Session Objectives
Discover that multiplication and division are, respectively, composing and decomposing equal units.
Understand how the part to whole and whole to part relationships apply to multiplication and division.
Curriculum Map for A Story of Units

Agenda

- **Foundations of Multiplication and Division**
- Grade 3 Progression of Multiplication and Division
- Grade 4 Progression of Multiplication and Division
- Grade 5 Progression of Multiplication and Division
- Continuation of Multiplication and Division in the Middle Grades

Grade 2 Module 6: Foundations of Multiplication and Division

Equal groups and repeated addition

- Tape diagrams and repeated addition

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Grade 2 Module 6: Foundations of Multiplication and Division

- Arrays and repeated addition

- Tiled rectangular arrays

Grade 2 – Module 6 – Lessons 7 and 11

Grade 2 Module 6: Foundations of Multiplication and Division

- Doubles

- Even and Odd

Grade 2 – Module 6 – Lessons 17 and 18

Models

Concrete ➔ Pictorial ➔ Abstract

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Strategies
Simple → Complex
Level 1: One object is one unit.
Level 2: A group is a unit.
Level 3: Multiple groups are units.

Agenda
• Foundations of Multiplication and Division
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Grade 3: Multiplication and Division in A Story of Units
By the end of Grade 3, students will:
Use multiplication and division within 100 to solve word problems. (3.OA.3)
Find the unknown number in a multiplication or division equation. (3.OA.4)
Apply properties of operations as strategies to multiply and divide. (3.OA.5)
Fluently multiply within 100. (3.OA.7)
Identify and explain patterns in the multiplication table. (3.OA.9)
Multiply one-digit whole numbers by multiples of 10. (3.NBT.3)
Relate area to multiplication. (3.MD.7)
Grade 3 Fluency
- Group Counting
- Use the Commutative Property
- Find the Common Products
- "Multiply By" Pattern Sheets
- Multiply by Different Units
- Sprints

Multiplication
Repeated addition

\[ 2 + 2 + 2 + 2 + 2 = 10 \]
\[ 4 \times 2 = 8 \]

Equal groups pictures

\[ 4 + 4 = 8 \]
\[ 2 \text{ fours} = 8 \]
\[ 2 \times 4 = 8 \]

Relate Multiplication to the Array Model.

Equal Groups Pictures → Array → Multiplication Number Sentence

\[ 4 \text{ threes} = 12 \]
\[ 4 \times 3 = 12 \]
Relate Multiplication to Number Bonds.

Number Bond
Grade 3 – Module 1 – Lesson 3

4 threes = 12
4 \times 3 = 12

RDW Process
Read, Draw, Write
1. Read the problem.
2. Draw a picture representing the problem.
3. Write a number sentence to solve a problem and a statement answering the problem.

Jordan uses 3 lemons to make 1 pitcher of lemonade. He makes 4 pitchers. How many lemons does he use altogether? Use the RDW process to show your solution.

\[ 4 \times 3 = 12 \]

Jordan uses 12 lemons altogether.
Try Problem # 1.
Arthur has 4 boxes of chocolates. Each box has 6 chocolates inside. How many chocolates does Arthur have altogether?

Division

Arrays Used to Relate Multiplication and Division
Rick puts 15 tennis balls into cans. Each can has 3 tennis balls. How many cans does Rick use?
Children sit in 2 rows of 9 on the carpet for math time. Erin says, “We make 2 equal groups.” Vittesh says, “We make 9 equal groups.” Who is correct? Explain how you know using models, numbers, and words.
Types of Division: Measurement

12 ÷ 4 = ?

Types of Division: Partitive

12 ÷ 4 = ?

Tape Diagrams and Types of Division

2 students equally share 8 crackers. How many crackers does each student get?

Partitive Division: number of groups is known

Grade 3 – Module 1 – Lesson 12
Tape Diagrams and Types of Division

There are 8 crackers, but this time each student gets 2. How many students get crackers?

Measurement Division: size of each group is known

Grade 3 – Module 1 – Lesson 12

Use a Tape Diagram to Model and Solve Multiplication.

Grade 3 – Module 1 – Lessons 14 and 15

Try Problems # 4 and # 5.

Grade 3 – Module 1 – Lessons 12 and 15
Model the 5 + n Pattern as a Strategy for Multiplying.

$$8 \times 4 = (5 \times 4) + (3 \times 4) = (5 + 3) \times 4$$

Apply the Distributive Property to Decompose Units to Divide.

$$28 \div 4 = \frac{28 \div 4}{20 \div 4} = \frac{8 \div 4}{2} = \frac{5}{2} = 7$$

Try Problem # 6.

$$32 \div 4 = \frac{32 \div 4}{20 \div 4} \cdot \frac{12 \div 4}{3} = \frac{5}{3} = 8$$
Multiply and Divide with Familiar Facts Using a Letter to Represent the Unknown.

Twenty-four people line up to use the canoes at the park. Three people are assigned to each canoe. How many canoes are used for all 24 people?

Grade 3 – Module 3 – Lesson 3

Count by Units of 6 and 7 to Multiply and Divide.

Grade 3 – Module 3 – Lesson 4

Model the Distributive Property with Tape Diagrams and Number Bonds.

Grade 3 – Module 3 – Lesson 6
Model the Associative Property as a Strategy to Multiply.

\[ 16 \times 2 = (8 \times 2) \times 2 \]
\[ = 8 \times (2 \times 2) \]
\[ = 8 \times 4 \]
\[ = 32 \]

Grade 3 – Module 3 – Lesson 9

Try Problem # 7.

\[ \begin{array}{c}
\text{Is there a different way to use the associative property to solve this problem?}
\end{array} \]

Grade 3 – Module 3 – Lesson 9

Apply 9 = 10 – 1 as a Strategy to Multiply by 9.

\[ 9 = 10 - 1 \]
\[ 9 \times 4 \]

Grade 3 – Module 3 – Lesson 12

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Identify and Use Arithmetic Patterns to Multiply.

$$3 \times 9 = \_\_\_$$

9 = 10 – 1
3 nines = 3 tens – 3 ones

$$3 \times 9 = 30 – 3$$
$$3 \times 9 = 27$$

Grade 3 – Module 3 – Lesson 14

Multiply and Divide Using Units of 1 and 0.

$$1 \times 4 = 4$$
$$1 \times n = n$$

$$7 \div n = 1$$
No value of $$n$$ will work!

$$0 \div n = 0$$
$$0 \times n = 0$$
$$n$$ can be any value!

Grade 3 – Module 3 – Lesson 16

Multiply by Multiples of 10 Using Place Value Disks.

$$2 \times 3 \text{ ones} = 6 \text{ ones}$$
$$2 \times 3 = 6$$

$$2 \times 3 \text{ tens} = 6 \text{ tens}$$
$$2 \times 30 = 60$$

Grade 3 – Module 3 – Lesson 19
Multiply by Multiples of 10 Using a Place Value Chart.

<table>
<thead>
<tr>
<th>tens</th>
<th>ones</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>⬤⬤⬤</td>
</tr>
</tbody>
</table>

2 × 3 ones = _____ ones
2 × 3 = _____

Use Place Value Strategies & the Associative Property to Multiply Multiples of 10.

Grade 3 – Module 3 – Lesson 20

d) 3 × (5 × 10)
   = [15 ones] × 10
   = 150

Grade 3 Module 4: Multiplication and Area

Grade 3 – Module 4 – Overview
Multiplying and Dividing With the Area Model

Grade 3 – Module 4 – Lesson 8

Apply the Distributive Property as a Strategy to Find Area.

Grade 3 – Module 4 – Lessons 10 and 11

Agenda

• Foundations of Multiplication and Division
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  • **Grade 4 Progression of Multiplication and Division**
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A Continuation of a Story of Units in Grade 4

By the end of Grade 4 students will:

- Multiply up to a four-digit number by a one-digit number. (4.NBT.5)
- Multiply two two-digit numbers. (4.NBT.5)
- Multiply a fraction by a whole number (4.NF.4)
- Divide up to four-digit numbers by one digit divisors finding whole number quotients with remainders. (4.NBT.6)
- Interpret multiplication as comparison. (4.OA.1)
- Solve word problems using multiplication and division. (4.OA.2,3)

Multiplication Models

Array ➔ Area Model

\[ A = l \times w \]

Grade 4 – Module 3 – Lesson 1

Multiplicative Comparison Word Problems

Grade 4 – Module 3 – Lesson 2

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Multiplication by 10, 100, and 1000

Place Value Chart

Number Disks

Unit Language

Multiplication by 10, 100, and 1000

Grade 4 – Module 3 – Topic B

40 × 20

4 × 2 tens  

(4 × 2) × 10  

8 × 10

Grade 4 – Module 3 – Topic B

Multiplication by 10, 100, and 1000

4 × 20  

(4 × 2) × 10  

8 × 10

Grade 4 – Module 3 – Topic B

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Multiplication of Up to 4 Digits by Single-Digit Whole Numbers

Place Value Chart & Number Disks → Partial Products

Grade 4 – Module 3 – Lesson 7

Multiplication of Up to 4 Digits by Single-Digit Whole Numbers

Place Value Chart
Number Disks
Algorithm

Grade 4 – Module 3 – Lessons 8-10

Multiplication of Up to 4 Digits by Single-Digit Whole Numbers

Algorithm

Grade 4 – Module 3 – Lessons 9-10

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Multiplication of Up to 4 Digits by Single-Digit Whole Numbers

Area Model

Distributive Property

Partial Products

Try Problem # 8.

Use RDW to solve.

The Turner family uses 548 liters of water each day. The Hill family uses 3 times as much water as the Turner family per day. How much water does the Hill family use each week?

Grade 4 – Module 3 – Lesson 12

Division of Tens and Ones with Successive Remainders

There are 12 students in PE class separated into 4 teams. How many students are on each team?

There are 12 students in PE class. The teacher makes teams of 3 students each. How many teams are there?

Grade 4 – Module 3 – Lesson 14

Array
Division of Tens and Ones with Successive Remainders

There are 13 students in PE class separated into 4 teams. How many students are on each team?

13 ÷ 4

Array ➔ Area Model

Tape Diagram

Grade 4 – Module 3 – Lesson 14

Division of Tens and Ones with Successive Remainders

6 ÷ 3

36 ÷ 3

Place Value Chart

Number Disks

Algorithm

Grade 4 – Module 3 – Lesson 16

Division of Tens and Ones with Successive Remainders

5 ÷ 4

45 ÷ 4

Place Value Chart

Number Disks

Algorithm

Grade 4 – Module 3 – Lesson 16

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Division of Tens and Ones with Successive Remainders

4 ÷ 3

42 ÷ 3

44 ÷ 3

Grade 4 – Module 3 – Lesson 17

Try Problem # 9.
Use RDW to solve:
The Grand Market sells 3 pounds of oranges for 87 cents. How much does 1 pound of oranges cost at Grand Market?

Grade 4 – Module 3 – Lesson 18

Division of Thousands, Hundreds, Tens, and Ones

Grade 4 - Module 3 – Lesson 29
Try Problem #10.
There are twice as many cows as goats on a farm. All the cows and goats have a total of 1,116 legs. How many goats are there?

Solution A:
- 312 legs × 3 goats = 936 legs
- 184 legs × 6 goats = 1,104 legs
- Total = 2,040 legs

Solution B:
- 312 legs × 2 goats = 624 legs
- 184 legs × 6 goats = 1,104 legs
- Total = 1,728 legs

Then on 93 goats.

Try Problem #11.
Solve 1,584 ÷ 2:
- a) Using number disks.
- b) Use the algorithm.

The Area Model for Division

A = l × w
A = w = l

Grade 4 – Module 3 – Lesson 29

Grade 4 – Module 3 – Lesson 33

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Try Problem #12.
Solve: 84 × 73.

Grade 4 – Module 3 – Lesson 38

Multiplication of Fractions

1 fourth + 1 fourth + 1 fourth = 3 fourths

3 × (1 fourth) = 3 fourths

(Times 3) fourths = 3 fourths

Grade 4 – Module 5 – Topic A

Multiplication of Fractions

Grade 4 – Module 5 – Topic E
Try Problem #13.

Use RDW to solve.

A bricklayer places 12 bricks along an outside wall of a shed. Each brick is \( \frac{3}{4} \) foot long. How many feet long is that wall of the shed?

\[
12 \times \frac{3}{4} = \frac{36}{4} = \frac{9}{1} = 9
\]

The wall of the shed is 9 feet long.
Try Problem #14.

Use RDW to solve.

Kelly’s new puppy weighed \( 4 \frac{7}{10} \) pounds when she brought him home. Now the puppy weighs six times as much as he did when he came home. How much does the dog weigh now?

\[
6 \times 4 \frac{7}{10} = (6 \times 4) + (6 \times \frac{7}{10}) = 24 + \frac{42}{10} = 24 + 4 \frac{2}{10} = 28 \frac{2}{10} \text{ lb}
\]

The dog weighs \( 28 \frac{2}{10} \) pounds now.

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A Continuation of a Story of Units in Grade 5

By the end of Grade 5 students will:

• Use whole-number exponents to denote power of 10. (5.NBT.2)
• Fluently multiply multi-digit whole numbers. (5.NBT.5)
• Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors. (5.NBT.6)
• Multiply and divide decimals to hundredths. (5.NBT.7)
• Interpret a fraction as division of the numerator by the denominator. (5.NF.3)
• Mutiply a fraction or whole number by a fraction. (5.NF.4)
• Solve real world problems involving multiplication of fractions and mixed numbers. (5.NF.6)
• Divide unit fractions by whole numbers and whole numbers by unit fractions. (5.NF.7)
Patterns on the Place Value Chart: Multiplication

Place Value Chart
2.16 \times 10^3 =
2.16 \times 10 \times 10 \times 10 = 2,160.

Unit Language
2.16 \times 10^3 =
216 \text{ hundredths} \times 10^3 =
216,000 \text{ hundredths} =
2,160.

Patterns on the Place Value Chart: Division

Place Value Chart
754 \div 10^3 =
754 \div 10 \div 10 \div 10 = .754

Decimal Multiplication

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Try Problem #15.

The total weight of 5 pieces of butter is 3.445 kg. What is the weight of each piece of butter?

*Use the place value chart and algorithm to solve.*
Multi-Digit Whole Number Multiplication
4509 x 326

Area Model → Standard Algorithm

Grade 5 - Module 2 – Lesson 7

Multi-Digit Multiplication with Decimals
41 x 7.38

Area Model → Algorithm
Compensation Strategy

Grade 5 – Module 2 – Lesson 11

Division with 2-digit Divisors
590 divided by 17

Estimates: 60 ÷ 20 = 3 ten, 80 ÷ 20 = 4 tens

Solution:

\[
\begin{array}{c}
\frac{590}{17} \\
34 \quad 578 \\
590 \\
\hline
0 \\
590
\end{array}
\]

Check:

\[
\begin{array}{c}
34 \times 17 = 578 \\
-590 \\
\hline
0
\end{array}
\]

Grade 5 – Module 2 – Lesson 22
Multi-Digit Decimal Division

Decompose the divisor.

Grade 5 – Module 2 – Lesson 24

Try Problem # 16.

$54 \div 900 \quad 5.4 \div 900$

Hint: Decompose the divisor so these problems become "mental math".

Grade 5 – Module 2 – Lesson 24

Fractions as Division

Grade 5 – Module 4 – Lesson 2

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Fractions as Division

Grade 5 – Module 4 – Lesson 2

Fractions as Division

Grade 5 – Module 3 – Lesson 3

Fractions as Division

Grade 5 – Module 4 – Lesson 4

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Fraction of a Set

\[ \frac{1}{4} \text{ of } 12 = \]

Array

(Similar to Area Model)

Tape Diagram

Grade 5 – Module 4 – Lessons 6 & 7

Fraction x Whole Number

\[ 2 \times 6 \]

• \( 6 + 6 \rightarrow 2 + 2 + 2 + 2 + 2 + 2 \)

• 6 copies of 2

• 6 times as much as 2

Grade 5 – Module 4 – Lessons 6 & 7

Fraction x Whole Number

\[ \frac{2}{3} \times 6 \]

Fraction of a Set:

2-thirds of 6

Repeated Addition 6 copies of 2/3

Grade 5 – Module 4 – Lesson 8

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Fraction x Whole Number

Grade 5 – Module 4 – Lesson 8 – Concept Development

Fraction x Fraction: Pictorial

Grade 5 – Module 4 – Lessons 13 & 15

Try Problem # 17.

1/2 of 1/3

2/3 x 3/4

Grade 5 – Module 2 – Lesson 24
Fraction x Fraction

\[
\frac{7}{9} \times \frac{3}{7}
\]

Mrs. Onusko made 60 cookies for a bake sale. She sold \( \frac{2}{3} \) of them and gave \( \frac{3}{4} \) of the remaining cookies to the students working at the table. How many cookies did she have left?

Fraction Multiplication

Fraction x Fraction with Decimal Notation

Fraction Notation → Decimal Notation
Fraction x Fraction with Decimal Notation

Fraction Notation \[ \frac{7}{10} \times \frac{2}{10} = \frac{14}{100} = 0.14 \]

Decimal Notation

Grade 5 – Module 4 – Lesson 17

Fraction x Fraction with Decimal Notation

\[ 2.3 \times 1.8 = \frac{23}{10} \times \frac{18}{10} = \frac{414}{100} = 4.14 \]

2.3 (tenths)
\[ \times \frac{18}{10} = 4.14 \] (hundredths)

Grade 5 – Module 4 – Lesson 18

Area with Fractional Side Lengths

\[ 3 \text{ units} \times 2 \text{ units} = 6 \text{ units}^2 \]

Grade 5 – Module 5 – Lesson 12

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Area with Fractional Side Lengths

Grade 5 – Module 5 – Lesson 12

Area with Fractional Side Lengths

Grade 5 – Module 5 – Lesson 13

Area with Fractional Side Lengths

Grade 5 – Module 5 – Lesson 13
Whole Number ÷ Unit Fraction
Jenny has 2 pounds of pecans. If she puts \( \frac{1}{2} \) pound in each bag, how many bags can she make?

Measurement Division & Tape Diagrams

2 ÷ \( \frac{1}{2} \) = 4
She can make 4 bags.

Grade 5 – Module 4 – Lesson 25

Whole Number ÷ Unit Fraction
Jenny has 2 pounds of pecans. If this is \( \frac{1}{2} \) the number she needs to make pecan pies, how many pounds will she need?

Partitive Division & Number Line

Grade 5 – Module 4 – Lesson 25

Try Problem # 18.

If 2 is \( \frac{1}{4} \), the whole is __. 1 whole is __.

Grade 5 – Module 4 – Lesson 25
Unit Fraction ÷ Whole Number
Nolan gives some pans of brownies to his 3 friends to share equally. If he has \( \frac{1}{2} \) a pan of brownies, how much of a pan will each friend get?

Each friend will get \( \frac{1}{6} \) pan of brownies.

Grade 5 – Module 4 – Lesson 26

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Continuation of Multiplication and Division in the Middle Grades
The students will complete their work with fraction operations in Grade 6 by dividing fractions by fractions. (6.NS.1)

Grade 6 – Module 2 – Lesson 4
Continuation of Multiplication and Division in the Middle Grades

\[ \frac{3}{4} \div \frac{1}{4} \]

Drew a model to support your answer.

There are 3 one-fourths in three-fourths.

Grade 6 – Module 2 – Lesson 5

Key Points

• Extends the work of K-2.
• Use of unit language.
• Familiar models support understanding.
• Fractions operate as whole numbers.
• Supports “Story of Ratios”.

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