Scientific Notation (ECR)

Overview

This instructional task requires students to simplify expressions and answer questions with scientific notation.

Standards

Work with radicals and integer exponents

8.EE.A.3 Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9, and determine that the world population is more than 20 times larger.*

8.EE.A.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade Level Standards	The Following Standards Will Prepare Them:	Items to Check for Task Readiness:	Sample Remediation Items :
8.EE.A.3	• 4.OA.A.2 • 5.NBT.A.2	1. Write 0.0045 in scientific notation. a. 4.5×10^{-3} 2. http://www.illustrativemathematics.org/illustrations/476 3. http://www.illustrativemathematics.org/illustrations/1593	• http://www.illustrativemathematics.org/illustrations/263 • http://www.illustrativemathematics.org/illustrations/1524 • http://www.illustrativemathematics.org/illustrations/1620 • http://learnzillion.com/lessonsets/272-estimate-and-compare-with-integers-to-the-power-of-10
8.EE.A.4	• 7.EE.B.3	1. $4 \times 10^{-3} \cdot 2 \times 10^6$ a. 8×10^3 2. http://www.illustrativemathematics.org/illustrations/113	• http://www.illustrativemathematics.org/illustrations/108 • http://www.illustrativemathematics.org/illustrations/478 • http://learnzillion.com/lessonsets/276-perform-operations-with-numbers-expressed-in-scientific-notation-including-decimals

Real-world preparation: The following questions will prepare students for some of the real-world components of this task:

What is a roundtrip? A roundtrip means that you will travel from one location to another and then return to the original location, generally following the same route.

After the Task:

Be sure to remind students that radical and integer exponents continue to be used throughout 8th grade and high school math courses.

Common Errors/Misunderstandings by Problem Number:

- Some students may forget to multiply each distance by 2 to account for the roundtrip.

- Some students may divide the smaller number by the larger number instead of the larger number by the smaller number.

- Remind students to treat 10^5 as like terms.

- Students may make a mistake when subtracting the exponents. Remind them that $5 - 8 = -3$. Discuss with students what this answer means in terms of the situation described (the trip would take less than one second).

- Students may leave their answers as 25×10^{-5}. Remind them that they should only have one non-zero digit in front of the decimal in scientific notation.

- Students may give their answers as 1×10^2 drops instead of 100 drops. Although this answer is correct, ask them how many drops that would be. Ask them which representation of the number of drops would be easier for most people to understand.

- Students may leave their answers as 9462×10^{-5} liters. Remind them that they should only have one non-zero digit in front of the decimal in scientific notation.

Student Extended Constructed Response

City 1	City 2	Approximate Distance Between Cities
New Orleans, Louisiana	Washington, DC	2×10^6 meters
New Orleans, Louisiana	Nashville, Tennessee	9×10^5 meters
New Orleans, Louisiana	Seattle, Washington	4×10^6 meters
New Orleans, Louisiana	Shreveport, Louisiana	5×10^5 meters

1. What is the combined distance that you would travel if you completed a roundtrip from New Orleans, Louisiana, to Washington, DC, and a roundtrip from New Orleans, Louisiana, to Seattle, Washington? Write your answer in scientific notation.

2. How many times farther is the trip from New Orleans, Louisiana, to Washington, DC, than the trip from New Orleans, Louisiana, to Nashville, Tennessee?

3. How much shorter is the trip from New Orleans, Louisiana, to Shreveport, Louisiana, than the trip from New Orleans, Louisiana, to Nashville, Tennessee?

4. If a car were invented that could travel at the speed of light, approximately 3×10^8 meters per second, how long would it take to drive from Shreveport, Louisiana, to New Orleans, Louisiana?

The volume of a drop of gasoline is approximately 5×10^{-5} liter.

5. You stopped to put gasoline in your car. As you finished filling your gas tank, five drops of gas dripped to the ground. What is the total volume of leaked gasoline? Write your answer in scientific notation.

6. How many drops would have to fall for 5×10^{-3} liter of gas to be lost?

7. You already have a 9457×10^{-5} l puddle of gasoline. If one more drop falls into the puddle, how much gasoline would be in the puddle? Write your answer in scientific notation.

Extended Constructed Response Exemplar Response

City 1	City 2	Approximate Distance Between Cities
New Orleans, Louisiana	Washington, DC	2×10^6 meters
New Orleans, Louisiana	Nashville, Tennessee	9×10^5 meters
New Orleans, Louisiana	Seattle, Washington	4×10^6 meters
New Orleans, Louisiana	Shreveport, Louisiana	5×10^5 meters

1. What is the combined distance that you would travel if you completed a roundtrip from New Orleans, Louisiana, to Washington, DC, and a roundtrip from New Orleans, Louisiana, to Seattle, Washington? Write your answer in scientific notation.

Roundtrip from New Orleans, Louisiana, to Washington, DC: $2 \times 2 \times 10^6 = 4 \times 10^6 meters$

Roundtrip from New Orleans, Louisiana, to Seattle, Washington: $2 \times 4 \times 10^6 = 8 \times 10^6$ *meters*

Combined Travel: $4 \times 10^6 meters + 8 \times 10^6$ *meters*

12×10^6 *meters*

$1.2 \times 10 \times 10^6$ *meters*

1.2×10^7 *meters*

The combined distance is 1.2×10^7 *meters.*

2. How many times farther is the trip from New Orleans, Louisiana, to Washington, DC, than the trip from New Orleans, Louisiana, to Nashville, Tennessee?

$$\frac{2 \times 10^6}{9 \times 10^5}$$

$$\frac{2}{9} \times \frac{10^6}{10^5}$$

$$\frac{2}{9} \times 10^1$$

$$.22 \times 10$$

$$2.2$$

The trip from New Orleans, Louisiana, to Washington, DC, is about twice as long as the trip from New Orleans, Louisiana, to Nashville, Tennessee.

3. How much shorter is the trip from New Orleans, Louisiana, to Shreveport, Louisiana, than the trip from New Orleans, Louisiana, to Nashville, Tennessee?

$$9 \times 10^5 - 5 \times 10^5$$
$$4 \times 10^5 \ meters$$

The trip from New Orleans, Louisiana, to Shreveport, Louisiana, is 4×10^5 meters shorter than the trip from New Orleans, Louisiana, to Nashville, Tennessee.

4. If a car were invented that could travel at the speed of light, approximately 3×10^8 meters per second, how long would it take to drive from Shreveport, Louisiana, to New Orleans, Louisiana?

$$\frac{5 \times 10^5 \ meters}{3 \times 10^8 meters \ per \ second}$$
$$\frac{5}{3} \times \frac{10^5}{10^8} \ seconds$$
$$\frac{5}{3} \times 10^{-3} \ seconds$$
$$1.67 \times 10^{-3} \ seconds$$

It would take 1.67×10^{-3} seconds to drive from Shreveport to New Orleans.

The volume of a drop of gasoline is approximately 5×10^{-5} liter.

5. You stopped to put gasoline in your car. As you finished filling your gas tank, five drops of gas dripped to the ground. What is the total volume of leaked gasoline? Write your answer in scientific notation.

$$5 \times 5 \times 10^{-5}$$
$$25 \times 10^{-5}$$
$$2.5 \times 10^{-4}$$

2.5×10^{-4} liter of gasoline leaked.

6. How many drops would have to fall for 5×10^{-3} liter of gas to be lost?

$$\frac{5 \times 10^{-3} \ liter}{5 \times 10^{-5} \ liter \ per \ drop}$$
$$\frac{5}{5} \times \frac{10^{-3}}{10^{-5}} \ drops$$
$$\frac{5}{5} \times 10^2 \ drops$$

$$1 \times 10^2 \; drops$$

$$100 \; drops$$

100 drops would have to fall for 5×10^{-3} *liter of gas to be lost.*

7. You already have a 9457×10^{-5} liter puddle of gasoline. If one more drop falls into the puddle, how much gasoline would be in the puddle? Write your answer in scientific notation.

$$9457 \times 10^{-5} liter + 5 \times 10^{-5} \; liter$$

$$9462 \times 10^{-5} \; liter$$

$$9.462 \times 10^3 \times 10^{-5} \; liter$$

$$9.462 \times 10^{-2} \; liter$$

The puddle would be 9.462×10^{-2} *liter.*

Dan's Leaking Bottle (ECR)

Overview
Students write a function from a given situation. They must graph the function and determine the rate of change and initial value.

Standards
Use functions to model relationships between quantities.

8.F.B.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (*x, y*) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models and in terms of its graph or a table of values.

Prior to the Task
Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade Level Standard	The Following Standards Will Prepare Them:	Items to Check for Task Readiness:	Sample Remediation Items :

8.F.B.4	• 7.RP.A.2
	• 8.F.A.3
		1. State the slope and y-intercept of the following equation:
		
		$$y = \frac{2}{3}x + 5$$
		
		  a. *Slope:* $\frac{2}{3}$
		    y-intercept: (0, 5)
		2. http://www.illustrativemathematics.org/illustrations/120
		3. http://www.illustrativemathematics.org/illustrations/247
			• http://www.illustrativemathematics.org/illustrations/813
			• http://www.illustrativemathematics.org/illustrations/180
			• http://www.illustrativemathematics.org/illustrations/181
			• http://learnzillion.com/lessonsets/686-construct-functions-determine-slope-and-initial-value-and-interpret-in-terms-of-a-situation
			• http://learnzillion.com/lessonsets/357-construct-functions-to-model-linear-relationships-between-two-quantities
			• http://learnzillion.com/lessonsets/52-construct-and-compare-linear-functions

Real-world preparation: The following questions will prepare students for some of the real-world components of this task:

What is a water dispenser? A water dispenser is a type of large bottle that holds water. You can use a nozzle to get the water out. If there is a hole in the dispenser, the water will leak out. The rate at which the water leaks will depend on the size of the hole.
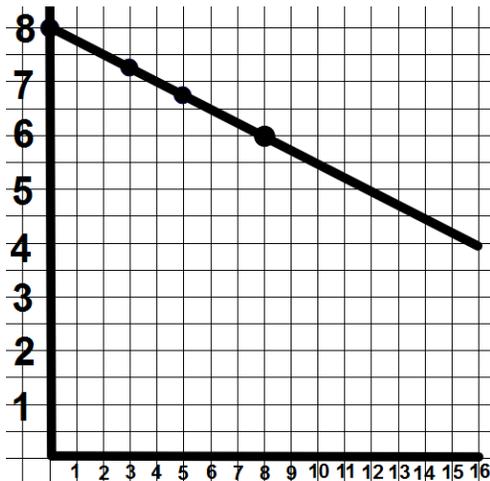
After the Task:
Students may struggle with graphing the function. They may want to graph the points by how much water has leaked out rather than subtracting from the original 8-inch water line. Students who struggle to find the rate of change may need additional practice in finding the unit rate.

Student Extended Constructed Response

Dan has an automatic water dispenser he uses to give his dogs water. The bottle holds 64 ounces of water when it is full. Lately, Dan notices that the bottle is leaking. Dan measures the height of the water in the dispenser bottle and finds that the level of water is 8 inches from the bottom of the bottle. After 3 minutes he finds that the water level has dropped ¾ of an inch because of the leak. After another 5 minutes, he finds that the water level has dropped another 1 ¼ inches. If the water is leaking at a constant rate, answer the following questions.

1. Using the description above, write a function with defined variables to show the relationship between the time that passes and the water level in the bottle. Graph the function.

2. Identify the rate of change and the initial value. Explain what the rate of change and initial value mean in the context of Dan's leaking bottle.

Extended Constructed Response Exemplar Response

Dan has an automatic water dispenser he uses to give his dogs water. The bottle holds 64 ounces of water when it is full. Lately, Dan notices that the bottle is leaking. Dan measures the height of the water in the dispenser bottle and finds that the level of water is 8 inches from the bottom of the bottle. After 3 minutes he finds that the water level has dropped ¾ of an inch because of the leak. After another 5 minutes, he finds that the water level has dropped another 1 ¼ inches. If the water is leaking at a constant rate, answer the following questions.

1. Using the description above, write a function with defined variables to show the relationship between the time that passes and the water level in the bottle. Graph the function.

8 – ¼ m = w; where m is the number of minutes that have passed, w is the amount of water left in the bottle

The graph should be similar to this. The line must go through the points (0, 8), (3, 7 ¼), and (8, 6). These are the points they can solve for with the provided information.

2. Identify the rate of change and the initial value. Explain what the rate of change and initial value mean in the context of Dan's leaking bottle.
The rate of change is ¼ of an inch per minute. This means the water level drops ¼ of an inch every minute of time that passes.
The initial value is 8. 8 inches; this is the original height of the water from the bottom of the bottle at which Dan begins checking the level.

Summer Jobs (ECR)

Overview

Students are asked to write functions when given the pay for two different people. They must also look at a graph of a person's pay and describe the features of the graph.

Standards

Define, evaluate, and compare functions.

8.F.A.2 Compare properties of two functions, each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*

Use functions to model relationships between quantities.

8.F.B.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models and in terms of its graph or a table of values.

8.F.B.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade Level Standard	The Following Standards Will Prepare Them:	Items to Check for Task Readiness:	Sample Remediation Items :
8.F.A.2	• 7.RP.A.2 • 8.EE.B.5 • 8.EE.B.6 • 8.F.A.1	1. Which has a greater rate of change? 5x + 9 = y or 9x + 9 = y a. 9x + 9 = y 2. Which would be cheaper for x = $6? 4x + 8 = y or 3x + 12 = y a. 3x + 12 = y 3. http://www.illustrativemathematics.org/illustrations/641	• http://www.illustrativemathematics.org/illustrations/129 • http://www.illustrativemathematics.org/illustrations/1537 • http://www.illustrativemathematics.org/illustrations/184 • http://www.illustrativemathematics.org/illustrations/1165 • http://learnzillion.com/lessonsets/271-understand-and-compare-functions • http://learnzillion.com/lessonsets/52-construct-and-compare-linear-functions

Grade Level Standard	The Following Standards Will Prepare Them:	Items to Check for Task Readiness:	Sample Remediation Items :

8.F.B.4	• 7.RP.A.2
	• 8.F.A.3
		1. What is the initial value and rate of change for 3x + 8 = y?
		   a. Initial value: 8; rate of change: 3
		2. What is the rate of change for the ratio $18 for 3 hours?
		   a. 6
		3. http://www.illustrativemathematics.org/illustrations/120
		4. http://www.illustrativemathematics.org/illustrations/247
			• http://www.illustrativemathematics.org/illustrations/813
			• http://www.illustrativemathematics.org/illustrations/180
			• http://www.illustrativemathematics.org/illustrations/181
			• http://learnzillion.com/lessonsets/686-construct-functions-determine-slope-and-initial-value-and-interpret-in-terms-of-a-situation
			• http://learnzillion.com/lessonsets/357-construct-functions-to-model-linear-relationships-between-two-quantities
			• http://learnzillion.com/lessonsets/52-construct-and-compare-linear-functions

8.F.B.5	• 8.F.A.1
	• 8.F.A.2
	• 8.F.A.3
		1. Describe what a graph for the linear equation 2x + 9 = y would look like.
		   a. The slope would be 2, meaning it would go up two units for every unit moved to the right. 9 is the initial value and would be the y-intercept.
		2. Describe what a graph for 9.25x = y looks like.
		   a. The initial value is 0. The graph would begin at the origin. The slope is 9.25, meaning it would go up 9.25 units for every unit moved to the right.
		3. http://www.illustrativemathematics.org/illustrations/628
		4. http://www.illustrativemathematics.org/illustrations/632
		5. http://www.illustrativemathematics.org/illustrations/633
		6. http://www.illustrativemathematics.org/illustrations/674
			• http://www.illustrativemathematics.org/illustrations/1165
			• http://www.illustrativemathematics.org/illustrations/641
			• http://www.illustrativemathematics.org/illustrations/813
			• http://learnzillion.com/lessonsets/705-describe-functions-by-analyzing-and-building-graphs
			• http://learnzillion.com/lessonsets/358-describe-the-functional-relationship-between-two-quantities-by-analyzing-a-graph

Real-world preparation: The following questions will prepare students for some of the real-world components of this task:

What does "earnings per hour" mean? It is the amount of money a person earns for each hour worked.
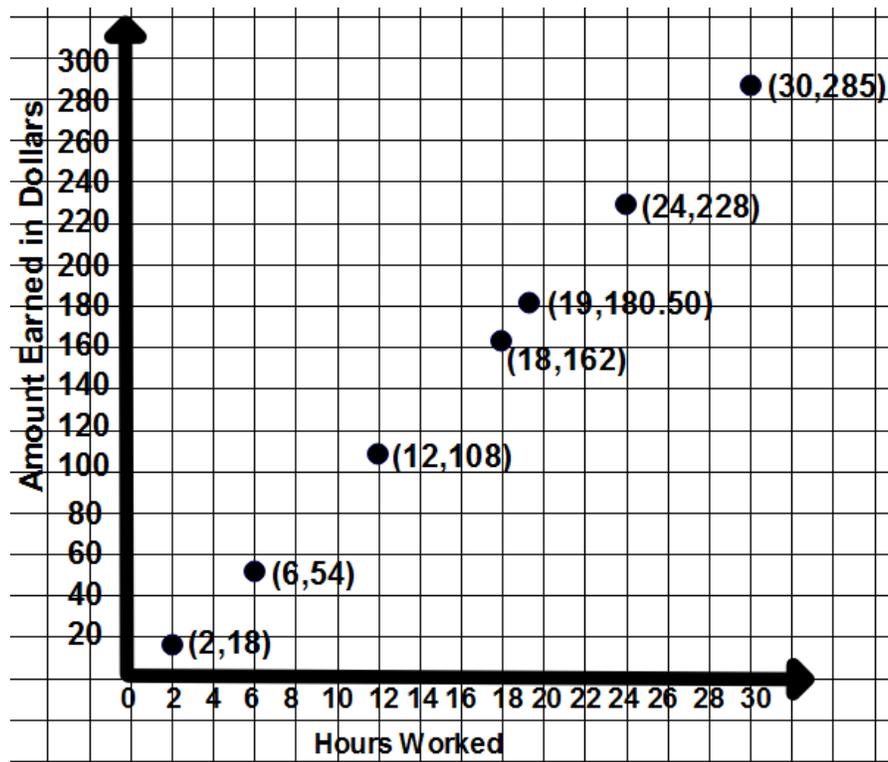
After the Task:

Students may struggle with the initial value for the two functions. This could be from not really understanding the parts of a function written in y-intercept form. Additional practice may be required in writing equations in y-intercept form and then defining the parts.

Student Extended Constructed Response

Lonnie and Tony get summer jobs. Lonnie makes $9.25 per hour. The table below shows how Tony is paid.

Tony's hours worked	4	9	13	18
Tony's earnings (dollars)	$35	$78.75	$113.75	$157.50

1. Who earns more money per hour? How much more does that person make per hour?

2. Write a function to model the relationship for Tony's pay shown in the table. Explain the rate of change and initial value of the function in terms of the given situation.

3. Is it possible for Lonnie and Tony to work the same number of hours and make the same amount of money? Explain.

4. Marcus works for the same company. The graph represents his pay for a 30-hour work week.

Graph with y-axis labeled "Amount Earned in Dollars" (0 to 300) and x-axis labeled "Hours Worked" (0 to 30). Plotted points: (2,18), (6,54), (12,108), (18,162), (19,180.50), (24,228), (30,285).

Explain whether this graph represents a linear or non-linear function. Justify your answer with information from the graph.

Extended Constructed Response Exemplar Response

Lonnie and Tony get summer jobs. Lonnie makes $9.25 per hour. The table below shows how Tony is paid.

Tony's hours worked	4	9	13	18
Tony's earnings (dollars)	$35	$78.75	$113.75	$157.50

1. Who earns more money per hour? How much more does that person make per hour?
   Lonnie earns more money per hour. He earns $0.50 per hour more than Tony.

2. Write a function to model the relationship for Tony's pay shown in the table. Explain the rate of change and initial value of the function in terms of the given situation.
   Tony   y = 8.75x + 0 or y = 8.75x; the rate of change is 8.75 representing the amount of money Tony earns per hour. The initial value is 0, which means that at 0 hours Tony makes $0.

3. Could Lonnie and Tony work the same number of hours and make the same amount of money? Explain.
   They could never make the same amount of money, because they earn different rates. They both earn $0 for working 0 hours.

1. Marcus works for the same company. The graph represents his pay for a 30-hour work week.
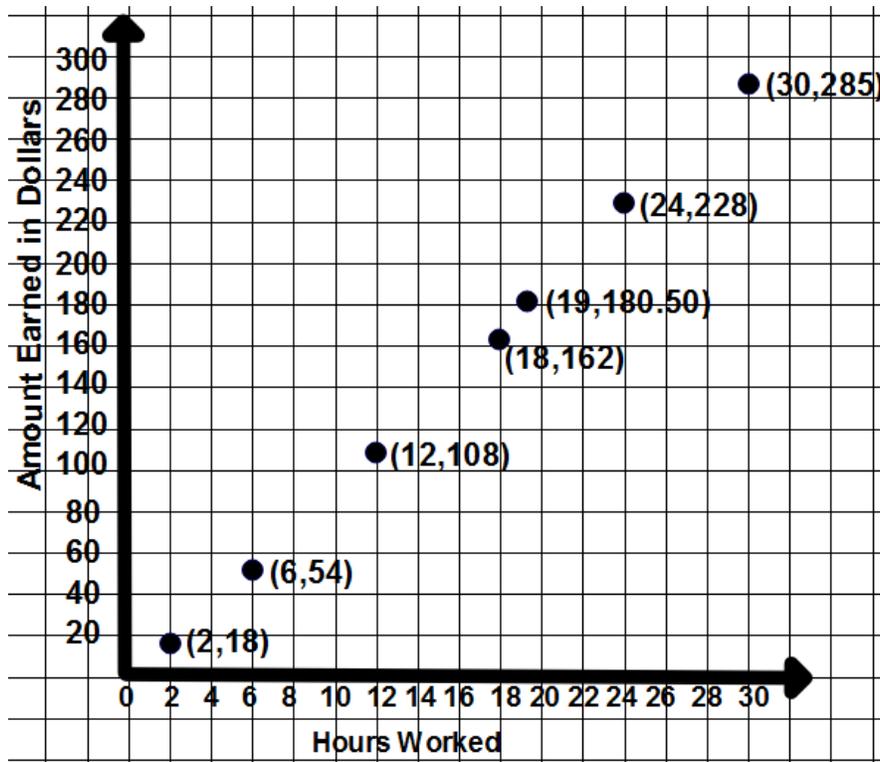
Explain whether this graph represents a linear or non-linear function. Justify your answer with information from the graph.

For the first 18 hours, Marcus makes $9 per hour. The rate of change for the points (2, 18) through (18, 162) is $9. At 19 hours, the rate of change is no longer $9. It is $9.50. This rate of change continues through 30 hours. This graph is non-linear since the rate of change is not the same for all points.

Party Zone Palace (ECR)

Overview

In this task, students are applying the definition of a function, identifying linear functions, and comparing the rates of change on different functions to plan a birthday party.

Standards

Define, evaluate, and compare functions.

8.F.A.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.[1]

8.F.A.2 Compare properties of two functions, each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*

8.F.A.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4), and (3,9), which are not on a straight line.*

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade Level Standard	The Following Standards Will Prepare Them:	Items to Check for Task Readiness:	Sample Remediation Items :
8.F.A.1	• 7.RP.A.2	1. Does this table represent a function? | Time (hours)	Distance (miles)
---	---
2	65
3	97.5
4	130
8	260	 a. Yes, because there is exactly one output for each input. 2. http://www.illustrativemathematics.org/illustrations/715 3. http://www.illustrativemathematics.org/illustrations/713	• http://www.illustrativemathematics.org/illustrations/104 • http://learnzillion.com/lessonsets/420-understand-functions-and-their-graphs • http://learnzillion.com/lessonsets/271-understand-and-compare-functions
8.F.A.2	• 8.EE.B.5 • 8.EE.B.6 • 8.F.A.1 • 7.RP.A.2	1. Determine the unit rate for each situation. a. | # of People	Total Cost
---	---
6	24
12	48
24	96		• http://www.illustrativemathematics.org/illustrations/129 • http://www.illustrativemathematics.org/illustrations/1537 • http://www.illustrativemathematics.org/illustrations/184 • http://www.illustrativemathematics.org/illustrations/1165

Grade Level Standard	The Following Standards Will Prepare Them:	Items to Check for Task Readiness:	Sample Remediation Items :
		i. *Unit Rate: $4 per person* b. *d = 65t,* where *t* is the time in hours and *d* is the distance in miles. i. *Unit rate is 65 miles per hour* 2. http://www.illustrativemathematics.org/illustrations/641	• http://learnzillion.com/lessonsets/271-understand-and-compare-functions • http://learnzillion.com/lessonsets/52-construct-and-compare-linear-functions
8.F.A.3	• 8.F.A.2 • 8.EE.B.6 • 8.F.A.1	1. Determine which functions are linear. A. *y = 4x + 7* B. *y = 3x²* C. $y = \frac{1}{5}x - 12$ i. *A and C are linear. B is not linear because x is squared.* 2. http://www.illustrativemathematics.org/illustrations/813	• http://www.illustrativemathematics.org/illustrations/713 • http://www.illustrativemathematics.org/illustrations/641 • http://learnzillion.com/lessonsets/561-interpret-the-equation-y-mx-b • http://learnzillion.com/lessonsets/277-interpret-the-equation-y-mx-b-as-defining-a-linear-function

Real-world preparation: The following discussion will prepare students for some of the real-world components of this task:

When reserving a birthday party, many venues offer different packages for people to choose from. These packages are usually based on the number of people attending the party.

After the Task:

Students may struggle when trying to find the unit rate for each package. If students do struggle, they may need remediation with 7.RP.2 in determining unit rates from equations, tables, and graphs. Package C does not have a unit rate, as the price is based upon groups of people. This causes package C to not represent a *linear function*. Students may need help remembering what determines if a function is linear or not.

This problem could be extended by adding in different elements for students to consider like whether a package includes the cake, balloons, invitations, or tickets for the games. Then students could make their own assumptions when determining the better deal for Benjamin's parents.

Student Extended Constructed Response

Benjamin is turning 9 and wants to have his birthday party at Party Zone Palace. Party Zone Palace offers several different birthday packages. The pricing for three packages are displayed below.

Package A	Package B		Package C	
$C = 8p + 12$ C = Total cost P = Number of people	**Number of people**	**Total cost**	**Number of people**	**Cost per person**
	3	$33	1-15	$15
	4	$44	16-30	$12
	5	$55	more than 30	$9
	9	$99		

1. Does Package C represent a function? Explain your reasoning.

2. Explain how you know that Package A and Package B represent linear functions.

3. Does package A or Package B have the greater cost per person? Explain your reasoning.

4. Benjamin wants to invite 36 people to his party. Determine which package his parents should buy for his birthday party in order to get the better deal. Justify your reasoning with equations and/or tables.

Extended Constructed Response Exemplar Response

Benjamin is turning 9 and wants to have his birthday party at Party Zone Palace. Party Zone Palace offers several different birthday packages. The pricing for three packages are displayed below.

Package A	Package B		Package C	
$C = 8p + 12$ C = Total cost p = Number of people	**Number of people**	**Total cost**	**Number of people**	**Cost per person**
	3	$33	1-15	$15
	4	$44	16-30	$12
	5	$55	more than 30	$9
	9	$99		

1. Does Package C represent a function? Explain your reasoning.
   a. *Yes, Package C represents a function because there is exactly one output for each input.*

2. Explain how you know that Package A and Package B represent linear functions.
   a. *Package A and Package B represent linear functions because they both have a constant rate of change.*

3. Does package A or Package B have the greater cost per person? Explain your reasoning.
   a. *Package A: C = 8p + 12   Package B: Find the Unit Rate: 33 ÷ 3 = 11*

   b. *Package A charges $8 per person, while Package B costs $11 per person. Therefore, Package B has a greater price per person.*

4. Benjamin wants to invite 36 people to his party. Determine which package his parents should buy for his birthday party in order to get the better deal. Justify your reasoning with equations and/or tables.

   Package A: C = 8(36) + 12   Package B: C = 11(36)   Package C: C = 9(36)

   C = $300   C = $396   C = $324

   Package A is the better deal costing only $300 for 36 people.

Students may choose to extend the table for Package B to show their reasoning.

Congruence Transformations (ECR)

Overview

Students will practice performing various congruence transformations and describing the effects of the transformations.

Standards

Understand congruence and similarity using physical models, transparencies, or geometry software.

8.G.A.1 Verify experimentally the properties of rotations, reflections, and translations:

  a. Lines are taken to lines, and line segments to line segments, of the same length.
  b. Angles are taken to angles of the same measure.
  c. Parallel lines are taken to parallel lines.

Understand congruence and similarity using physical models, transparencies, or geometry software.

8.G.A.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

8.G.A.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade Level Standard	The Following Standards Will Prepare Them:	Items to Check for Task Readiness:	Sample Remediation Items :

8.G.A.1	• 7.G.A.2
	 • 7.G.B.5
		1. $\overline{AB}$ is reflected over line *l*. What must be true about the resulting image $\overline{A'B'}$?
		   a. *The resulting image, $\overline{A'B'}$, is the same length as $\overline{AB}$.*
		 2. ΔABC is rotated around point *A*. What is true about angle *B'C'A'*?
		   a. *Angle B'C'A' is congruent to angle BCA.*
			• http://learnzillion.com/lessonsets/473-verify-properties-of-rotations-reflections-and-translations

8.G.A.2	• 8.G.A.1	1. Two triangles are drawn on a coordinate plane. How can you tell if they are congruent? a. The two triangles are congruent if one triangle can be obtained from the other through a sequence of rotations, reflections, and/or translations. 2. http://www.illustrativemathematics.org/illustrations/646	• http://learnzillion.com/lessonsets/528-understand-congruency-in-twodimensional-figures • http://learnzillion.com/lessonsets/466-assess-congruence-using-rotations-reflections-and-translations

| Grade Level Standard	The Following Standards Will Prepare Them:	Items to Check for Task Readiness:		Sample Remediation Items :
		3.	http://www.illustrativemathematics.org/illustrations/1228	
		4.	http://www.illustrativemathematics.org/illustrations/1231	
8.G.A.3	• 6.G.A.3 • 8.G.A.1	1.	Graph rectangle *RSTU* with vertices $R(2,3), S(3,1), T(-1,-1)$, and $U(-2,1)$. Rotate rectangle *RSTU* 90° clockwise around the origin. What are the coordinates of *R'S'T'U'*? a. *R'(3, -2), S'(1, -3), T'(-1, 1), and U'(1, 2)* 2. http://www.illustrativemathematics.org/illustrations/1243	• http://www.illustrativemathematics.org/illustrations/1188 • http://learnzillion.com/lessonsets/534-describe-the-effect-of-dilations-translations-rotations-and-reflections-on-twodimensional-figures-using-coordinates • http://learnzillion.com/lessonsets/476-describe-the-effects-of-dilations-translations-rotations-and-reflections-using-coordinates

After the Task:

For Part c in Question 1, students may use the original figure when performing the reflection. This would affect their answer to Part e as well.

In the second question, students may struggle with finding a starting point to map one triangle to the other. This requires a higher cognitive demand than performing given transformations. Have students practice performing various transformations on triangle *ABC* and describe how the transformations move the triangle to help students understand which transformations would be best to use. The following are sample questions that may help students with this portion of the task:

1. In which quadrant is triangle *PQR*? Which transformations would move a figure from one quadrant to another?
2. What do you notice about the orientation of the triangle? Which transformations would change the orientation of the triangle?

Student Extended Constructed Response

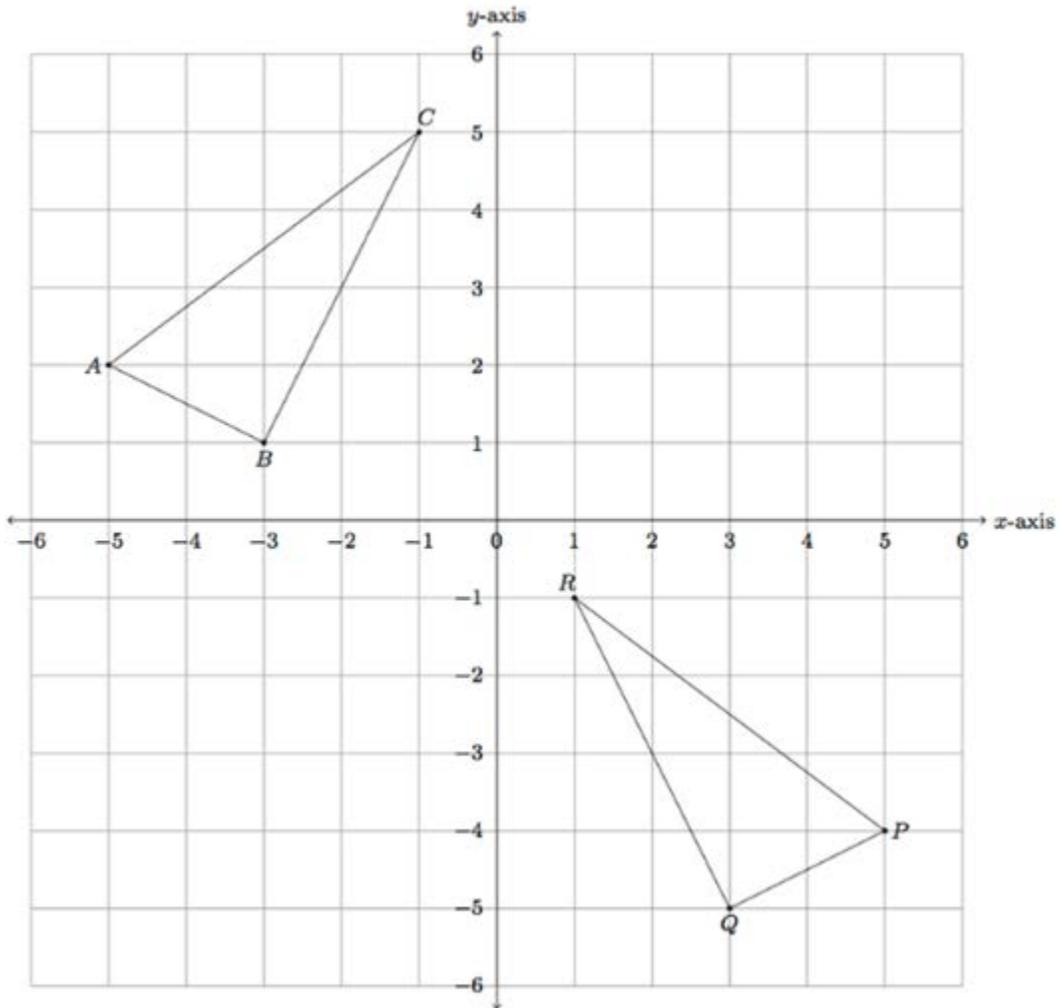1. Complete all parts. You may use a ruler and/or protractor to complete the task.
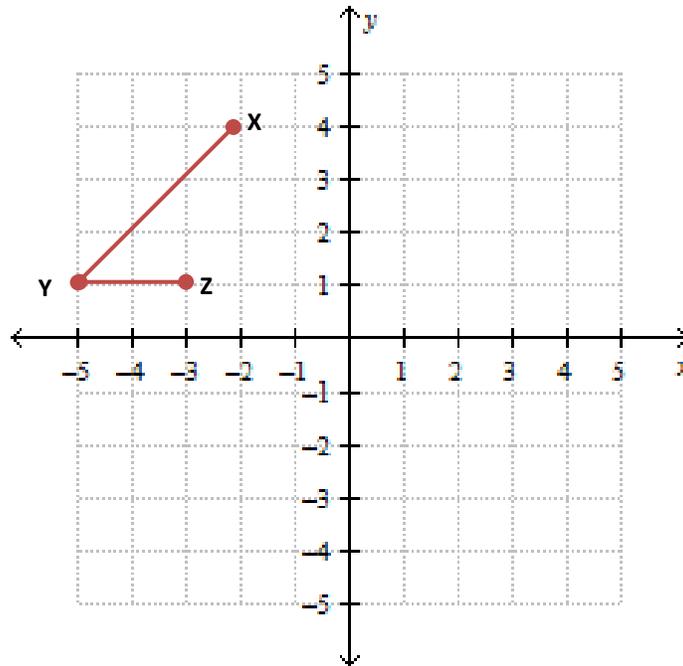
a. Rotate ∠XYZ 90˚counterclockwise about the origin. Draw and label the image ∠X'Y'Z'.

b. Name the ordered pairs for: X' _____

                                   Y' _____

                                   Z' _____

c. Reflect ∠X'Y'Z' over the y-axis. Draw and label ∠X"Y"Z".

d. What is the $m∠$XYZ? _____ What is the $m∠$X"Y"Z"?_____

How do the measures of these angles compare?
Explain why this is reasonable for an angle that is rotated and reflected.

e. Sarah thinks that she can produce the ∠X"Y"Z" from ∠XYZ with one transformation, a 180˚ rotation about the origin. Is she correct? Explain why or why not.

2. Triangles ABC and PQR are shown below in the coordinate plane.

a. Show that $\triangle ABC$ is congruent to $\triangle PQR$ with a reflection followed by a translation.

b. If you reverse the order of your reflection and translation in Part a does it still map $\triangle ABC$ to $\triangle PQR$?

c. Find a second way, different from your work in Parts a or b, to map $\triangle ABC$ to $\triangle PQR$ using translations, rotations, and/or reflections.

Task adapted from http://www.illustrativemathematics.org/illustrations/1232.

Extended Constructed Response Exemplar Response

1. Complete all parts. You may use a ruler and/or protractor to complete the task.
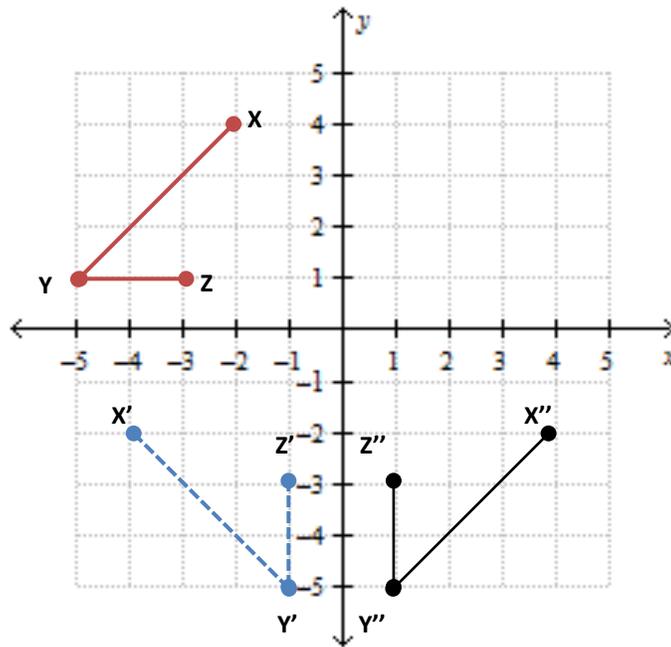
a. Rotate ∠XYZ 90˚counterclockwise about the origin. Draw and label the image ∠X'Y'Z'.

b. Name the ordered pairs for: X' __(-4, -2)_____
   Y' __(-1, -5)_____

   Z' __(-1, -3)_____

c. Reflect ∠X'Y'Z' over the y-axis. Draw and label ∠X"Y"Z".

d. What is the *m*∠XYZ? _____45˚_____ What is the *m*∠X"Y"Z"?_____45˚_____

   How do the measures of these angles compare? *The angles are congruent.*
   Explain why this is reasonable for an angle that is rotated and reflected. *Rotations and reflections produce congruent transformations, so the angles should be congruent after transformation.*

e. Sarah thinks that she can produce the ∠X"Y"Z" from ∠XYZ with one transformation, a 180˚ rotation about the origin. Is she correct? Explain why or why not.
   No, the coordinates for ∠X"Y"Z" are X"(4, -2), Y"(1, -5), and Z"(1, -3), but the coordinates of the angle if it is rotated 180˚ are X(2, -4), Y(5, -1), and Z(3, -1).
   Or
   In order to produce 180˚ rotation, two reflections are necessary.
   Or

Any other student responses that are accurate and demonstrate strong mathematical reasoning.

2. Triangles ABC and PQR are shown below in the coordinate plane.
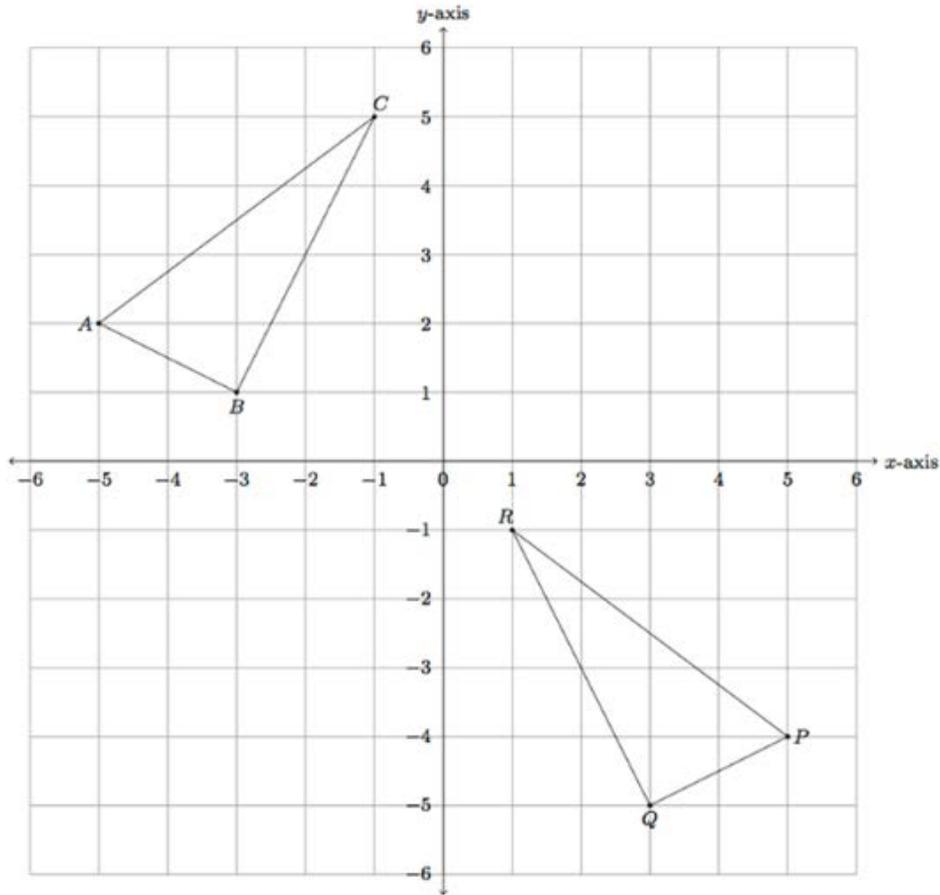
a. Show that $\triangle ABC$ is congruent to $\triangle PQR$ with a reflection followed by a translation.
   Reflect across the y-axis, then translate down 6 units.
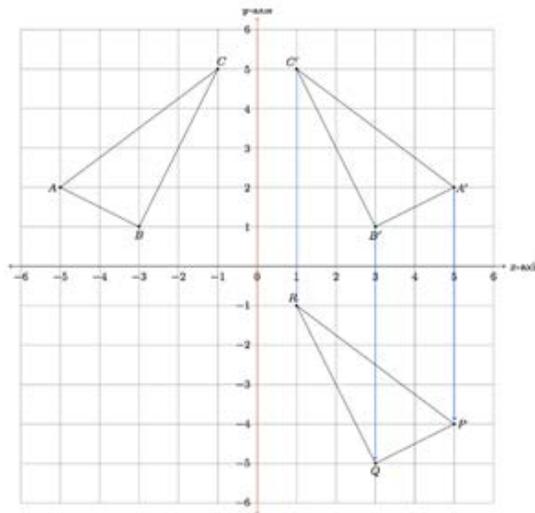
There are other possible combinations of reflections and translations, but this is the one the students will most likely choose. However, other solutions will need to be checked for accuracy.

b. If you reverse the order of your reflection and translation in Part a does it still map ΔABC to ΔPQR?

This answer is dependent on the answer in Part a. Using the reflection and translation chosen above, the answer would be yes. ΔABC translated down 6 units and then reflected over the y-axis would result in ΔPQR. See below.
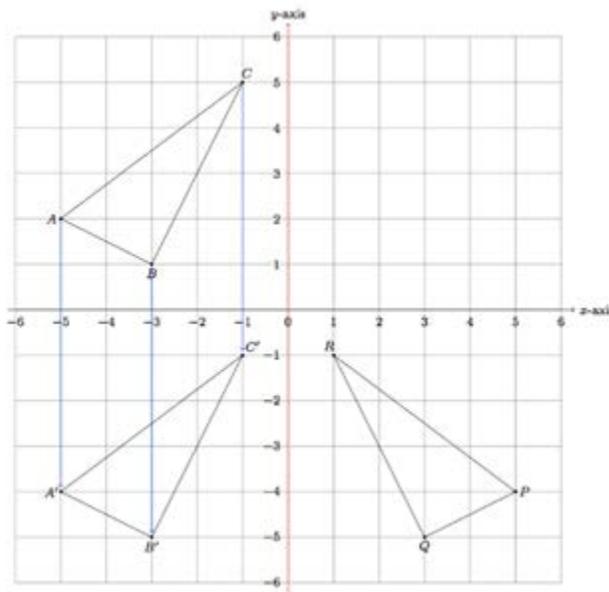
c. Find a second way, different from your work in Part a, to map ΔABC to ΔPQR using translations, rotations, and/or reflections.

There are several possible answers. Below are a few samples.