Louisiana STUDENT STANDARDS MATHEMATICS

Algebra II

Louisiana Student Standards: Companion Document for Teachers 2.0

This document is designed to assist educators in interpreting and implementing the Louisiana Student Standards for Mathematics. Found here are descriptions of each standard which answer questions about the standard's meaning and application to student understanding. Also included are the intended level of rigor and coherence links to prerequisite and corequisite standards. Examples are samples only and should not be considered an exhaustive list.

Additional information on the Louisiana Student Standards for Mathematics, including how to read the standards' codes, a listing of standards for each grade or course, and links to additional resources, is available on the Louisiana Department of Education K-12 Math Planning Page. Please direct any questions to STEM@la.gov.

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DEPARTMENT of EDUCATION
Louisiana Believes

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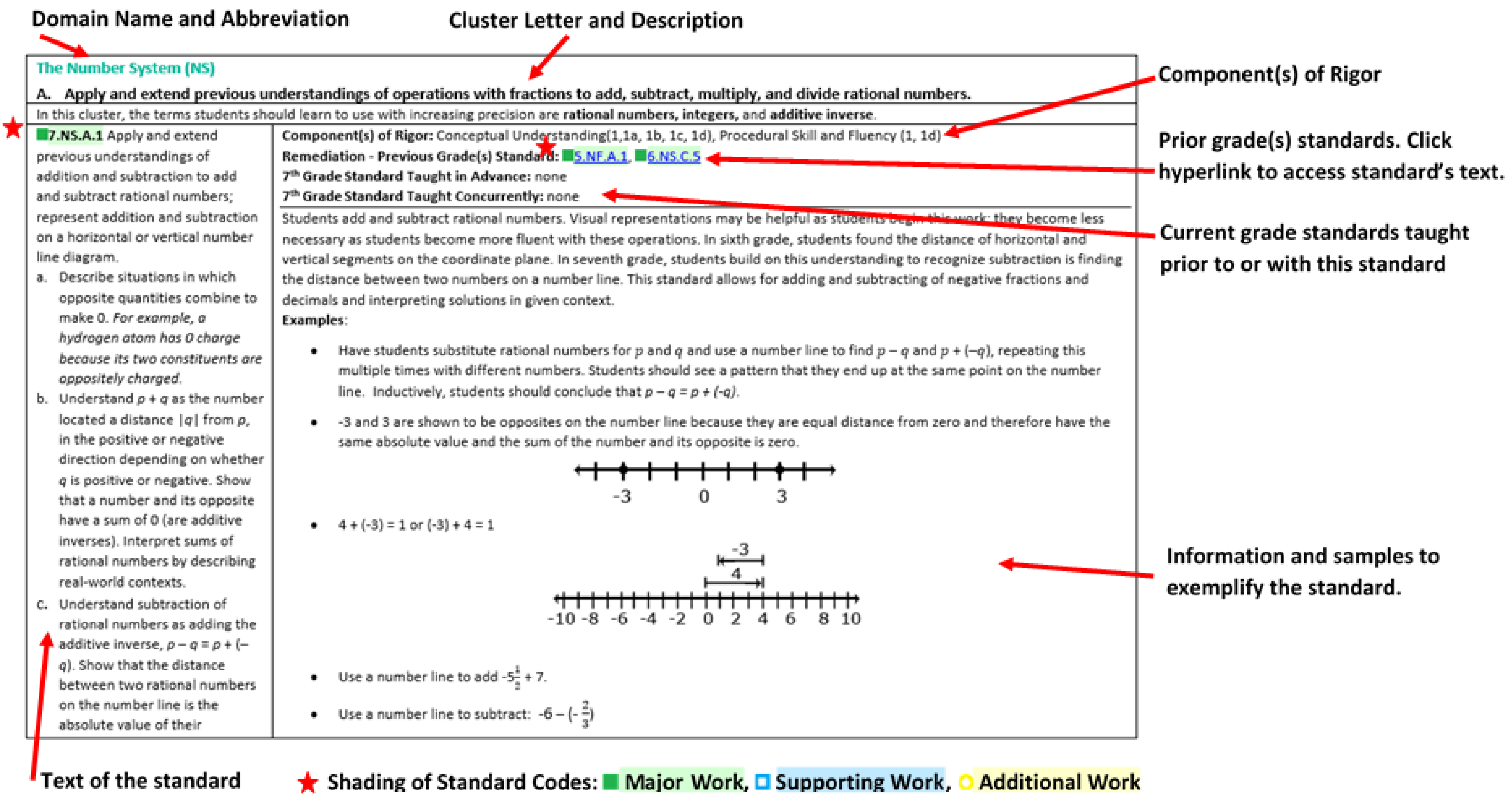
How-to-Read Guide

The diagram below provides an overview of the information found in all companion documents. Definitions and more complete descriptions are provided on the next page.



The Number System (NS)

A. Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

In this cluster, the terms students should learn to use with increasing precision are **rational numbers, integers,** and **additive inverse**.

🟩**7.NS.A.1** Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

a. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.*

b. Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

c. Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their

Component(s) of Rigor: Conceptual Understanding(1,1a, 1b, 1c, 1d), Procedural Skill and Fluency (1, 1d)

Remediation - Previous Grade(s) Standard: 🟩5.NF.A.1, 🟩6.NS.C.5

7th Grade Standard Taught in Advance: none

7th Grade Standard Taught Concurrently: none

Students add and subtract rational numbers. Visual representations may be helpful as students begin this work; they become less necessary as students become more fluent with these operations. In sixth grade, students found the distance of horizontal and vertical segments on the coordinate plane. In seventh grade, students build on this understanding to recognize subtraction is finding the distance between two numbers on a number line. This standard allows for adding and subtracting of negative fractions and decimals and interpreting solutions in given context.

Examples:

- Have students substitute rational numbers for p and q and use a number line to find $p - q$ and $p + (-q)$, repeating this multiple times with different numbers. Students should see a pattern that they end up at the same point on the number line. Inductively, students should conclude that $p - q = p + (-q)$.
- -3 and 3 are shown to be opposites on the number line because they are equal distance from zero and therefore have the same absolute value and the sum of the number and its opposite is zero.
- $4 + (-3) = 1$ or $(-3) + 4 = 1$
- Use a number line to add $-5\frac{1}{2} + 7$.
- Use a number line to subtract: $-6 - (-\frac{2}{3})$

1. **Domain Name and Abbreviation:** A grouping of standards consisting of related content that are further divided into clusters. Each domain has a unique abbreviation and is provided in parentheses beside the domain name.
2. **Cluster Letter and Description:** Each cluster within a domain begins with a letter. The description provides a general overview of the focus of the standards in the cluster.
3. **Previous Grade(s) Standards:** One or more standards that students should have mastered in previous grades to prepare them for the current grade standard. If students lack the pre-requisite knowledge and remediation is required, the previous grade standards provide a starting point.
4. **Standards Taught in Advance:** These current grade standards include skills or concepts on which the target standard is built. These standards are best taught before the target standard.
5. **Standards Taught Concurrently:** Standards which should be taught with the target standard to provide coherence and connectedness in instruction.
6. **Component(s) of Rigor:** See full explanation on components of rigor.
7. **Sample Problem:** The sample provides an example how a student might meet the requirements of the standard. Multiple examples are provided for some standards. However, sample problems should not be considered an exhaustive list. Explanations, when appropriate, are also included.
8. **Text of Standard:** The complete text of the targeted Louisiana Student Standards of Mathematics is provided.

Classification of Major, Supporting, and Additional Work

Students should spend the large majority of their time on the 🟩 major work of the grade. 🟦 Supporting work and, where appropriate, 🟡 additional work can engage students in the major work of the grade. Each standard is color-coded to quickly and simply determine how class time should be allocated. Furthermore, standards from previous grades that provide foundational skills for current grade standards are also color-coded to show whether those standards are classified as 🟩 major, 🟦 supporting, or 🟡 additional in their respective grades.

Components of Rigor

The K-12 mathematics standards lay the foundation that allows students to become mathematically proficient by focusing on conceptual understanding, procedural skill and fluency, and application.

Conceptual Understanding refers to understanding mathematical concepts, operations, and relations. It is more than knowing isolated facts and methods. Students should be able to make sense of why a mathematical idea is important and the kinds of contexts in which it is useful. It also allows students to connect prior knowledge to new ideas and concepts.

Procedural Skill and Fluency is the ability to apply procedures accurately, efficiently, and flexibly. It requires speed and accuracy in calculation while giving students opportunities to practice basic skills. Students' ability to solve more complex application tasks is dependent on procedural skill and fluency.

Application provides a valuable content for learning and the opportunity to solve problems in a relevant and a meaningful way. It is through real-world application that students learn to select an efficient method to find a solution, determine whether the solution makes sense by reasoning, and develop critical thinking skills.

Standards for Mathematical Practice

The Louisiana Standards for Mathematical Practice are expected to be integrated into every mathematics lesson for all students in grades K-12. Below are a few examples of how these practices may be integrated into tasks that high school students complete.

Louisiana Standard	Explanations and Examples
HS.MP.1 Make sense of problems and persevere in solving them.	High school students start to examine problems by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. By high school, students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. They check their answers to problems using different methods and continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.
HS.MP.2 Reason abstractly and quantitatively.	High school students seek to make sense of quantities and their relationships in problem situations. They abstract a given situation and represent it symbolically, manipulate the representing symbols, and pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Students use quantitative reasoning to create coherent representations of the problem at hand; consider the units involved; attend to the meaning of quantities, not just how to compute them; and know and flexibly use different properties of operations and objects.
HS.MP.3 Construct viable arguments and critique the reasoning of others.	High school students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. High school students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. High school students learn to determine domains, to which an argument applies, listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
HS.MP.4 Model with mathematics.	High school students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. High school students make assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Louisiana Standard	**Explanations and Examples**
HS.MP.5 Use appropriate tools strategically.	High school students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. High school students should be sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. They are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.
HS.MP.6 Attend to precision.	High school students try to communicate precisely to others by using clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, specifying units of measure, and label axes to clarify the correspondence between quantities in a problem. They calculate accurately and efficiently, expressing numerical answers with a degree of precision appropriate for the problem context. By the time they reach high school they have learned to examine claims and make explicit use of definitions.
HS.MP.7 Look for and make use of structure.	By high school, students look closely to discern a pattern or structure. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y. High school students use these patterns to create equivalent expressions, factor and solve equations, and compose functions, and transform figures.
HS.MP.8 Look for and express regularity in repeated reasoning.	High school students notice if calculations are repeated, and look both for general methods and for shortcuts. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, derive formulas or make generalizations, high school students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Modeling Standards

Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★).

What is modeling?

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

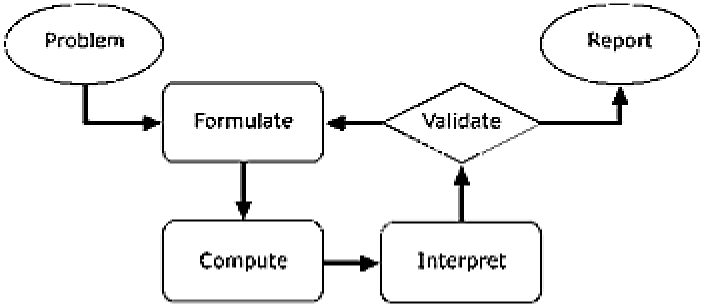
Some examples of such situations might include:

- Estimate how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Plan a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.
- Design the layout of the stalls in a school fair so as to raise as much money as possible.
- Analyze the stopping distance for a car.
- Model a savings account balance, bacterial colony growth, or investment growth.
- Engage in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.
- Analyze the risk in situations such as extreme sports, pandemics, and terrorism.
- Relate population statistics to individual predictions.

In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.

The basic modeling cycle is summarized in the diagram. It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.



In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model—for example, graphs of global temperature and atmospheric CO_2 over time.

Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters which are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems.

Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena (e.g., the behavior of polynomials) as well as physical phenomena).

Number and Quantity: The Real Number System (N-RN)

A. Extend the properties of exponents to rational exponents.

In this cluster, the terms students should learn to use with increasing precision are **radicals and rational exponents**

Louisiana Standard	Explanations and Examples
🟩 **A2: N-RN.A.1** Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.*	**Component(s) of Rigor:** Conceptual Understanding **Remediation - Previous Grade(s) Standard:** 🟩 8.EE.A.1, 🟩 8.EE.A.2 **Algebra II Standard Taught in Advance:** none **Algebra II Standard Taught Concurrently:** none The meaning of an exponent relates the frequency with which a number is used as a factor. So 5^3 indicates the product where 5 is a factor 3 times. Extend this meaning to a rational exponent, then $125^{1/3}$ indicates one of three equal factors whose product is 125. Students recognize that a fractional exponent can be expressed as a radical or a root. *For example*, an exponent of $^1/_3$ is equivalent to a cube root; an exponent of $^1/_4$ is equivalent to a fourth root. Students extend the use of the power rule, $(b^n)^m = b^{nm}$ from whole number exponents, i.e., $(7^2)^3 = 7^6$, to rational exponents. They compare examples, such as $\left(7^{1/2}\right)^2 = 7^{1/2*2} = 7^1 = 7$ to $(\sqrt{7})^2 = 7$ to establish a connection between radicals and rational exponents: $7^{1/2} = \sqrt{7}$ and, in general, $b^{1/2} = \sqrt{b}$. **Examples:** - Determine the value of *x* a. $64^{\frac{1}{2}} = 8^x$ b. $(12^5)^x = 12$ - Evaluating a special exponential expression: https://www.illustrativemathematics.org/content-standards/HSN/RN/A/1/tasks/1823 - Evaluating exponential expressions: https://www.illustrativemathematics.org/content-standards/HSN/RN/A/1/tasks/1866

🟩 **A2: N-RN.A.2** Rewrite expressions involving radicals and rational exponents using the properties of exponents.	**Component(s) of Rigor:** Conceptual Understanding, Procedural Skill and Fluency **Remediation - Previous Grade(s) Standard:** none **Algebra II Standard Taught in Advance:** 🟩 A2: N-RN.A.1 **Algebra II Standard Taught Concurrently:** none **Examples:** • Using the properties of exponents, simplify a. $\left(\sqrt[4]{32^3}\right)^2$ b. $\frac{\sqrt[5]{b^3}}{b^{\frac{4}{3}}}$ • What is an equivalent exponential expression for $8^{\frac{2}{3}}$? *Solution:* $8^{\frac{2}{3}} = (8^2)^{\frac{1}{3}} = \left(8^{\frac{1}{3}}\right)^2 = (\sqrt[3]{8}\,)^2 = 2^2$ • Rewrite $81^{\frac{3}{4}}$ without a rational exponent. *Solution:* $81^{\frac{3}{4}} = \sqrt[4]{81^3} = \left(\sqrt[4]{81}\right)^3 = 3^3$ • Determine whether each equation is true or false using the properties of exponents. $\sqrt{32} = 2^{\frac{5}{2}}$ $\quad 4^{\frac{1}{2}} = \sqrt[4]{64}$ $\quad (\sqrt{64}\,)^{\frac{1}{3}} = 8^{\frac{1}{6}}$ $16^{\frac{3}{2}} = 8^2$ $\quad 2^8 = (\sqrt[3]{16}\,)^6$

Number and Quantity: Quantities ★ (N-Q)

A. Use properties of rational and irrational numbers.

In this cluster, the terms students should learn to use with increasing precision are **quantities and descriptive modeling.**

Louisiana Standard	Explanations and Examples
▣ **A2: N-Q.A.2** Define appropriate quantities for the purpose of descriptive modeling.	**Component(s) of Rigor:** Conceptual Understanding **Remediation - Previous Grade(s) Standard:** ▣ A1: N-Q.A.1, ▣ A1: N-Q.A.2 **Algebra II Standard Taught in Advance:** none **Algebra II Standard Taught Concurrently:** none **Examples:** • Giving raises: https://www.illustrativemathematics.org/content-standards/HSN/Q/A/2/tasks/1850

<table>
<tr><td colspan="2">Number and Quantity: The Complex Number System (N-CN)
A. Perform arithmetic operations with complex numbers.</td></tr>
<tr><td colspan="2">In this cluster, the terms students should learn to use with increasing precision are complex number, real number, <i>I</i>, and i^2.</td></tr>
<tr><td>Louisiana Standard</td><td>Explanations and Examples</td></tr>
<tr><td>○ A2: N-CN.A.1 Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.</td><td>
Component(s) of Rigor: Conceptual Understanding
Remediation - Previous Grade(s) Standard: 🟩 8.EE.A.2
Algebra II Standard Taught in Advance: none
Algebra II Standard Taught Concurrently: ○ A2.N-CN.A.2
<p>Students will review the structure of the complex number system, realizing that every number is a complex number that can be written in the form $a + bi$ where a and b are real numbers. If $a = 0$, then the number is a pure imaginary number; however, when $b = 0$, the number is a real number.</p>
<p>The square root of a negative number is a complex number.</p>
<p>Example:</p>
<table>
<tr><th></th><th>Problem</th><th>Solution</th></tr>
<tr><td>1.</td><td>$\sqrt{-36}$</td><td>$\sqrt{-36} = \sqrt{-1} \cdot \sqrt{36} = 6i$</td></tr>
<tr><td>2.</td><td>$2\sqrt{-49}$</td><td>$2\sqrt{-49} = 2\sqrt{-1} \cdot \sqrt{49} = 2 \cdot 7i = 14i$</td></tr>
<tr><td>3.</td><td>$-3\sqrt{-10}$</td><td>$-3\sqrt{-10} = -3\sqrt{-1} \cdot \sqrt{10} = -3 \cdot i \cdot \sqrt{10} = -3i\sqrt{10}$</td></tr>
<tr><td>4.</td><td>$5\sqrt{-8}$</td><td>$5\sqrt{-8} = 5\sqrt{-1} \cdot \sqrt{8} = 5 \cdot i \cdot 2\sqrt{2} = 10i\sqrt{2}$</td></tr>
</table>
<p>Example:</p>
Complex number patterns: https://www.illustrativemathematics.org/content-standards/HSN/CN/A/1/tasks/722
</td></tr>
</table>

○ **A2: N-CN.A.2** Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

Component(s) of Rigor: Procedural Skill and Fluency
Remediation - Previous Grade(s) Standard: 🟩 7.EE.A.1
Algebra II Standard Taught in Advance: none
Algebra II Standard Taught Concurrently: ○ A2.N-CN.A.1

The complex number system possesses the same basic properties as the real number system: that addition and multiplication are commutative and associative; the existence of additive identity and multiplicative identity; the existence of an additive inverse for every complex number and the existence of multiplicative inverse or reciprocal for every non-zero complex number; and the distributive property of multiplication over addition. An awareness of the properties minimizes students' rote memorization and links the rules for manipulations with the complex number system to the rules for manipulations with binomials with real coefficients of the form $a + bx$.

Example:

- Simplify the following expression. Justify each step using the commutative, associative and distributive properties.

$$(3-2i)(-7+4i)$$

Solutions may vary; one solution follows:

$(3-2i)(-7+4i)$	
$3(-7+4i)-2i(-7+4i)$	Distributive Property
$-21+12i+14i-8i^2$	Distributive Property
$-21+(12i+14i)-8i^2$	Associative Property
$-21+i(12+14)-8i^2$	Distributive Property
$-21+26i-8i^2$	Computation
$-21+26i-8(-1)$	$i^2=-1$
$-21+26i+8$	Computation
$-21+8+26i$	Commutative Property
$-13+26i$	Computation

Number and Quantity: The Complex Number System (N-CN)

C. Use complex numbers in polynomial identities and equations.

In this cluster, the terms students should learn to use with increasing precision are **quadratic, real coefficients, discriminant, and complex solution.**

Louisiana Standard	Explanations and Examples
○ **A2: N-CN.C.7** Solve quadratic equations with real coefficients that have complex solutions.	**Component(s) of Rigor:** Procedural Skill and Fluency **Remediation - Previous Grade(s) Standard:** none **Algebra II Standard Taught in Advance:** ○ A2.N-CN.A.1, ○ A2.N-CN.A.2 **Algebra II Standard Taught Concurrently:** none In Algebra I students learned to recognize when a quadratic equation had no real solutions; however, they were not equipped with the skilla nor understanding of how to find the complex solutions to such am equation. With knowledge and skills from previously taught Algebra II standards, students can now solve any quadratic equation regardless if the solutions are real or complex. **Examples:** • Solve $x^2 = -2$. • Find all solutions of $2x^2 + 5 = 2x$ and express them in the form $a + bi$. • Completing the square: https://www.illustrativemathematics.org/content-standards/HSN/CN/C/7/tasks/1690

Algebra: Seeing Structure in Expressions (A-SSE)

A. Interpret the structure of expressions.

In this cluster, the terms students should learn to use with increasing precision are **factor, greatest common factor, linear factor, quadratic factor, and equivalent form.**

Louisiana Standard	**Explanations and Examples**
🟩 **A2: A-SSE.A.2** Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$*.*	**Component(s) of Rigor:** Conceptual Understanding, Procedural Skill and Fluency **Remediation - Previous Grade(s) Standard:** 🟩 A1: A-SSE.A.1, 🟩 A1: A-SSE.A.2 **Algebra II Standard Taught in Advance:** none **Algebra II Standard Taught Concurrently:** none Students should extract the greatest common factor (whether a constant, a variable, or a combination of each). If the remaining expression is quadratic, students should factor the expression further. **Example:** • Factor: $x^3 - 2x^2 - 35x$ • Rewrite $m^{2x} + m^x - 6$ into an equivalent form. • Factor: $x^3 - 8$ • Sum of even and odd: https://www.illustrativemathematics.org/content-standards/HSA/SSE/A/2/tasks/198 • Computations with complex numbers: https://www.illustrativemathematics.org/content-standards/HSA/SSE/A/2/tasks/617

Algebra: Seeing Structure in Expressions (A-SSE)

B. Write expressions in equivalent forms to solve problems.

In this cluster, the terms students should learn to use with increasing precision are **equivalent expression, properties of exponents, and finite geometric series.**

Louisiana Standard	**Explanations and Examples**
🟩 **A2: A-SSE.B.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. ★ c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression* 1.15^t *can be rewritten as* $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*	**Component(s) of Rigor:** Conceptual Understanding (3), Procedural Skill and Fluency (3, 3c) **Remediation - Previous Grade(s) Standard:** 🟦 A1: A-SSE.B.3 **Algebra II Standard Taught in Advance:** 🟩 A2: A-SSE.A.2 **Algebra II Standard Taught Concurrently:** none Students will use the properties of operations to create equivalent expressions. **Example:** Forms of Exponential Expressions: https://www.illustrativemathematics.org/content-standards/HSA/SSE/B/3/tasks/1305
🟩 **A2: A-SSE.B.4** Apply the formula for the sum of a finite geometric series (when the common ratio is not 1) to solve problems. *For example, calculate mortgage payments.* ★	**Component(s) of Rigor:** Procedural Skill and Fluency, Application **Remediation - Previous Grade(s) Standard:** none **Algebra II Standard Taught in Advance:** 🟩 A2: A-SSE.B.3 **Algebra II Standard Taught Concurrently:** none This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. Students understand that a geometric series is the sum of terms in a geometric sequence and can be used to solve real-world problems. The sum of a finite geometric series with common ratio not equal to 1 can be written as the simple formula $S_n = \frac{a(1-r^n)}{1-r}$ where *r* is the common ratio, *a* is the initial value, and *n* is the number of terms in the series. **Examples:** • Course of antibiotics: https://www.illustrativemathematics.org/content-standards/HSA/SSE/B/4/tasks/805 • YouTube explosion: https://www.illustrativemathematics.org/content-standards/HSA/SSE/B/4/tasks/1797

Algebra: Arithmetic with Polynomials and Rational Expressions (A-APR)

B. Perform arithmetic operations on polynomials.

In this cluster, the terms students should learn to use with increasing precision are **Remainder Theorem, factorizations, and zeros of polynomials,**

Louisiana Standard	Explanations and Examples
🟩 **A2: A-APR.B.2** Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x – a$ is $p(a)$, so $p(a) = 0$ if and only if $(x – a)$ is a factor of $p(x)$.	**Component(s) of Rigor:** Conceptual Understanding, Procedural Skill and Fluency **Remediation - Previous Grade(s) Standard:** ▣ A1: A-SSE.B.3 **Algebra II Standard Taught in Advance:** 🟩 A2: A-APR.B.3 **Algebra II Standard Taught Concurrently:** ▣ A2: A-APR.D.6 The Remainder theorem says that if a polynomial $p(x)$ is divided by $x – a$, then the remainder is the constant $p(a)$. That is, $p(x) = q(x)(x - a) + p(a)$. So if p(a) = 0 then p(x) = q(x)(x-a). **Example:** • The missing coefficient: https://www.illustrativemathematics.org/content-standards/HSA/APR/B/2/tasks/592 • Zeros and factorization of a quadratic polynomial I: https://www.illustrativemathematics.org/content-standards/HSA/APR/B/2/tasks/787
🟩 **A2: A-APR.B.3** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	**Component(s) of Rigor:** Conceptual Understanding, Procedural Skill and Fluency **Remediation - Previous Grade(s) Standard:** ▣ A1: A-SSE.B.3, ▣ A1: A-APR.B.3 **Algebra II Standard Taught in Advance:** 🟩 A2: A-SSE.A.2 **Algebra II Standard Taught Concurrently:** none Students identify the multiplicity of the zeroes of a factored polynomial and explain how the multiplicity of the zeroes provides a clue as to how the graph will behave when it approaches and leaves the x-intercept. Graphing calculators or programs can be used to generate graphs of polynomial functions. **Example:** • For a certain polynomial function, $x = 3$ is a zero with multiplicity two, $x = 1$ is a zero with multiplicity three, and $x = -3$ is a zero with multiplicity one. Write a possible equation for this function and sketch its graph. • Graphing from factors I: https://www.illustrativemathematics.org/content-standards/HSA/APR/B/3/tasks/2139 • Graphing from factors II: https://www.illustrativemathematics.org/content-standards/HSA/APR/B/3/tasks/2140

Algebra: Arithmetic with Polynomials and Rational Expressions (A-APR)

C. Use polynomial identities to solve problems.

In this cluster, the terms students should learn to use with increasing precision are **polynomial identity.**

Louisiana Standard	Explanations and Examples
○ **A2: A-APR.C.4** Use polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2+y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.*	**Component(s) of Rigor:** Conceptual Understanding, Procedural Skill and Fluency **Remediation - Previous Grade(s) Standard:** none **Algebra II Standard Taught in Advance:** 🟩 A2: A-SSE.A.2 **Algebra II Standard Taught Concurrently:** none Polynomial identities should include but are not limited to: • The product of the sum and difference of two terms, • The difference of two squares, • The sum and difference of two cubes, • The square of a binomial **Examples:** • Use the distributive law to explain why $x^2 - y^2 = (x - y)(x + y)$ for any two numbers x and y. • Derive the identity $(x - y)^2 = x^2 - 2xy + y^2$ from $(x + y)^2 = x^2 + 2xy + y^2$ by replacing y with $-y$. • Use an identity to explain the pattern $2^2 - 1^2 = 3$ $3^2 - 2^2 = 5$ $4^2 - 3^2 = 7$ $5^2 - 4^2 = 9$ [Answer: $(n + 1)^2 - n^2 = 2n + 1$ for any whole number n.]

Algebra: Arithmetic with Polynomials and Rational Expressions (A-APR)

D. Rewrite rational expressions.

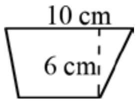
In this cluster, the terms students should learn to use with increasing precision are **different form of a rational expression, degree, inspection, and long division.**

Louisiana Standard	Explanations and Examples
☐ **A2: A-APR.D.6** Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.	**Component(s) of Rigor:** Conceptual Understanding, Procedural Skill and Fluency **Remediation - Previous Grade(s) Standard:** 🟩 7.NS.A.2 **Algebra II Standard Taught in Advance:** 🟩 A2: A-SSE.A.2 **Algebra II Standard Taught Concurrently:** 🟩 A2: A-APR.B.2 The polynomial $q(x)$ is called the quotient, and the polynomial $r(x)$ is called the remainder. Expressing a rational expression in this form allows one to see different properties of the graph, such as horizontal asymptotes. **Examples:** • Find the quotient and remainder for the rational expression $\frac{x^3-3x^2+x-6}{x^2+2}$ and use them to write the expression in a different form. • Egyptian fractions II: https://www.illustrativemathematics.org/content-standards/HSA/APR/D/6/tasks/1346

Algebra: Creating Equations ★ (A-CED)

A. Create equations that describe numbers or relationships.

In this cluster, the terms students should learn to use with increasing precision are **linear function, quadratic function, rational function, and exponential function.**

Louisiana Standard	Explanations and Examples
🟦 **A2: A-CED.A.1** Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*	**Component(s) of Rigor:** Conceptual Understanding, Procedural Skill and Fluency, Application **Remediation - Previous Grade(s) Standard:** 🟩 A1: A-CED.A.1, 🟩 A1: A-REI.B.4 **Algebra II Standard Taught in Advance:** none **Algebra II Standard Taught Concurrently:** 🟩 A2: A-REI.A.1 **Examples:** • Given that the following trapezoid has area 54 cm^2, set up an equation to find the length of the base, and solve the equation.  • Lava coming from the eruption of a volcano follows a parabolic path. The height h in feet of lava t seconds after it is ejected from the volcano is given by $h(t) = -t^2 + 16t + 936$. After how many seconds does the lava reach its maximum height of 1000 feet? • Basketball: https://www.illustrativemathematics.org/content-standards/HSA/CED/A/1/tasks/702 • If the world population at the beginning of 2008 was 6.7 billion and growing at a rate of 1.16% each year, in what year will the population be double?

Reasoning with Equations and Inequalities (A-REI)

A. Understand solving equations as a process of reasoning and explain the reasoning.

In this cluster, the terms students should learn to use with increasing precision are **rational equation, radical equation, solution of an equation, extraneous solution, viable argument, and justify.**

Louisiana Standard	Explanations and Examples
🟩 **A2: A-REI.A.1** Explain each step in solving an equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	**Component(s) of Rigor:** Conceptual Understanding **Remediation - Previous Grade(s) Standard:** 🟩 A1: A-REI.A.1 **Algebra II Standard Taught in Advance:** none **Algebra II Standard Taught Concurrently:** 🟦 A2: A-CED.A.1 Properties of operations can be used to change expressions on either side of the equation to equivalent expressions. In addition, adding the same term to both sides of an equation or multiplying both sides by a non-zero constant produces an equation with the same solutions. Other operations, such as squaring both sides, may produce equations that have extraneous solutions. **Examples:** • Show that *x* = 2 and *x* = -3 are solutions to the equation $x^2 + x = 6$. Write the equation in a form that shows these are the only solutions, explaining each step in your reasoning. • Prove $(\mathrm{x}^3 - \mathrm{y}^3) = (\mathrm{x} - \mathrm{y})(\mathrm{x}^2 + \mathrm{xy} + \mathrm{y}^2)$. Justify each step. • Explain the steps involved in solving each of the following: a. $5(2^t) + 20 = 60$ $5(2^t) = 40$ $(2^t) = 8$ $2^t = 2^3$ $t = 3$ b. $8(1.5)^x = 200$ $(1.5)^x = 25$ $(10^{log1.5})^x = 10^{\log 25}$ $\mathrm{x} \cdot \log 1.5 = \log 25$ $x = \frac{\log 25}{\log 1.5} \approx 7.94$ c. $2000(1.06)^x = 10000$ $(1.06)^x = 5$ $e^{\mathrm{x} \cdot \ln 1.06} = e^{\ln 5}$ $x \cdot \ln 1.06 = \ln 5$ $x = \frac{\ln 5}{\ln 1.06}$ ≈ 27.62

🟩 **A2: A-REI.A.2** Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	**Component(s) of Rigor:** Conceptual Understanding, Procedural Skill and Fluency **Remediation - Previous Grade(s) Standard:** none **Algebra II Standard Taught in Advance:** 🟩 A2: A-REI.A.1 **Algebra II Standard Taught Concurrently:** none **Examples:** • Mary solved $x = \sqrt{2-x}$ for x and got $x = -2$ and $x = 1$. Evaluate her solutions and determine if she is correct. Explain your reasoning. • Solve $\frac{3}{x-3} = \frac{x}{x-3} - \frac{3}{2}$. Can x have a value of 3? Explain your reasoning. • Solve for x: o $\sqrt{x+2} = 5$ o $\frac{7}{8}\sqrt{2x-5} = 21$ o $\frac{x+2}{x+3} = 2$ o $\sqrt{3x-7} = -4$ o $5 - \sqrt{-(x-4)} = 2$

Reasoning with Equations and Inequalities (A-REI)

B. Solve equations and inequalities in one variable.

In this cluster, the terms students should learn to use with increasing precision are **solve a quadratic, inspection, find square root, complete the square, quadratic formula, factor, root, real root, and complex root.**

Louisiana Standard

▣ **A2: A-REI.B.4** Solve quadratic equations in one variable.

b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

Explanations and Examples

Component(s) of Rigor: Conceptual Understanding (4b), Procedural Skill and Fluency (4, 4b)
Remediation - Previous Grade(s) Standard: 🟩 A1: A-REI.B.4
Algebra II Standard Taught in Advance: none
Algebra II Standard Taught Concurrently: none

Students should solve by factoring, completing the square, and using the quadratic formula. The zero product property is used to explain why the factors are set equal to zero. Students should relate the value of the discriminant to the type of root to expect. A natural extension would be to relate the type of solutions to $ax^2 + bx + c = 0$ to the behavior of the graph of $y = ax^2 + bx + c$.

Value of Discriminant	Nature of Roots	Nature of Graph
$b^2 – 4ac = 0$	1 real roots	intersects *x*-axis once
$b^2 – 4ac > 0$	2 real roots	intersects *x*-axis twice
$b^2 – 4ac < 0$	2 complex roots	does not intersect *x*-axis

Examples:

- Are the roots of $2x^2 + 5 = 2x$ real or complex? How many roots does it have? Find all solutions of the equation.
- What is the nature of the roots of $x^2 + 6x + 10 = 0$? Solve the equation using the quadratic formula and completing the square. How are the two methods related?

Reasoning with Equations and Inequalities (A-REI)

C. Solve systems of equations.

In this cluster, the terms students should learn to use with increasing precision are **system of linear equations, system of a linear and a quadratic equation in two variables, solve exactly, solve approximately, and solve graphically.**

Louisiana Standard	Explanations and Examples
○ **A2: A-REI.C.6** Solve systems of linear equations exactly and approximately (e.g., with graphs), limited to systems of at most three equations and three variables. With graphic solutions, systems are limited to two variables.	**Component(s) of Rigor:** Procedural Skill and Fluency **Remediation - Previous Grade(s) Standard:** ○ A1: A-REI.C.6 **Algebra II Standard Taught in Advance:** none **Algebra II Standard Taught Concurrently:** 🟩 A2: A-REI.D.11 The system solution methods can include but are not limited to graphical, elimination/linear combination, substitution, and modeling. Systems can be written algebraically or can be represented in context. Students may use graphing calculators, programs, or applets to model and find approximate solutions for systems of equations. **Examples:** • Solve the system of equations: $x - 2y + 3z = 5$, $x + 3z = 11$, $5y - 6z = 9$.
○ **A2: A-REI.C.7** Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line* $y = -3x$ *and the circle* $x^2 + y^2 = 3$.	**Component(s) of Rigor:** Procedural Skill and Fluency **Remediation - Previous Grade(s) Standard:** none **Algebra II Standard Taught in Advance:** ○ A2: A-REI.C.6 **Algebra II Standard Taught Concurrently:** 🟩 A2: A-REI.D.11 **Example:** • The circle and the line: https://www.illustrativemathematics.org/content-standards/HSA/REI/C/7/tasks/223 A linear and quadratic system: https://www.illustrativemathematics.org/content-standards/HSA/REI/C/7/tasks/576

Reasoning with Equations and Inequalities (A-REI)

D. Represent and solve equations and inequalities graphically.

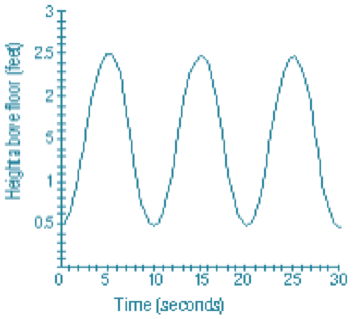
In this cluster, the terms students should learn to use with increasing precision are **solve for the solution *of f(x) = g(x)* for linear, polynomial, rational, absolute value, exponential, and logarithmic functions.**

Louisiana Standard	Explanations and Examples
🟩 **A2: A-REI.D.11** Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.	**Component(s) of Rigor:** Conceptual Understanding, Procedural Skill and Fluency **Remediation - Previous Grade(s) Standard:** 🟩A1: A-REI.D.11 **Algebra II Standard Taught in Advance:** none **Algebra II Standard Taught Concurrently:** ○ A2: A-REI.C.6, ○ A2: A-REI.C.7 This standard is not about systems of equations in two variables; rather, this standard is about using knowledge about solving a system of equations to solve a single-variable equation defined by two expressions in terms of the same variable. This standard allows for students to solve equations that, algebraically, would be too advanced. Students should have mastered this understanding in Algebra I; thus, this standard simply extends the work to include more types of expressions (e.g., polynomial, rational, etc.). **Example:** • Two Squares are Equal: https://www.illustrativemathematics.org/content-standards/HSA/REI/D/11/tasks/618

Functions: Interpreting Functions (F-IF)

B. Interpret functions that arise in applications in terms of the context.

In this cluster, the terms students should learn to use with increasing precision are **linear, piecewise linear, absolute value, quadratic, and exponential functions; model a relationship, interpret key features (intercepts, increase, decrease, positive, negative, relative minimum and maximums, symmetries, end behavior), calculate average rate of change, estimate rate of change in graph, and specified interval.**

Louisiana Standard	**Explanations and Examples**
🟩 **A2: F-IF.B.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* ★	**Component(s) of Rigor:** Conceptual Understanding **Remediation - Previous Grade(s) Standard:** ▣ A1: N-Q.A.1, 🟩 A1: F-IF.A.1, 🟩 A1: F-IF.B.4 **Algebra II Standard Taught in Advance:** none **Algebra II Standard Taught Concurrently:** 🟩 A2: F-BF.A.1, ▣ A2: F-IF.C.7 **Examples:** • The story of a flight: https://www.illustrativemathematics.org/content-standards/HSF/IF/B/4/tasks/2095 • Lake Sonoma: https://www.illustrativemathematics.org/content-standards/HSF/IF/B/4/tasks/2086 • Logistic growth model; abstract version: https://www.illustrativemathematics.org/content-standards/HSF/IF/B/4/tasks/800 • Jumper horses on carousels move up and down as the carousel spins. Suppose that the back hooves of such a horse are six inches above the floor at their lowest point and two-and-one-half feet above the floor at their highest point. Draw a graph that could represent the height of the back hooves of this carousel horse during a half-minute portion of a carousel ride. Sample Graph 

🟩 **A2: F-IF.B.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★

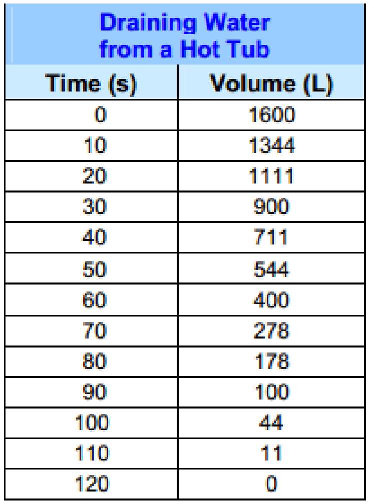
Component(s) of Rigor: Conceptual Understanding, Procedural Skill and Fluency
Remediation - Previous Grade(s) Standard: 🟩 A1: F-IF.A.2, 🟩 A1: IF.B.6
Algebra II Standard Taught in Advance: none
Algebra II Standard Taught Concurrently: none

The average rate of change of a function $y = f(x)$ over an interval [a,b] is $\frac{\Delta y}{\Delta x} = \frac{f(b)-f(a)}{b-a}$

In addition to finding average rates of change from functions given symbolically, graphically, or in a table, students may collect data from experiments or simulations (ex. falling ball, velocity of a car, etc.) and find average rates of change for the function modeling the situation.

Examples:

- The plug is pulled in a small hot tub. The table gives the volume of water in the tub from the moment the plug is pulled, until it is empty. What is the average rate of change between:
    - 60 seconds and 100 seconds?
    - 0 seconds and 120 seconds?
    - 70 seconds and 110 seconds?

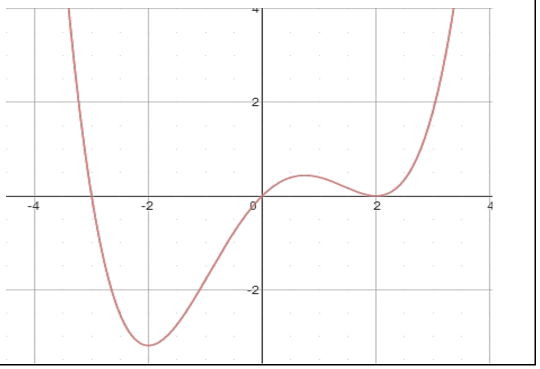


Draining Water from a Hot Tub

Time (s)	Volume (L)
0	1600
10	1344
20	1111
30	900
40	711
50	544
60	400
70	278
80	178
90	100
100	44
110	11
120	0

<table>
<tr><td colspan="2">Functions: Interpreting Functions (F-IF)
C. Analyze functions using different representations.</td></tr>
<tr><td colspan="2">In this cluster, the terms students should learn to use with increasing precision are square root, cube root, piecewise, step absolute value, polynomial, exponential, logarithmic and trigonometric functions; intercepts, end behavior, period, midline, amplitude, properties of exponents, different representations, and compare properties of two functions.</td></tr>
<tr><td>Louisiana Standard</td><td>Explanations and Examples</td></tr>
<tr><td>◻ A2: F-IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ★
b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.</td><td>Component(s) of Rigor: Conceptual Understanding (7, 7c, 7e, Procedural Skill and Fluency (7, 7b, 7c, 7e)
Remediation - Previous Grade(s) Standard: ◻ A1: A-APR.B.3, 🟩 A1: F-IF.A.1, ◻ A1: F-IF.C.7, ◻ A1: F-IF.C.8
Algebra II Standard Taught in Advance: none
Algebra II Standard Taught Concurrently: 🟩 A2: F-IF.B.4, ◻ A2: F-IF.C.8, ○ A2: F-BF.B.3
Key characteristics include but are not limited to maxima, minima, intercepts, symmetry, end behavior, and asymptotes. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to graph functions.
Examples:
• Describe key characteristics of the graph of $f(x) = |x - 3| + 5$.
• Sketch the graph and identify the key characteristics of the function described below.
$$F(x) = \begin{cases} x + 2 \text{ for } x \geq 0 \\ -x^2 \text{for } x < -1 \end{cases}$$
• Graph the function $f(x) = 2^x$ by creating a table of values. Identify the key characteristics of the graph.</td></tr>
</table>

🟦**A2: F-IF.C.8** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay.*	**Component(s) of Rigor:** Conceptual Understanding (8, 8b), Procedural Skill and Fluency (8) **Remediation - Previous Grade(s) Standard:** 🟦A1: F-IF.C.8 **Algebra II Standard Taught in Advance:** 🟩 A2: N-RN.A.1 **Algebra II Standard Taught Concurrently:** 🟦 A2: F-IF.C.7, 🟡 A2: F-BF.B.3 Students can determine if an exponential function models growth or decay. Students can also identify and interpret the growth or decay factor. Students can rewrite an expression in the form $a(b)^{kx}$ as $a(b^k)^x$. They can identify b^k as the growth or decay factor. Students recognize that when the factor is greater than 1, the function models growth and when the factor is between 0 and 1 the function models decay. **Examples:** • The projected population of Delroysville is given by the function $p(t) = 1500(1.08)^{2t}$ where *t* is the number of years since 2010. You have been selected by the city council to help them plan for future growth. Explain what the function $p(t) = 1500(1.08)^{2t}$ means to the city council members. • Suppose a single bacterium lands on one of your teeth and starts reproducing by a factor of 2 every hour. If nothing is done to stop the growth of the bacteria, write a function for the number of bacteria as a function of the number of days. • The expression $50(0.85)^x$ represents the amount of a drug in milligrams that remains in the bloodstream after *x* hours. a. Describe how the amount of drug in milligrams changes over time. b. What would the expression $50(0.85)^{12x}$ represent? *c.* What new or different information is revealed by the changed expression?

◻ **A2: F-IF.C.9** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*	**Component(s) of Rigor:** Conceptual Understanding, Procedural Skill and Fluency **Remediation - Previous Grade(s) Standard:** ◻ A1: F-IF.C.9 **Algebra II Standard Taught in Advance:** 🟩 A2: F-IF.B.4 **Algebra II Standard Taught Concurrently:** none Students compare properties of two functions. The representations of the functions should vary: table, graph, algebraically, or verbal description. **Example:**  • If $f(x) = -(x+7)^2(x-2)$ and $g(x)$ is represented on the graph, what is the difference between the zero with the least value of $f(x)$ and the zero with the least value of $g(x)$? Which has the largest relative maximum? Describe their end behaviors. Why are they different? What can be said about each function?

Functions: Building Functions (F-BF)

A. Build a function that models a relationship between two quantities.

In this cluster, the terms students should learn to use with increasing precision are **arithmetic sequence, geometric sequence, explicit formula, recursive process, model a situation, and translate between two forms.**

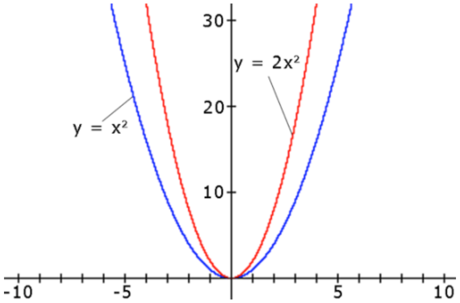
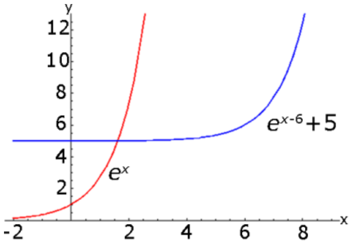
Louisiana Standard	Explanations and Examples
🟩 **A2: F-BF.A.1** Write a function that describes a relationship between two quantities.★ a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*	**Component(s) of Rigor:** Conceptual Understanding (1, 1a, 1b), Procedural Skill and Fluency (1, 1a,1b), Application (1b) **Remediation - Previous Grade(s) Standard:** ◻ A1: F-BF.A.1 **Algebra II Standard Taught in Advance:** none **Algebra II Standard Taught Concurrently:** 🟩 A2: F-IF.B.4, ◻ A2: F-LE.A.2 **Examples:** • Skeleton tower: https://www.illustrativemathematics.org/content-standards/HSF/BF/A/1/tasks/75 • Susita's account: https://www.illustrativemathematics.org/content-standards/HSF/BF/A/1/tasks/218 • A sum of functions: https://www.illustrativemathematics.org/content-standards/HSF/BF/A/1/tasks/230

🟩 **A2: F-BF.A.2** Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★	**Component(s) of Rigor:** Conceptual Understanding, Procedural Skill and Fluency, Application **Remediation - Previous Grade(s) Standard:** none **Algebra II Standard Taught in Advance:** 🟩 A2: F-BF.A.1 **Algebra II Standard Taught Concurrently:** 🟦 A2: F-LE.A.2 An explicit rule for the *n*th term of a sequence gives a_n as an expression in the term's position *n*; a recursive rule gives the first term of a sequence, and a recursive equation relates a_n to the preceding term(s). Both methods of presenting a sequence describe a_n as a function of *n*. **Examples:** • Generate the 5th-11th terms of a sequence if A_1= 2 and $A_{(n+1)} = (A_n)^2 - 1$ • Given the sequence defined by the function $a_{n+1} = \frac{3}{4}a_n$ with $a_0 = 424$. Write an explicit function rule. • Snake on a plane: https://www.illustrativemathematics.org/content-standards/HSF/BF/A/2/tasks/1695

Functions: Building Functions (F-BF)

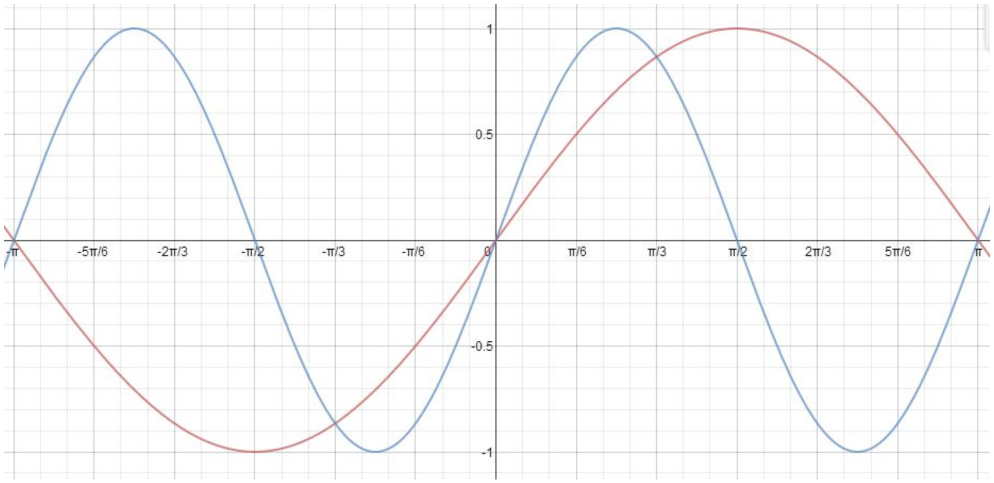
B. Build new functions from existing functions.

In this cluster, the terms students should learn to use with increasing precision are **effect of a transformation on a graph and even and odd functions.**

Louisiana Standard	Explanations and Examples
○ **A2: F-BF.B.3** Identify the effect on the graph of replacing *f*(*x*) by *f*(*x*) + *k*, *k f*(*x*), *f*(*kx*), and *f*(*x* + *k*) for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*	**Component(s) of Rigor:** Conceptual Understanding, Procedural Skill and Fluency **Remediation - Previous Grade(s) Standard:** ○ A1: F-BF.B.3 **Algebra II Standard Taught in Advance:** none **Algebra II Standard Taught Concurrently:** □ A2: F-IF.C.7, □ A2: F-IF.C.8 Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to graph functions. **Examples:** • Is *f*(*x*) = $x^3 - 3x^2 + 2x + 1$ even, odd, or neither? Explain your answer orally or in written format.  • Describe effect of varying the parameters *a, h,* and *k* have on the shape and position of the graph of $f(x) = a(x-h)^2 + k$. • Compare the shape and position of the graphs of $f(x) = e^x$ to $g(x) = e^{x-6} + 5$, and explain the differences, orally or in written format, in terms of the algebraic expressions for the functions. 

<table>
<tr>
<td>🟡 A2: F-BF.B.3 <i>continued</i></td>
<td>

- Compare the shape and position of the graphs of *y* = sin x to *y* = 2 sin x.



</td>
</tr>
<tr>
<td>🟡 A2: F-BF.B.4 Find inverse functions.
a. Solve an equation of the form <i>f</i>(<i>x</i>) = <i>c</i> for a simple function <i>f</i> that has an inverse and write an expression for the inverse. <i>For example, f(x) =2 x^3 or f(x) = (x+1)/(x-1) for x ≠ 1.</i></td>
<td>

Component(s) of Rigor: Procedural Skill and Fluency (4, 4a)
Remediation - Previous Grade(s) Standard: none
Algebra II Standard Taught in Advance: 🟩 A2: A-REI.A.2
Algebra II Standard Taught Concurrently: none

Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model functions.

Examples:

- For the function *h*(*x*) = (*x* – 2)^3, defined on the domain of all real numbers, find the inverse function if it exists or explain why it doesn't exist.
- Find the inverse of the function $g(x) = 2^x$ and demonstrate it is the inverse using input – output pairs.
- Find a domain for f(*x*) = 3*x*^{2} + 12*x* - 8 on which it has an inverse. Explain why it is necessary to restrict the domain of the function.

</td>
</tr>
</table>

Functions: Linear, Quadratic, and Exponential Models (F-LE)

A. Construct and compare linear, quadratic, and exponential models and solve problems.

In this cluster, the terms students should learn to use with increasing precision are **construct a function, arithmetic sequence, and geometric sequence.**

Louisiana Standard

◻ **A2: F-LE.A.2** Given a graph, a description of a relationship, or two input-output pairs (include reading these from a table), construct linear and exponential functions, including arithmetic and geometric sequences to solve multi-step problems.

Explanations and Examples

Component(s) of Rigor: Conceptual Understanding, Procedural Skill and Fluency, Application

Remediation - Previous Grade(s) Standard: ◻ A1: F-LE.A.1, ◻ A1: F-LE.A.2

Algebra II Standard Taught in Advance: none

Algebra II Standard Taught Concurrently: 🟩 A2: F-BF.A.1, 🟩 A2: F-BF.A.2

Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to construct linear and exponential functions.

Examples:

- Determine an exponential function of the form $f(x) = ab^x$ using data points from the table. Graph the function and identify the key characteristics of the graph.

x	$f(x)$
0	1
1	3
3	27

- Sara's starting salary is $32,500. Each year she receives a $700 raise. Write a sequence in explicit form to describe the situation.
- After a record setting winter storm, there are 10 inches of snow on the ground! Now that the sun is finally out, the snow is melting. At 7 am there were 10 inches and at 12 pm there were 6 inches of snow.

  a. Construct a linear function rule to model the amount of snow.
  b. Construct an exponential function rule to model the amount of snow.
  c. Which model best describes the amount of snow? Provide reasoning for your choice.

🟦 **A2: F-LE.A.4** For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. ★	**Component(s) of Rigor:** Conceptual Understanding, Procedural Skill and Fluency **Remediation - Previous Grade(s) Standard:** none **Algebra II Standard Taught in Advance:** 🟩 A2: A-SSE.B.3, 🟦 A2: F-IF.C.8 **Algebra II Standard Taught Concurrently:** none Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to analyze exponential models and evaluate logarithms. **Example**: • Solve $200\ e^{0.04t} = 450$ for t. *Solution:* We first isolate the exponential part by dividing both sides of the equation by 200. $e^{0.04t} = 2.25$ Now we take the natural logarithm of both sides. $ln\ e^{0.04t} = ln\ 2.25$ The left hand side simplifies to $0.04t$, by logarithmic identity 1. $0.04t = ln\ 2.25$ Lastly, divide both sides by 0.04. $t = ln\ (2.25) / 0.04$ $t \approx 20.3$

<table>
<tr><td colspan="2">Functions: Linear, Quadratic, and Exponential Models (F-LE)
B. Interpret expressions for functions in terms of the situation they model.</td></tr>
<tr><td colspan="2">In this cluster, the terms students should learn to use with increasing precision are interpret parameters in terms of a context.</td></tr>
<tr><td>Louisiana Standard</td><td>Explanations and Examples</td></tr>
<tr><td>○ A2: F-LE.B.5 Interpret the parameters in a linear, quadratic, or exponential function in terms of a context.★</td><td>Component(s) of Rigor: Conceptual Understanding
Remediation - Previous Grade(s) Standard: ■ A1: F-LE.B.5
Algebra II Standard Taught in Advance: ■ A2: F-IF.C.7, ■ A2: F-LE.A.2
Algebra II Standard Taught Concurrently: none
<hr>
<p>Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model and interpret parameters in linear, quadratic or exponential functions.</p>
<p>Examples:</p>

A function of the form $f(n) = P(1 + r)^n$ is used to model the amount of money in a savings account that earns 5% interest, compounded annually, where n is the number of years since the initial deposit. What is the value of r? What is the meaning of the constant P in terms of the savings account? Explain either orally or in written format.
Lauren keeps records of the distances she travels in a taxi and what it costs:
<table>
<tr><th>Distance <i>d</i> in miles</th><th>Fare <i>f</i> in dollars</th></tr>
<tr><td>3</td><td>8.25</td></tr>
<tr><td>5</td><td>12.75</td></tr>
<tr><td>11</td><td>26.25</td></tr>
</table>
<ol type="a">
If you graph the ordered pairs (d, f) from the table, they lie on a line. How can this be determined without graphing them?
Show that the linear function in part a. has equation $F = 2.25d + 1.5$.
What do the 2.25 and the 1.5 in the equation represent in terms of taxi rides.

</td></tr>
</table>

Functions: Trigonometric Functions (F-TF)

A. Extend the domain of trigonometric functions using the unit circle.

In this cluster, the terms students should learn to use with increasing precision are **radian measure, arc length, unit circle, arc subtended by an angle, and counterclockwise.**

Louisiana Standard	Explanations and Examples
○ **A2: F-TF.A.1** Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.	**Component(s) of Rigor:** Conceptual Understanding **Remediation - Previous Grade(s) Standard:** ○ GM: G-C.B.5 **Algebra II Standard Taught in Advance:** none **Algebra II Standard Taught Concurrently:** none Students know that if the length of an arc subtended by an angle is the same length as the radius of the circle, then the measure of the angle is 1 radian. Students should also determine the radian measures of angles subtended around the circle. **Examples:** • What xactly is a radian: https://www.illustrativemathematics.org/content-standards/HSF/TF/A/1/tasks/1874 • Bicycle wheel: https://www.illustrativemathematics.org/content-standards/HSF/TF/A/1/tasks/1873
○ **A2: F-TF.A.2** Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	**Component(s) of Rigor:** Conceptual Understanding **Remediation - Previous Grade(s) Standard:** 🟩 GM: G-SRT.C.8, ○ GM: G-GPE.A.1 **Algebra II Standard Taught in Advance:** ○ A2: F-TF.A.1 **Algebra II Standard Taught Concurrently:** none Students understand that one complete rotation around the unit circle, starting at (0,1), restricts the domain of trigonometric functions to $0 \leq \theta \leq 2\pi$. As more rotations are considered, the domain extends to all real numbers since the radian measure of any angle is a real number and there is no limit to the number of times one can travel around the unit circle. **Examples:** • Trigonometric functions for arbitrary angles (radians): https://www.illustrativemathematics.org/content-standards/HSF/TF/A/2/tasks/1692 • Trig functions and the unit circle: https://www.illustrativemathematics.org/content-standards/HSF/TF/A/2/tasks/1820

Functions: Trigonometric Functions (F-TF)

B. Model periodic phenomena with trigonometric functions.

In this cluster, the terms students should learn to use with increasing precision are **periodic phenomena, amplitude, frequency, and midline.**

Louisiana Standard	Explanations and Examples
○**A2: F-TF.B.5** Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. ★	**Component(s) of Rigor:** Conceptual Understanding, Procedural Skill and Fluency **Remediation - Previous Grade(s) Standard:** none **Algebra II Standard Taught in Advance:** ○ A2: F-BF.B.3 **Algebra II Standard Taught Concurrently:** none **Example:** • Foxes and rabbits 2: https://www.illustrativemathematics.org/content-standards/HSF/TF/B/5/tasks/816 • Foxes and rabbits 3: https://www.illustrativemathematics.org/content-standards/HSF/TF/B/5/tasks/817

Functions: Trigonometric Functions (F-TF)

C. Prove and apply trigonometric identities.

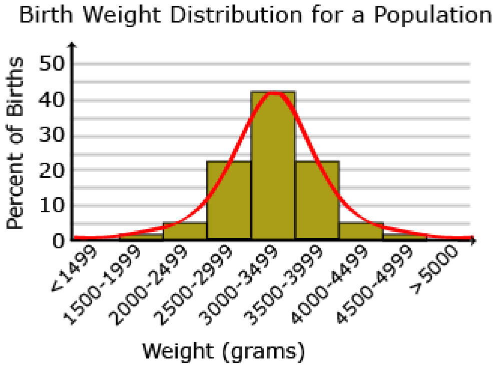
In this cluster, the terms students should learn to use with increasing precision are **proof, Pythagorean Identity, trigonometric functions sin(θ), cos(θ), and tan(θ), and quadrant of an angle.**

Louisiana Standard	Explanations and Examples
○ **A2: F-TF.C.8** Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle.	**Component(s) of Rigor:** Conceptual Understanding, Procedural Skill and Fluency **Remediation - Previous Grade(s) Standard:** none **Algebra II Standard Taught in Advance:** ○ A2: F-TF.A.2 **Algebra II Standard Taught Concurrently:** none Students prove $sin^2(\theta) + cos^2(\theta) = 1$. In the unit circle, the cosine is the *x*-value, while the sine is the *y*-value. Since the hypotenuse is always 1, the Pythagorean relationship $sin^2(\theta) + cos^2(\theta) = 1$ is always true. **Example:** Given $\cos\theta = \frac{\sqrt{3}}{2}$ and $\frac{3\pi}{2} \le \theta \le 2\pi$ find $sin\ (\theta)$ and $tan\ (\theta)$.

Statistics and Probability: Interpreting Categorical and Quantitative Data ★(S-ID)

A. Summarize, represent, and interpret data on a single count or measurement variable.

In this cluster, the terms students should learn to use with increasing precision are **mean deviation, standard deviation, normal distribution, and normal curve.**

Louisiana Standard	Explanations and Examples
A2: S-ID.A.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.	**Component(s) of Rigor:** Conceptual Understanding, Procedural Skill and Fluency, Application **Remediation - Previous Grade(s) Standard:** 6.SP.B.5, A1: S-ID.A.2 **Algebra II Standard Taught in Advance:** none **Algebra II Standard Taught Concurrently:** none Students use the normal distribution to make estimates of frequencies (which can be expressed as probabilities**).** They recognize that only some data are well described by a normal distribution. They use the 68-95-99.7 rule to estimate the percent of a normal population that falls within 1, 2, or 3 standard deviations of the mean. **Examples:** • The bar graph below gives the birth weight of a population of 100 chimpanzees. The line shows how the weights are normally distributed about the mean, 3250 grams. Estimate the percent of baby chimps weighing 3000-3999 grams.  Birth Weight Distribution for a Population o Scores on a history test have a mean of 80 with standard deviation of 6. How many standard deviations from the mean is the student that scores a 90.

Statistics and Probability: Interpreting Categorical and Quantitative Data ★ (S-ID)

B. Summarize, represent, and interpret data on a two categorical and quantitative variables.

In this cluster, the terms students should learn to use with increasing precision are **scatter plot, fit a function to data, and context of data.**

Louisiana Standard	Explanations and Examples
🟦 **A2: S-ID.B.6** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. *Emphasize exponential models.*	**Component(s) of Rigor:** Conceptual Understanding (6, 6a), Procedural Skill and Fluency (6, 6a), Application (6a) **Remediation - Previous Grade(s) Standard:** 🟦 A1: S-ID.B.6 **Algebra II Standard Taught in Advance:** 🟩 A2: F-BF.A.1, 🟦 A2: F-LE.A.2 **Algebra II Standard Taught Concurrently:** none **Examples:** • Used Subaru Foresters I: https://www.illustrativemathematics.org/content-standards/HSS/ID/B/6/tasks/941

Statistics and Probability: Making Inferences and Justifying Conclusions ★ (S-IC)

A. Understand and evaluate random processes underlying statistical experiments.

In this cluster, the terms students should learn to use with increasing precision are **inference, population parameters, random sample, and model consistent with results.**

Louisiana Standard	**Explanations and Examples**
A2: S-IC.A.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population. ★	**Component(s) of Rigor:** Conceptual Understanding **Remediation - Previous Grade(s) Standard:** 7.SP.A.2 **Algebra II Standard Taught in Advance:** none **Algebra II Standard Taught Concurrently:** none Student should be able to define populations, population parameter, random sample, and inference. • A *population* consists of everything or everyone being studied in an inference procedure. It is rare to be able to perform a census of every individual member of the population. Due to constraints of resources it is nearly impossible to perform a measurement on every subject in a population. • A *parameter* is a value, usually unknown (and which therefore has to be estimated), used to represent a certain population characteristic. • *Inferential statistics* considers a subset of the population. This subset is called a statistical sample often including members of a population selected in a random process. The measurements of the individuals in the sample tell us about corresponding measurements in the population. Students demonstrate an understanding of the different kinds of sampling methods. **Example:** From a class containing 12 girls and 10 boys, three students are to be selected to serve on a school advisory panel. Here are four different methods of making the selection. i. Select the first three names on the class roll. ii. Select the first three students who volunteer. iii. Place the names of the 22 students in a hat, mix them thoroughly, and select three names from the mix. iv. Select the first three students who show up for class tomorrow. Which is the best sampling method, among these four, if you want the school panel to represent a fair and representative view of the opinions of your class? Explain the weaknesses of the three you did not select as the best.

◻ **A2: S-IC.A.2** Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin will fall heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?* ★	**Component(s) of Rigor:** Conceptual Understanding, Procedural Skill and Fluency, Application **Remediation - Previous Grade(s) Standard:** ◻ 7.SP.C.7 **Algebra II Standard Taught in Advance:** none **Algebra II Standard Taught Concurrently:** none Possible data-generating processes include (but are not limited to): flipping coins, spinning spinners, rolling a number cube, and simulations using the random number generators. Students may use graphing calculators, spreadsheet programs, or applets to conduct simulations and quickly perform large numbers of trials. The law of large numbers states that as the sample size increases, the experimental probability will approach the theoretical probability. Comparison of data from repetitions of the same experiment is part of the model building verification process. **Example:** • Have multiple groups flip coins. One group flips a coin 5 times, one group flips a coin 20 times, and one group flips a coin 100 times. Which group's results will most likely approach the theoretical probability? • Block Scheduling: http://www.illustrativemathematics.org/illustrations/125

Statistics and Probability: Making Inferences and Justifying Conclusions ★ (S-IC)

B. Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

In this cluster, the terms students should learn to use with increasing precision are **sample survey, experiment, observational study, randomization, population mean or proportion, margin of error, treatment(s), and evaluate reports.**

Louisiana Standard	Explanations and Examples
🟩 **A2: S-IC.B.3** Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.★	**Component(s) of Rigor:** Conceptual Understanding **Remediation - Previous Grade(s) Standard:** none **Algebra II Standard Taught in Advance:** 🟦 A2: S-IC.A.1 **Algebra II Standard Taught Concurrently:** none Students understand the different methods of data collection, specifically the difference between an observational study and a controlled experiment, and know the appropriate use for each. • *Observational study* – a researcher collects information about a population by measuring a variable of interest, but does not impose a treatment on the subjects. (I.e. examining the health effects of smoking) • *Experiment* – an investigator imposes a change or treatments on one or more group(s), often called treatment group(s). A comparative experiment is where a control group is given a placebo to compare the reaction(s) between the treatment group(s) and the control group. Students understand the role that randomization plays in eliminating bias from collected data**.** **Example:** Students in a high school mathematics class decided that their term project would be a study of the strictness of the parents or guardians of students in the school. Their goal was to estimate the proportion of students in the school who thought of their parents or guardians as "strict". They do not have time to interview all 1000 students in the school, so they plan to obtain data from a sample of students. a. Describe the parameter of interest and a statistic the students could use to estimate the parameter. b. Is the best design for this study a sample survey, an experiment, or an observational study? Explain your reasoning. c. The students quickly realized that, as there is no definition of "strict", they could not simply ask a student, "Are your parents or guardians strict?" Write three questions that could provide objective data related to strictness. d. Describe an appropriate method for obtaining a sample of 100 students, based on your answer in part (a) above.

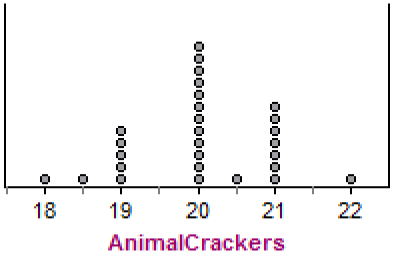
🟩 **A2: S-IC.B.4** Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.★

Component(s) of Rigor: Procedural Skill and Fluency, Application
Remediation - Previous Grade(s) Standard: none
Algebra II Standard Taught in Advance: 🟦 A2: S-IC.A.2, 🟩 A2: S-IC.B.3
Algebra II Standard Taught Concurrently: none

Students estimate a sample mean or sample proportion given data from a sample survey. Estimate the population value.

Examples:

- The label on a Barnum's Animal Cracker box claims that there are 2 servings per box and a serving size is 8 crackers. The graph displays the number of animal crackers found in a sample of 28 boxes. Use the data from the 28 samples to estimate the average number of crackers in a box with a margin of error. Explain your reasoning or show your work.



- Margin of Error for Estimating a Population Mean:
https://www.illustrativemathematics.org/content-standards/HSS/IC/B/4/tasks/1956

🟩 **A2: S-IC.B.5** Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. ★

Component(s) of Rigor: Conceptual Understanding, Procedural Skill and Fluency, Application
Remediation - Previous Grade(s) Standard: none
Algebra II Standard Taught in Advance: 🟦 A2: S-IC.A.2, 🟩 A2: S-IC.B.3
Algebra II Standard Taught Concurrently: none

Contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.

Examples:

- Sal purchased two types of plant fertilizer and conducted an experiment to see which fertilizer would be best to use in his greenhouse. He planted 20 seedlings and used Fertilizer A on ten of them and Fertilizer B on the other ten. He measured the height of each plant after two weeks. Use the data below to determine which fertilizer Sal should use.

Fertlizer A	23.4	30.1	28.5	26.3	32.0	29.6	26.8	25.2	27.5	30.8
Fertlizer B	19.8	25.7	29.0	23.2	27.8	31.1	26.5	24.7	21.3	25.6

a. Use the data to generate simulated treatment results by randomly selecting ten plant heights from the twenty plant heights listed.

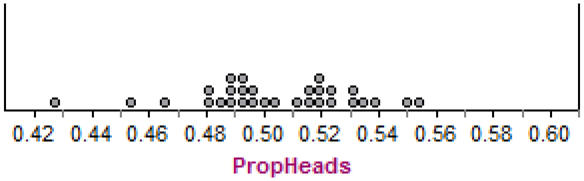
🟩 **A2: S-IC.B.5** *continued*

b. Calculate the average plant height for each treatment of ten plants.

c. Find the difference between consecutive pairs of treatment averages and compare. Does your simulated data provide evidence that the average plant heights using Fertlizer A and Fertilizer B is significant?

- “Are Starbucks customers more likely to be female?” To answer the question, students decide to randomly select 30-minute increments of time throughout the week and have an observer record the gender of every tenth customer who enters the Starbucks store. At the end of the week, they had collected data on 260 customers, 154 females and 106 males. This data seems to suggest more females visited Starbucks during this time than males.

  To determine if these results are statistically significant, students investigated if they could get this proportion of females just by chance if the population of customers is truly 50% females and 50% males. Students simulated samples of 260 customers that are 50-50 females to males by flipping a coin 260 then recording the proportion of heads to represent the number of women in a random sample of 260 customers (e.g., 0.50 means that 130 of the 260 flips were heads). Their results are displayed in the graph at the right.



  Use the distribution to determine if the class’s data is statistically significant enough to conclude that Starbucks customers are more likely to be female.

🟩 **A2: S-IC.B.6** Evaluate reports based on data.★

Component(s) of Rigor: Conceptual Understanding
Remediation - Previous Grade(s) Standard: none
Algebra II Standard Taught in Advance: 🟩 A2: S-IC.B.4, 🟩 A2: S-IC.B.5
Algebra II Standard Taught Concurrently: none

Contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.

Example:

Read the article below from NPR.org then answer the following questions.

Kids and Screen Time: What Does the Research Say?
By Juana Summers
August 28, 2014

Kids are spending more time than ever in front of screens, and it may be inhibiting their ability to recognize emotions, according to new research out of the University of California, Los Angeles.

🟩 **A2: S-IC.B.6** *continued*	The study, published in the journal *Computers in Human Behavior*, found that sixth-graders who went five days without exposure to technology were significantly better at reading human emotions than kids who had regular access to phones, televisions and computers. The UCLA researchers studied two groups of sixth-graders from a Southern California public school. One group was sent to the Pali Institute, an outdoor education camp in Running Springs, Calif., where the kids had no access to electronic devices. For the other group, it was life as usual. At the beginning and end of the five-day study period, both groups of kids were shown images of nearly 50 faces and asked to identify the feelings being modeled. Researchers found that the students who went to camp scored significantly higher when it came to reading facial emotions or other nonverbal cues than the students who continued to have access to their media devices. "We were pleased to get an effect after five days," says Patricia Greenfield, a senior author of the study and a distinguished professor of psychology at UCLA. "We found that the kids who had been to camp without any screens but with lots of those opportunities and necessities for interacting with other people in person improved significantly more." If the study were to be expanded, Greenfield says, she'd like to test the students at camp a third time — when they've been back at home with smartphones and tablets in their hands for five days. "It might mean they would lose those skills if they weren't maintaining continual face-to-face interaction," she says. a. Was this an experiment or an observational study? b. What can you conclude? c. Are there any limitations or concerns with this statistical study?

Grade 6 Standards

○ **6.SP.B.5** Summarize numerical data sets in relation to their context, such as by:

a. Reporting the number of observations.

b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.

c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.

d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

Return to ○ A2: S-ID.A.4

Grade 7 Standards

🟩 **7.NS.A.2** Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (-1)(-1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.

c. Apply properties of operations as strategies to multiply and divide rational numbers.

d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

Return to ◻ A2: A-APR.D.6

🟩 **7.EE.A.1** Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients to include multiple grouping symbols (e.g., parentheses, brackets, and braces). *Return to* ○ A2: N-CN.A.2

◻ **7.SP.A.2** Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be*. *Return to* ◻ A2: S-IC.A.1

▣ **7.SP.C.7** Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.*

b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*

Return to ▣ A2: S-IC.A.2

Grade 8 Standards

🟩 **8.EE.A.1** Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.
Return to 🟩 A2: N-RN.A.1

🟩 **8.EE.A.2** Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where *p* is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational. *Return to* 🟩 A2: N-RN.A.1, 🟩 A2: N-CN.A.1

Algebra I Course Standards

▣ **A1: N-Q.A.1** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. *Return to* ▣ A2: N-Q.A.2, 🟩 A2: F-IF.B.4

▣ **A1: N-Q.A.2** Define appropriate quantities for the purpose of descriptive modeling. *Return to* ▣ A2: N-Q.A.2

🟩 **A1: A-SSE.A.1** Interpret expressions that represent a quantity in terms of its context.

a. Interpret parts of an expression, such as terms, factors, and coefficients.

b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*

Return to 🟩 A2: A-SSE.A.2

🟩 **A1: A-SSE.A.2** Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, or see $2x^2 + 8x$ as $(2x)(x) + 2x(4)$, thus recognizing it as a polynomial whose terms are products of monomials and the polynomial can be factored as $2x(x+4)$.* *Return to* 🟩 A2: A-SSE.A.2

🟦 **A1: A-SSE.B.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

a. Factor a quadratic expression to reveal the zeros of the function it defines.
b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
c. Use the properties of exponents to transform expressions for exponential functions emphasizing integer exponents. *For example, the growth of bacteria can be modeled by either* $f(t) = 3^{(t+2)}$ *or* $g(t) = 9(3^t)$ *because the expression* $3^{(t+2)}$ *can be rewritten as* $(3^t)(3^2) = 9(3^t)$.

Return to 🟩 A2: A-SSE.B.3, 🟩 A2: A-APR.B.2, 🟩 A2: A-APR.B.3

🟦 **A1: A-APR.B.3** Identify zeros of quadratic functions, and use the zeros to sketch a graph of the function defined by the polynomial.
Return to 🟩 A2: A-APR.B.3, 🟦 A2: F-IF.C.7

🟩 **A1: A-CED.A.1** Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear, quadratic, and exponential functions. Return to* 🟦 A2: A-CED.A.1

🟩**A1: A-REI.A.1** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. *Return to* 🟩 A2: A-REI.A.1

🟩 **A1: A-REI.B.4** Solve quadratic equations in one variable.

a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as "no real solution."

Return to 🟦 A2: A-CED.A.1, 🟦 A2: A-REI.B.4

🟡 **A1: A-REI.C.6** Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
Return to 🟡 A2: A-REI.C.6

🟩 **A1: A-REI.D.11** Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, piecewise linear (to include absolute value), and exponential functions. *Return to* 🟩 A2: A-REI.D.11

🟩 **A1: F-IF.A.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$. *Return to* 🟩 A2: F-IF.B.4, 🟦 A2: F-IF.C.7

🟩 **A1: F-IF.A.2** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
Return to 🟩 A2: F-IF.B.6

🟩 **A1: F-IF.B.4** For linear, piecewise linear (to include absolute value), quadratic, and exponential functions that model a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior*. *Return to* 🟩 A2: F-IF.B.4

🟩 **A1: F-IF.B.6** Calculate and interpret the average rate of change of a linear, quadratic, piecewise linear (to include absolute value), and exponential function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. *Return to* 🟩 A2: F-IF.B.6

▣ **A1: F-IF.C.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
b. Graph piecewise linear (to include absolute value) and exponential functions.

Return to ▣ A2: F-IF.C.7

▣ **A1: F-IF.C.8** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

Return to ▣A2: F-IF.C.7, ▣ A2: F-IF.C.8

▣ **A1: F-IF.C.9** Compare properties of two functions (linear, quadratic, piecewise linear [to include absolute value] or exponential) each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, determine which has the larger maximum*. *Return to* ▣ A2: F-IF.C.9,

▣ **A1: F-BF.A.1** Write a linear, quadratic, or exponential function that describes a relationship between two quantities.

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

Return to 🟩 A2: F-BF.A.1

○ **A1: F-BF.B.3** Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative). Without technology, find the value of k given the graphs of linear and quadratic functions. With technology, experiment with cases and illustrate an explanation of the effects on the graph that include cases where $f(x)$ is a linear, quadratic, piecewise linear (to include absolute value) or exponential function.
Return to ○ A2: F-BF.B.3

□ **A1: F-LE.A.1** Distinguish between situations that can be modeled with linear functions and with exponential functions.

- a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
- b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
- c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

Return to □ A2: F-LE.A.2

□ **A1: F-LE.A.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). *Return to* □ A2: F-LE.A.2

□ **A1: F-LE.B.5** Interpret the parameters in a linear or exponential function in terms of a context. *Return to* ○ A2: F-LE.B.5

○ **A1: S-ID.A.2** Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. *Return to* ○ A2: S-ID.A.4

□ **A1: S-ID.B.6** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

- *a.* Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear and quadratic models.*
- b. Informally assess the fit of a function by plotting and analyzing residuals.
- c. Fit a linear function for a scatter plot that suggests a linear association.

Return to □A2: S-ID.B.6

Geometry Course Standards

○ **GM: G-C.B.5** Use similarity to determine that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. *Return to* ○ A2: F-TF.A.1

■ **GM: G-SRT.C.8** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. *Return to* ○ A2: F-TF.A.2

○ **GM: G-GPE.A.1** Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. *Return to* ○ A2: F-TF.A.2