger Unknown ("Version with 'more'"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? ("Version with 'fewer'"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2+3=?$, $3+2=?$	Smaller Unknown ("Version with 'more'"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? ("Version with 'fewer'"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? $5-3=?$, $7+3=5$

⁴These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean adds or results in less always does mean is the same number as.

⁵Under addition can be subtracted, as there are those instances of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

⁶For the bigger Unknown or Smaller Unknown situations, one version shows the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other version are more difficult.

⁷Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

Table 2: Addition and subtraction situations by grade level.

sign (=, here with the meaning of becomes, rather than the more general "equals").

Add To	Result Unknown A bunions sat on the grass. <i>B</i> more bunions hopped there. How many bunions are on the grass now? $A+B=?$	Change Unknown A bunions were sitting on the grass. Some more bunions hopped there. Then there were <i>C</i> bunions. How many bunions hopped over to the first A bunions? $A+?=C$	Start Unknown Some bunions were sitting on the grass. <i>B</i> more bunions hopped there. Then there were <i>C</i> bunions. How many bunions were on the grass before? $?+B=C$
Take From	Total Unknown C apples were on the table. I ate <i>B</i> apples. How many apples are on the table now? $C-B=?$	Both Addends Unknown C apples were on the table. I ate some apples. Then there were <i>A</i> apples. How many apples did I eat? $C-?=A$	Added Unknown C apples were on the table. I ate <i>B</i> apples. Then there were <i>A</i> apples before? $?-B=A$
Put Together/ Take Apart	Total Unknown A red apples and <i>B</i> green apples are on the table. How many apples are there? $A+B=?$	Both Addends Unknown Grandma has <i>C</i> flowers. How many can she put in her red vase and how many in her blue vase? $?=+?$	Added Unknown C apples are on the table. <i>A</i> are red and the rest are green. How many apples are green? $A+?=C$
Compare	Difference Unknown "How many more?" version: Lucy has <i>A</i> apples. Julie has <i>C</i> apples. How many more apples does Julie have than Lucy? $A+?=C$ $C-A=?$	Bigger Unknown "More" version suggests operation: Lucy has <i>B</i> more apples than Lucy. Julie has <i>A</i> apples. How many apples does Julie have? $A+B=?$	Smaller Unknown "Fewer" version suggests wrong operation: Lucy has <i>B</i> fewer apples than Julie. Julie has <i>A</i> apples. How many apples does Lucy have? $C-B=?$ $?+B=C$

indicates the four kindergarten problem subtypes. Grade 1 and 2 students work with all subtypes and variants. Problems are the four difficult subtypes or variants that students should work with in Grade 1 but need not master (see the table in the next column). The table is based on Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity for All Children, National Research Council (2009, pp. 32-33).

show all decompositions of a given number, especially important for numbers within 10. Equations with totals on the right do not always mean "trades" or "results in" but always means "is the same number as." The problem subtype with one unknown, as is the Added Unknown subtype to the right. These problems are a productive extension of the basic situation. The other version are more difficult.

and both variations should be included.

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Progressions for Operations and Algebraic Thinking

“They represent these problems in increasingly sophisticated ways. And they learn and use increasingly sophisticated computation methods to find answers.”

Pg. 6, Progressions; K, Counting and Cardinality; K-5, Operations and Algebraic Thinking.

Methods used for solving single-digit addition and subtraction problems

Level 1. Direct Modeling by Counting All or Taking Away. Represent situation or numerical problem with groups of objects, a drawing, or fingers. Model the situation by composing two addend groups or decomposing a total group. Count the resulting total or addend.

Level 2. Counting On. Embed an addend within the total (the addend is perceived simultaneously as an addend and as part of the total). Count this total but abbreviate the counting by omitting the count of this addend; instead, begin with the number word of this addend. Some method of keeping track (fingers, objects, mentally imaged objects, body motions, other count words) is used to monitor the count.

For addition, the count is stopped when the amount of the remaining addend has been counted. The last number word is the total. For subtraction, the count is stopped when the total occurs in the count. The tracking method indicates the difference (seen as an unknown addend).

Level 3. Convert to an Easier Problem. Decompose an addend and compose a part with another addend.

See Appendix for examples and further details.

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Math work station 1

36 1

Appendix. Methods used for solving single-digit addition and subtraction problems

Level 1. Direct Modeling by Counting All or Taking Away.

Represent situation or numerical problem with groups of objects, a drawing, or fingers. Model the situation by composing two addend groups or decomposing a total group. Count the resulting total or addend.

Adding ($8 + 6 = \square$): Represent each addend by a group of objects. Put the two groups together. Count groups or decomposing a total group. Count the resulting total or addend.

Subtracting ($14 - 8 = \square$): Represent the total by a group of objects. Take the known addend number of objects away. Count the resulting group of objects to find the unknown added. Use this strategy for Take From/Result Unknown.

Levels	Count All	Count on	Retrieval: Make a Ten	Take Away
Level 1: Count all				
Level 2: Count on				
Level 3: Retrieval: Make a ten (general): Make a ten (general): Use addition facts apart to make 10 with the other addend				
Make a ten (from 5's within each addend): Double a n				

Note: Many children attempt to count down for subtraction, but counting down is difficult and error-prone. Children are much more successful with counting on; it makes subtraction as easy as addition.

Ornith, 5/29/2011, comment at commoncoretools.wordpress.com.

Work Station 1, Purple 1

Level 1: Direct Modeling by Counting All or Taking Away: Adding

Directions:

1. Read the highlighted sections of the OA Progression Appendix, page 36, Level 1: Direct Modeling by Counting All or Taking Away.
2. Examine the highlighted sections of the chart on page 36, Level 1: Count All.
3. Read word problem (a), and represent the problem using counters.
4. Use Level 1 methods to solve the problem, and write a solution in the space below the problem. Show your work using pictures, words, and numbers.
5. Repeat steps 3 and 4 for problem (b).
6. Discuss your understanding of the Count All strategy as pictured in the chart.

(a) Diva has 8 stickers. If Diva goes to the store and buys 6 stickers more, how many stickers will she have?

(b) Diva has 4 dog stickers and 7 cat stickers. How many stickers does she have all together?

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Sample representations – Work station 1

Level 1: Direct modeling by counting all – Adding

a

□ □ □ □ □ □ □ □ □ □ □ □ □ □ (14)

1 2 3 4 5 6 7 8 9 10 11 12 13 14

1 2 3 4 5 6 7 8 9 10 11 12 13 14

8 + 6 = 14

b

DDDD CCCCCC

4 dog stickers 7 cat stickers

DDDDCCCCC (11)

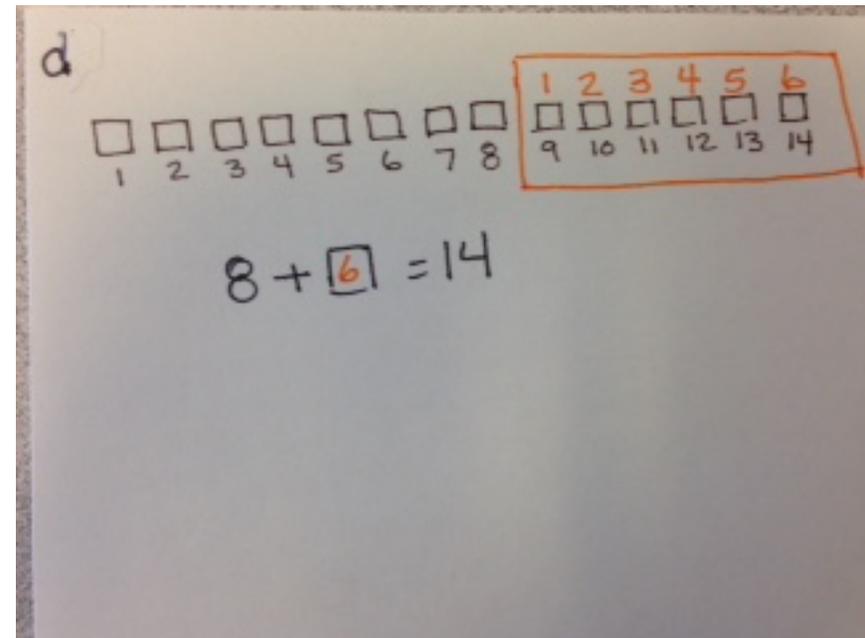
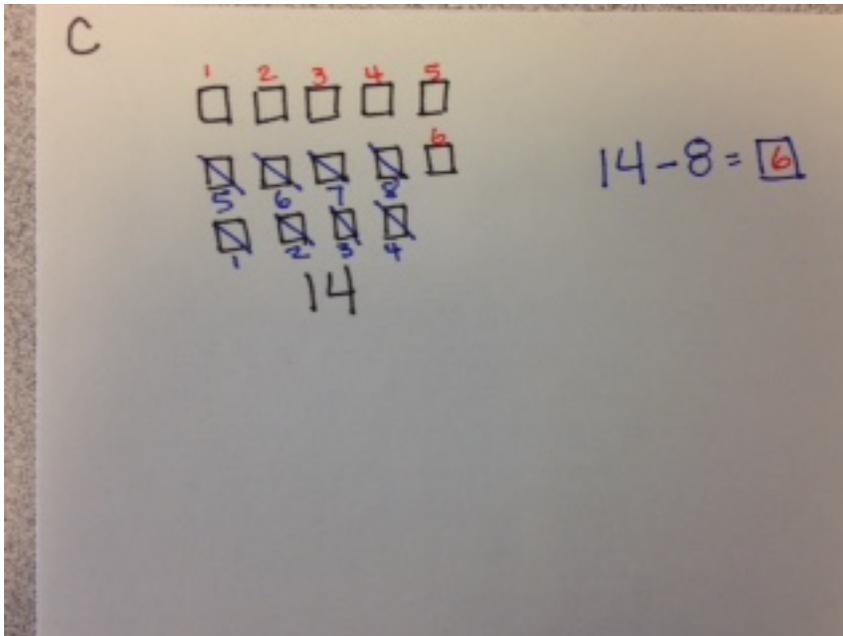
1 2 3 4 5 6 7 8 9 10 11

4 + 7 = 11

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Sample representations – Work station 2

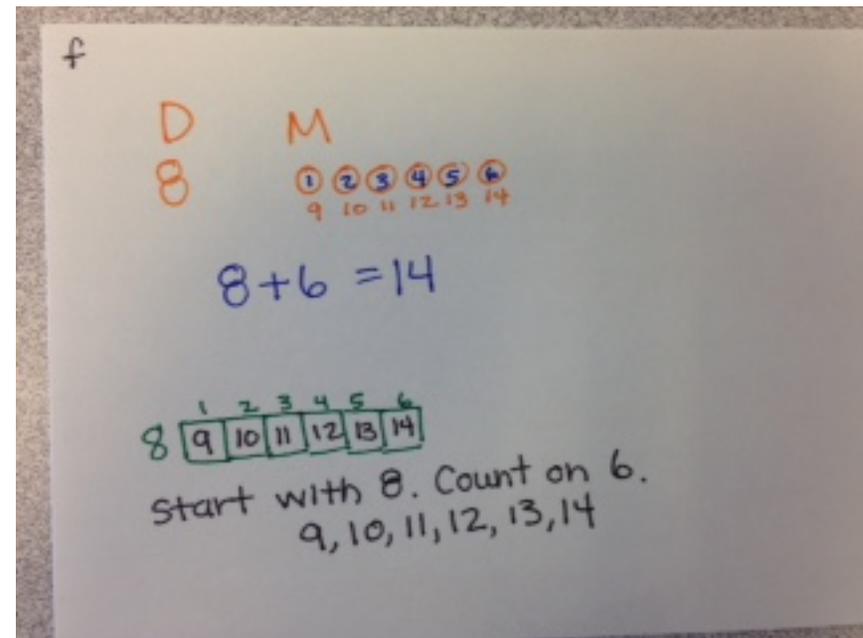
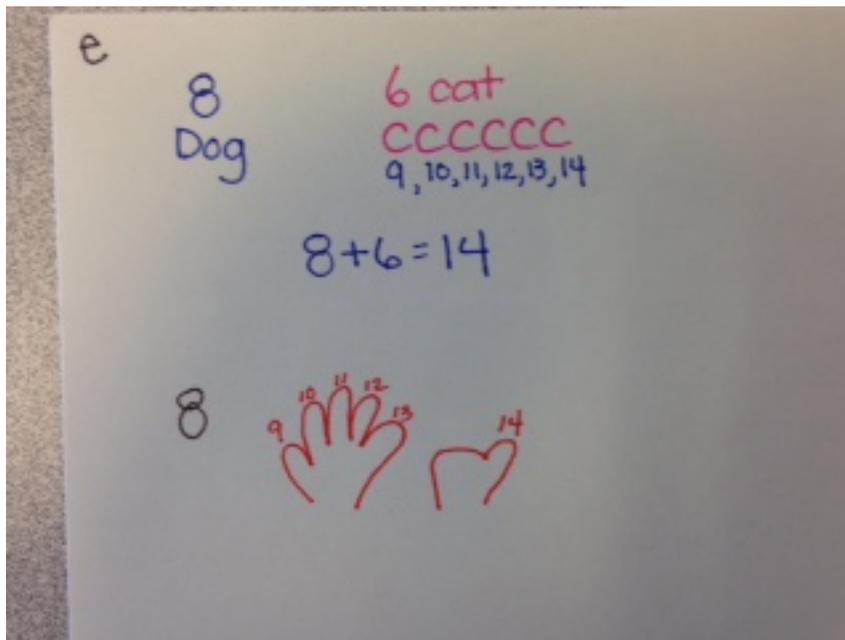
Level 1: Direct modeling by counting all or taking away – Subtracting



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Sample representations – Work station 3

Level 2: Counting on – Adding



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Sample representations – Work station 4

Level 2: Counting on – Finding an unknown addend

g

$$\boxed{6} + 8 = 14$$

Number Strip

1	2	3	4	5	6	7	8	9	10	11	12	13	14
---	---	---	---	---	---	---	---	---	----	----	----	----	----

Start with 8, count on. Stop on 14.

h

$$8 + \boxed{6} = 14$$

say 8 9, 10, 11, 12, 13, 14

1	2	3	4	5	6
---	---	---	---	---	---

8 14 + 1

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Levels 1 and 2 methods for addition and subtraction

- Why is it important to give students many opportunities to practice *counting all* when adding and subtracting?
- What makes a *start unknown or change unknown* problem more difficult than a *result unknown* problem?

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Work stations 6, 7 and 8

Work Station 6, Red
Level 3: Convert to an Easier Equivalent Problem:
Adding and Finding an Unknown Addend

6

Directions:

1. Read the highlighted sections of the OA Progression Appendix, page 38, Level 3: Convert to an Easier Equivalent Problem.
2. Examine the highlighted sections of the chart on page 36, Level 3: Recompose, Make a Ten, and Doubles +/- n.
3. Solve word problem (a) using make a ten and/or doubles plus-or-minus-1 method, and write a solution in the space below the problem.
4. Repeat step 3 for problem (d).
5. Discuss your understanding of the Recompose, Make a Ten, and Doubles +/- n methods as pictured in the chart.

(a) Diva has 8 stickers. If Diva goes to the store and buys 6 more stickers, how many stickers will she have?

(d) Diva had 8 stickers. Her friend Maggie gave her some more. Diva now has 14 stickers. How many stickers did Maggie give her?

Work Station 7, Yellow
Level 3: Convert to an Easier Equivalent Problem:

7

OA Progression Appendix, page 39, Level 3: Convert to an easier equivalent problem using the chart on page 36, Level 3: Recompose, Make a Ten, Make a Ten, and Doubles +/- n methods as pictured in the chart.

Recompose and Make a Ten methods as pictured in the chart.

8 to her friend Maggie. How many stickers does she have now?

Maggie gave her 8 more stickers. Now Diva has a total of 14 stickers. How many stickers did Diva have before?

Work Station 8, Black
Level 3: Convert to an Easier Equivalent Problem:
Computation

8

Solve the word problems on this page. Discuss your solutions, and discuss it with a partner.

Write a picture frame. Maggie has 8 more stickers than Diva. How many stickers does Maggie have if Diva has 14 stickers?

Write a picture frame. Maggie has 8 more yards of ribbon than Diva. How many yards of ribbon does Maggie have if Diva has 14 yards of ribbon?

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Sample representations – Work station 6

Level 3: Convert to an easier equivalent problem

6a

Make a ten

$$\begin{array}{c} 8 + 6 = \square \\ \swarrow \quad \searrow \\ 8 + 2 + 4 \\ \swarrow \quad \searrow \\ 10 + 4 = 14 \end{array}$$

Doubles +2

$$\begin{array}{c} 8 + 6 = \\ \swarrow \quad \searrow \\ 6 + 2 + 6 \\ \swarrow \quad \searrow \\ 12 + 2 = 14 \end{array}$$

6d

Make a ten

$$\begin{array}{l} 8 + \square = 14 \\ 8 + 2 = 10 \\ 10 + 4 = 14 \\ 8 + 6 = 14 \end{array}$$

Doubles \pm n

$$\begin{array}{l} 8 + \square = 14 \\ 8 + 8 = 16 \\ 16 - 2 = 14 \\ 8 - 2 = 6 \\ 8 + 6 = 14 \end{array}$$

I know $14 = 7 + 7$
 $7 + 1 = 8$
 $7 - 1 = 6$
So, $8 + 6 = 14$

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Sample representations – Work station 7

Level 3: Convert to an easier equivalent problem

7 C Make a ten

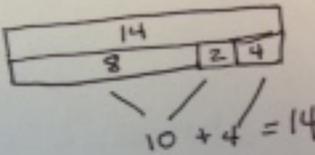
$14 - 8 = \boxed{6}$

$8 + \square = 14$

$8 + \boxed{2} = 10$

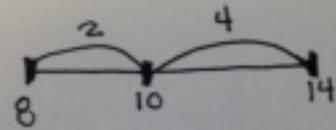
$10 + \boxed{4} = 14$

$8 + \boxed{6} = 14$



7 g Make a ten.

$\square + 8 = 14$

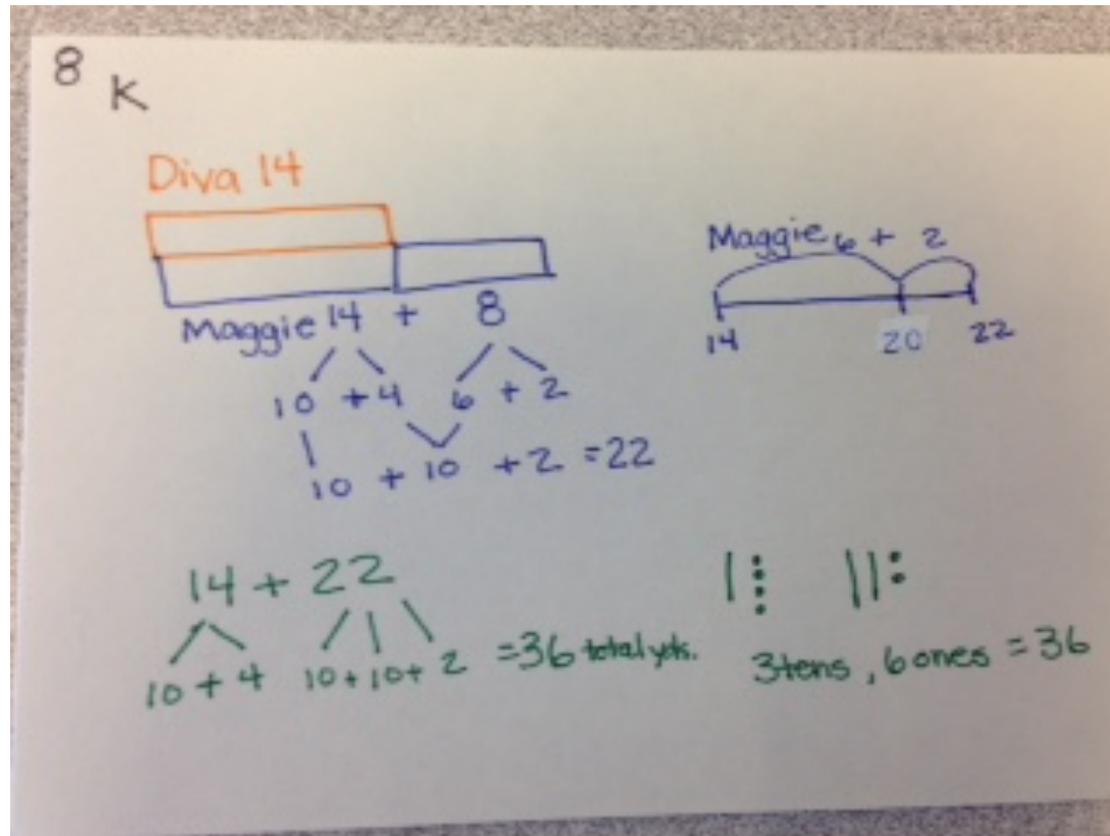


$8 + \boxed{6} = 14$

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Sample representations – Work station 8

Level 3: Convert to an easier equivalent problem –
Generalizing beyond single digit computation



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Reflection

How do these methods support and develop numerical fluency (*flexibility, efficiency, and accuracy*) and mathematical reasoning?

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Reflecting on the session

What?

What are two important ideas that you have learned today?

So What?

What is important about what you have learned?

Now What?

What actions will you take as a result of what you have learned?