Functions for ALL: Toward a Rigorous and Thorough Understanding
Louisiana Teacher Leader 2016
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What is a function? _____

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Excerpts from CCSS Functions Progressions

Grade 8

Define, evaluate, and compare functions*. Since the elementary grades, students have been describing patterns and expressing relationships between quantities. These ideas become semi-formal in Grade 8 with the introduction of the concept of function: a rule that assigns to each input exactly one output (8.F.1). Formal language, such as domain and range, and function notation may be postponed until high school. Building on their earlier experiences with graphs and tables in Grades 6 and 7, students explore functional relationships algebraically, graphically, numerically in tables, and through verbal descriptions. (8.F.2) They explain correspondences between equations, verbal descriptions, tables, and graphs (MP.1). Repeated reasoning about entries in tables or points on graphs results in equations for functional relationships (MP.8).*

Eureka Math **Grade 8 Module 5 End-of-Module Assessment Item 1**

a. We define x as a year between 2008 and 2013 and y as the total number of smartphones sold that year, in millions. The table shows values of x and corresponding y values.

Year (x)	2008	2009	2010	2011	2012	2013
Number of smartphones in millions (y)	3.7	17.3	42.4	90	125	153.2

i. How many smartphones were sold in 2009?

ii. In which year were 90 million smartphones sold?

iii. Is y a function of x? Explain why or why not.

b. Randy began completing the table below to represent a particular linear function. Write an equation to represent the function he was using and complete the table for him.

Input (x)	-3	-1	0	$1/2$	1	2	3
Output (y)	-5		4				13

c. Create the graph of the function in part (b).

d. At NYU in 2013, the cost of the weekly meal plan options could be described as a function of the number of meals. Is the cost of the meal plan a linear or nonlinear function? Explain.

8 meals: \$125/week 10 meals: \$135/week 12 meals: \$155/week 21 meals: \$220/week

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High School

Understand the concept of a function and use function notation. *Building on semi-formal notions of functions from Grade 8, students in high school begin to use formal notation and language for functions. Now the input/output relationship is a correspondence between two sets: the domain and the range (F-IF.1). The domain is the set of input values, and the range is the set of output values. A key advantage of function notation is that the correspondence is built into the notation. For example, $f(5)$ is shorthand for "the output value of f when the input value is 5."*

Although it is common to say "the function $f(x)$," the notation $f(x)$ refers to a single output value when the input value is x. To talk about the function as a whole, write f, or perhaps "the function f, where $f(x) = 3x + 4$." The x is merely a placeholder, so $f(t) = 3t + 4$ describes exactly the same function.

Eureka Math **High School Definition of Function from Algebra 1 Module 3 Lesson 8**

FUNCTION: A *function* is a correspondence between two sets, X and Y, in which each element of X is matched to one and only one element of Y. The set X is called the *domain of the function*.

The notation $f: X \to Y$ is used to name the function and describes both X and Y. If x is an element in the domain X of a function $f: X \to Y$, then x is matched to an element of Y called $f(x)$. We say $f(x)$ is the value in Y that denotes the *output* or *image* of f corresponding to the *input* x.

The *range (or image)* of a function $f: X \to Y$ is the subset of Y, denoted $f(X)$, defined by the following property: y is an element of $f(X)$ if and only if there is an x in X such that $f(x) = y$.

A sequence is a function whose domain is a subset of the integers (F-IF.3). In fact, many patterns explored in grades K–8 can be considered sequences. For example, the sequence $4, 7, 10, 13, 16, \ldots$ might be described as a "plus 3 pattern" because terms are computed by adding 3 to the previous term. To show how the sequence can be considered a function, we need an index that indicates which term of the sequence we are talking about, and which serves as an input value to the function. Deciding that the 4 corresponds to an index value of 1, we make a table showing the correspondence. The sequence can be described recursively by the rule $f(1) = 4$, $f(n+1) = f(n) + 3$, for $n \geq 2$. Notice that the recursive definition requires both a starting value and a rule for computing subsequent terms. The sequence can also be described with the closed formula $f(n) = 3n + 1$, for integers $n \geq 1$. Notice that the domain is included as part of the description. A graph of the sequence consists of discrete points because the specification does not indicate what happens "between the dots."

Eureka Math **Algebra 1 Module 3 Lesson 2: Exit Ticket**

1. Consider the sequence following a minus 8 pattern: $9, 1, -7, -15, \ldots$.
   a. Write an explicit formula for the sequence.

   b. Write a recursive formula for the sequence.

   c. Find the 38th term of the sequence.

2. Consider the sequence given by the formula $a(n+1) = 5a(n)$ and $a(1) = 2$ for $n \geq 1$.
   a. Explain what the formula means.

   b. List the first five terms of the sequence.

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***Eureka Math* Algebra 1 Module 3 Lesson 11 Exploratory Challenge 3**

Instead of Printing the set G to the screen, we can use another command, *Plot*, to plot the points on a Cartesian plane.

```
Declare x integer
  Initialize G as {}
  For all x from 2 to 8
          Append (x, 2x + 3) to G
  Next x
  Plot G
```

Output:

Build new functions from existing functions. *With a basis of experiences in building specific functions from scratch, students start to develop a notion of naturally occurring families of functions that deserve particular attention. It is possible to harden the curriculum too soon around these families, before students have enough experience to get a feel for the effects of different parameters. Students can start getting that feel by playing around with the effect on the graph of simple algebraic transformations of the input and output variables (F-BF.3). Quadratic and absolute value functions are good contexts for getting a sense of the effects of many of these transformations, but eventually students need to understand these ideas abstractly and be able to talk about them for any function f.*

***Eureka Math* Algebra 1 Module 3 Lesson 20**

Closing

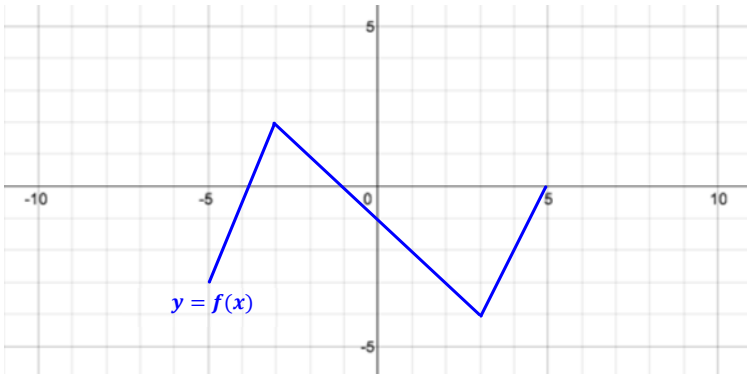
- The transformations that translate and scale familiar functions, like the absolute value function, also apply to piecewise functions and to any function in general.
- By focusing on strategic points in the graph of a piecewise function, the entire graph of the function can be translated and scaled by manipulating the coordinates of those few points.

Exit Ticket

The graph of a piecewise function f is shown below.

Let $p(x) = f(x-2)$, $q(x) = \frac{1}{2}f(x-2)$, and $r(x) = \frac{1}{2}f(x-2) + 3$.

Graph $y = p(x)$, $y = q(x)$, and $y = r(x)$ on the same set of axes as the graph of $y = f(x)$.



References

The Common Core Standards Writing Team. (1 March 2013). *Progressions for the Common Core State Standards in Mathematics (draft). Grade 8, High School, Functions.* Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

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