Zeros of Polynomials (ECR)

Overview

Apply the Remainder Theorem to identify zeros of a function and determine a missing coefficient. Students will also sketch the graph of a polynomial function using the zeros of the function.

Standards

Understand the relationship between zeros and factors of polynomials.

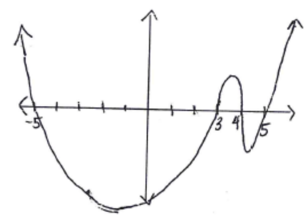
HSA-APR.B.2 Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x – a$ is $p(a)$, so $p(a) = 0$ if and only if $(x – a)$ is a factor of $p(x)$.

HSA-APR.B.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSA-APR.B.2	• HSA-SSE.B.3a	1. Use the Remainder Theorem to determine whether $(x-5)$ is a factor of $x^3+3x^2-25x-75$. a. $(x-5)$ is a factor of the polynomial because $p(5)=0$. 2. http://www.illustrativemathematics.org/illustrations/592	• http://www.illustrativemathematics.org/illustrations/388 • http://learnzillion.com/lessonsets/605-know-and-apply-the-remainder-theorem • http://learnzillion.com/lessonsets/504-know-and-apply-the-remainder-theorem
HSA-APR.B.3	• HSA-SSE.A.2 • HSA-SSE.B.3a	1. Find all zeros of $p(x)=(x^2-25)(x^2-7x+12)$. a. Zeros: –5, 3, 4, 5 2. Construct a rough graph of $p(x)=(x^2-25)(x^2-7x+12)$ showing all zeros of the function.	• http://www.illustrativemathematics.org/illustrations/87 • http://www.illustrativemathematics.org/illustrations/21 • http://learnzillion.com/lessonsets/606-identify-zeros-of-polynomials-and-construct-rough-graphs-of-polynomial-functions • http://learnzillion.com/lessonsets/559-identify-zeros-and-construct-graphs-of-

Grade Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
		a. 	2degree-polynomials

After the Task

Common errors/misconceptions by problem number:

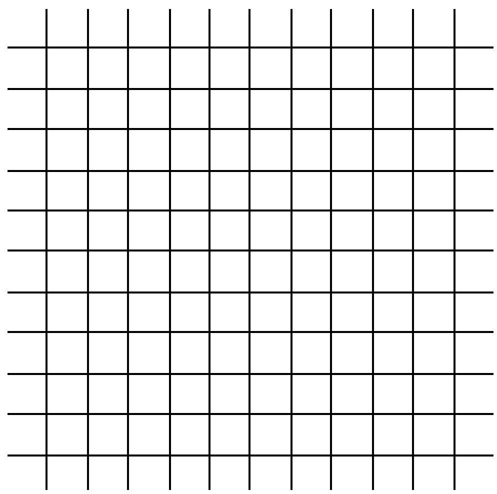
- ***Problem 1:*** Students may use 3 for the value of a instead of –3 when evaluating the polynomial. Remind students to rewrite the binomial $(x + 3)$ as $(x - (-3))$ to find the correct value of a.
- ***Problem 2:*** Students may struggle to find the remaining zeros. Remind students that one factor was given in problem 1. To find the remaining zeros, students will need to find the other factors. Ask students to think about methods used to find an unknown factor when given a product and one of the factors. Then guide students to recall the ways they know to divide polynomials. Students will also need to be reminded of how to factor a quadratic trinomial from Algebra I. Extra practice with division of polynomials and factoring may be necessary for struggling students.
- ***Problem 3:*** Students' responses should demonstrate that they are able to apply the Remainder Theorem. If students use long division or synthetic division, ask them if there is a more efficient method to find the remainder of the given polynomial.
- ***Problem 4:*** Students who are struggling with applying the Remainder Theorem will attempt to solve this by using other methods (e.g., long division or synthetic division). Ask students to identify what the remainder would be if the given polynomial were divided evenly by the binomial. Students should understand that the remainder would be zero and that $p(a)$ should be zero.

Student Extended Constructed Response

Use the polynomial function $p(x) = 7x^3 + 29x^2 + 25x + 3$ to answer questions 1 and 2 below.

1. Show that $(x + 3)$ is a factor of $p(x)$ using the Remainder Theorem. Explain your reasoning.

2. Sketch a graph of $p(x)$ showing all zeros of the function. Show all work to find the zeros.



3. Consider the polynomial $f(x) = x^4 + 2x^3 - 7x^2 - 8x + 12$. Determine the remainder of $f(x) \div k(x)$ if $k(x) = (x - 3)$. Show your work.

4. The polynomial function $q(x) = x^4 + ax^3 - 19x^2 - 46x + 120$ has a root of –4. What is the value of the missing coefficient, *a*? Show your work and explain your reasoning.

Extended Constructed Response Exemplar Response

Use the polynomial function $p(x) = 7x^3 + 29x^2 + 25x + 3$ to answer questions 1 and 2 below.

1. Show that $(x + 3)$ is a factor of $p(x)$ using the Remainder Theorem. Explain your reasoning.

   If $(x + 3)$ is a factor of $p(x)$, then $p(-3) = 0$. Substitute –3 for x and evaluate the expression.

$$p(-3) = 7(-3)^3 + 29(-3)^2 + 25(-3) + 3$$

$$p(-3) = -189 + 261 - 75 + 3$$

$$p(-3) = 0$$

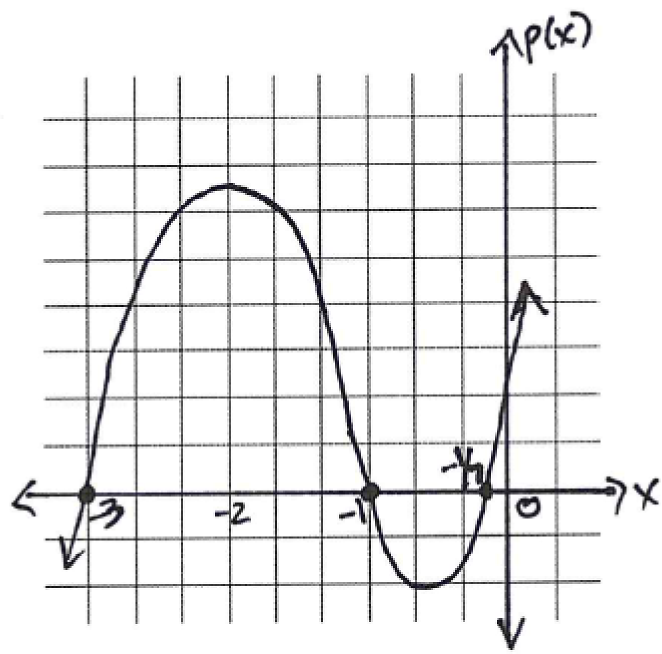
2. Sketch a graph of $p(x)$ showing all zeros of the function. Show all work to find the zeros.

   Find the remaining factors first.

-3	7	29	25	3
		-21	-24	-3
	7	8	1	0

$7x^2 + 8x + 1 = (7x + 1)(x + 1)$; *set factors equal to zero and solve for x.*

Zeros of the function: $-3,\ -1, -\frac{1}{7}$



3. Consider the polynomial $f(x) = x^4 + 2x^3 - 7x^2 - 8x + 12$. Determine the remainder of $f(x) \div k(x)$ if $k(x) = (x - 3)$. Show your work.

$$f(3) = (3)^4 + 2(3)^3 - 7(3)^2 - 8(3) + 12$$

$$f(3) = 60$$

The remainder of $f(x) \div k(x)$ if $k(x) = (x - 3)$ is 60.

4. The polynomial function $q(x) = x^4 + ax^3 - 19x^2 - 46x + 120$ has a root of –4. What is the value of the missing coefficient, *a*? Show your work and explain your reasoning.

If –4 is a zero of $q(x)$, then $q(-4) = 0$. So replace all x variables with –4, set the function equal to 0, and solve for a.

$$(-4)^4 + a(-4)^3 - 19(-4)^2 - 46(-4) + 120 = 0$$

$$256 - 64a - 304 + 184 + 120 = 0$$

$$-64a + 256 = 0$$

$$-64a = -256$$

$$a = 4$$

Solving Exponential Equations (ECR)

Overview

Using technology, students will solve exponential equations by finding the *x*-coordinate(s) of the point(s) of intersection.

Standard

Represent and solve equations and inequalities graphically.

HSA-REI.D.11 Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are prerequisites for student success with this task's standards.

Grade Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSA-REI.D.11	• HSA-REI.C.5 • HSA-REI.C.6 • HSA-REI.C.7	1. Using technology, approximate the solution to the equation $f(x) = g(x)$ if $f(x) = \left(\frac{1}{2}\right)^x$ and $g(x) = 3$. a. $x \approx -1.585$ 2. http://www.illustrativemathematics.org/illustrations/645	• http://www.illustrativemathematics.org/illustrations/1363 • http://www.illustrativemathematics.org/illustrations/1833 • http://www.illustrativemathematics.org/illustrations/223 • http://www.illustrativemathematics.org/illustrations/576 • http://learnzillion.com/lessonsets/692-identify-the-points-of-intersection-between-y-fx-and-y-gx-as-the-solutions-to-fx-gx • http://learnzillion.com/lessonsets/662-understand-relationships-between-functions-and-why-the-xcoordinates-of-where-the-graphs-of-fx-and-gx-intersect-are-the-solutions-of-fxgx

After the Task

In all three problems, students will be required to explain how they used technology to find their solution. If students do not include an explanation, guide them to write the steps they perform on the calculator to help with their explanation. Students may also be allowed to sketch what the screen of their calculator looks like to support the explanation. For problems 1 and 3, if students have learned how to solve exponential equations using logarithms, the task could be modified to ask students to solve the problem using pencil and paper in place of technology.

Student Extended Constructed Response

1. Consider the functions $f(x) = 5e^x$ and $g(x) = 25$. Using technology, approximate the *x*-coordinate(s) of the point(s) of intersection of the graphs of $y = f(x)$ and $y = g(x)$. Explain how you found your answer.

2. Approximate the solution of the equation $b(x) = c(x)$, given $b(x) = \log(2x)$ and $c(x) = 5 - e^x$. Explain how you found your answer.

3. A company estimates that the monthly cost of gradually implementing a new process in its factory can be modeled by the function $f(n) = \frac{1}{4}(2)^{0.25n}$ where *n* is the number of months since implementation began. This cost continues to change up to a maximum monthly cost of $m(n) = 200$ dollars. Once the monthly cost reaches the maximum of $200, the process is fully implemented.

    a. Write an equation to determine the number of months until full implementation.

    b. Determine the approximate number of months until the process is fully implemented. Explain how you found your answer.

Extended Constructed Response Exemplar Response

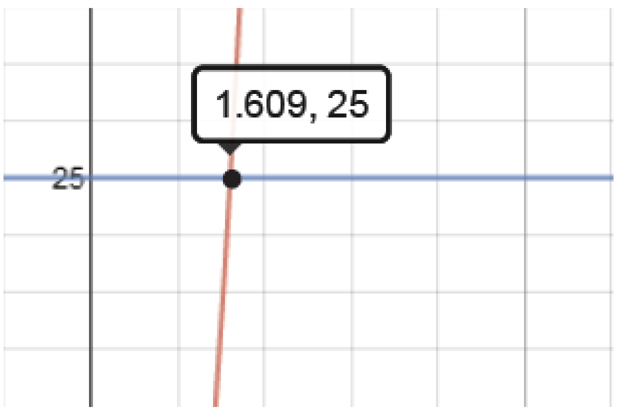
1. Consider the functions $f(x) = 5e^x$ and $g(x) = 25$. Using technology, approximate the *x*-coordinate(s) of the point(s) of intersection of the graphs of $y = f(x)$ and $y = g(x)$. Explain how you found your answer.

   Note: As students find the approximation, they may use technology to create a table of values or create a graph and find the point of intersection. The approximation may vary based on the chosen methods.

   Sample response:

   I graphed each function and used the calculate feature on the graphing calculator to find the point of intersection. The x-coordinate is approximately 1.609.

   The window used to create the graph below is:
   Xmin: -1; Xmax: 6; X scale: 1; Ymin: 21; Ymax: 28; Yscale: 1

   The online calculator at https://www.desmos.com/calculator/ was used to create this graph.

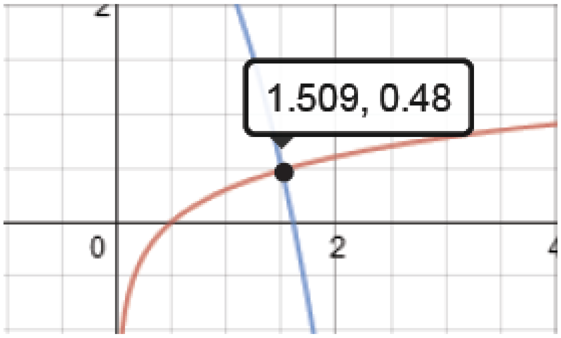
2. Approximate the solution of the equation $b(x) = c(x)$, given $b(x) = \log(2x)$ and $c(x) = 5 - e^x$. Explain how you found your answer.

   Note: As students find the approximation, they may use technology to create a table of values or create a graph and find the point of intersection. The approximation may vary based on the chosen methods.

   Sample response:

   I graphed each function and used the calculate feature on the graphing calculator to find the point of intersection. The x-coordinate of the point of intersection is the solution to the equation $b(x) = c(x)$. The solution is approximately 1.509.

The window used to create the graph below is:
Xmin: -1; Xmax: 4; X scale: 0.5; Ymin: -1; Ymax: 2; Yscale: 0.5



The online calculator at https://www.desmos.com/calculator/ was used to create this graph.

3. A company estimates that the monthly cost of gradually implementing a new process in their factory can be modeled by the function $f(n) = \frac{1}{4}(2)^{0.25n}$ where *n* is the number of months since implementation began. This cost continues to change up to a maximum monthly cost of $m(n) = 200$ dollars. Once the monthly cost reaches the maximum of $200, the process is fully implemented.

   a. Write an equation to determine the number of months until full implementation.

$$\frac{1}{4}(2)^{0.25n} = 200$$

   b. Determine the approximate number of months until the process is fully implemented. Explain how you found your answer.

   Note: As students find the approximation, they may use technology to create a table of values or create a graph and find the point of intersection. The approximation may vary based on the chosen methods.

   Sample response:

   The process will be fully implemented at approximately 38.6 months. I created a table of values with my calculator. I entered the function into Y_1 = and started the table at 1 with a change in x-value of 1. Next, I found that the function value was 181.02 at x = 38 and 215.27 at x = 39. Then I changed the table to start at 38 with a change in x-value of 0.1. At x = 38.5, the value of the function is 197.4. At x = 38.6, the value of the function is 200.85.

   Note: If students get a correct response based on an incorrect equation from part a, they should be awarded credit for this part.

Arithmetic and Geometric Sequences (ECR)

Overview

Students will write recursive and explicit formulas for arithmetic and geometric sequences and use the formulas to solve a mathematical problem.

Standard

Build a function that models a relationship between two quantities.

HSF-BF.A.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are prerequisites for student success with this task's standards.

Grade Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSF-BF.A.2	• HSF-BF.A.1a • HSF-IF.A.3	1. Are the following sequences geometric or arithmetic? a. $4, 2, 1, \frac{1}{2}, \frac{1}{4}, \ldots$ i. geometric b. 7, 16, 25, 34, 43, . . . i. arithmetic c. $\frac{1}{2}, 1, 1\frac{1}{2}, 2, 2\frac{1}{2}, \ldots$ i. arithmetic d. 2, 6, 18, 54, 162, . . . i. geometric e. 1, 2, 4, 8, 16, . . . i. geometric 2. Write a formula to represent the sequence: 7, 16, 25, 34, 43, . . . a. $a_n = 7 + 9(n - 1)$ 3. Write a formula to represent the sequence: 2, 6, 18, 54, 162, . . a. $a_n = (2)(3)^{n-1}$	• http://www.illustrativemathematics.org/illustrations/572 • http://www.illustrativemathematics.org/illustrations/573 • http://www.illustrativemathematics.org/illustrations/218

Real-World Preparation: The following question will prepare students for some of the real-world components of this task:

- ***What is a book of stamps?*** Stamps are available individually or in packets called *books*. Books of stamps commonly hold either 10 or 20 stamps.

After the Task

Students may struggle with writing the explicit and recursive formulas. Provide students with additional practice with writing both the explicit and recursive forms of arithmetic and geometric sequences.

Students may choose to use the recursive formula to answer part c on problem 1. Discuss with them how they could have found the answer more quickly using the explicit formula. Discuss different situations when both the recursive formula and the explicit formula would be useful.

Have students identify a real-world situation that can be modeled with a geometric and arithmetic sequence. Have students write the formulas for each situation and then use those formulas to answer questions.

Student Extended Constructed Response

Jerome works at the United States Post Office.

1. Jerome is selling books of 10 stamps. As he is selling them to customers, he notices that 2 books cost $9.80, 3 books cost $14.70, 4 books cost $19.60, and 5 books cost $24.50.

    a. Is this sequence arithmetic, geometric, or neither? Show or explain your thinking.

    b. Write the recursive and explicit formulas for this sequence.

    c. Use either of the formulas that you created in the step above to show how much 17 books of stamps would cost. Show your work.

2. A customer wants to hang a poster on Jerome's bulletin board. Jerome notices that the poster is too tall to fit in the space that he has available, so he decides to use his copier to reduce the height of the poster. Jerome knows that he will have to reduce the poster more than one time to make it fit in the space that he has available. The original height of the poster is 20 inches. For each reduction, the copier will reduce the poster's height to 64% of the previous height.

    a. Is this sequence arithmetic, geometric, or neither? Show or explain your thinking.

    b. Write the recursive and explicit formulas for this sequence.

    c. Use either of the formulas that you created in the step above to find the height of the photograph after three reductions. Show your work.

3. Last year, Jerome's town had a population of approximately 5,000 people. The population this year is 1.01 times the population last year.

    a. Assuming that the population continues to grow at this rate, write a formula for this sequence.

    b. Find the population on the third year of the sequence. Show your work.

    c. If each person uses approximately 20 stamps per year, how many stamps should Jerome order for the third year in the sequence? Show your work.

Extended Constructed Response Exemplar Response

Jerome works at the United States Post Office.

1. Jerome is selling books of 10 stamps. As he is selling them to customers, he notices that 2 books cost \$9.80, 3 books cost \$14.70, 4 books cost \$19.60, and 5 books cost \$24.50.

    a. Is this sequence arithmetic, geometric, or neither? Show or explain your thinking.

    Sample response:

    This sequence is arithmetic.

    - $14.70 - 9.80 = 4.90$
    - $19.60 - 14.70 = 4.90$
    - $24.50 - 19.60 = 4.90$

    The sequence is arithmetic because there is a common difference. The common difference is \$4.90.

    b. Write the recursive and explicit formulas for this sequence.

    Sample response:

    Explicit:

    $$a_n = a_1 + d(n-1)$$

    $$a_n = 4.90 + 4.90(n-1)$$

    Alternate explicit formula: $a_n = 4.90n$

    Recursive:

    $$\begin{cases} a_1 = start \\ a_n = a_{n-1} + d \end{cases}$$

    $$\begin{cases} a_1 = 4.90 \\ a_n = a_{n-1} + 4.90 \end{cases}$$

    c. Use either of the formulas that you created in the step above to show how much 17 books of stamps would cost. Show your work.

    $$a_n = 4.90 + 4.90(n-1)$$

    $$\begin{gathered} a_n = 4.90 + 4.90(17-1) \\ a_n = 4.90 + 4.90(16) \\ a_n = 4.90 + 78.40 \\ a_n = 83.30 \end{gathered}$$

    ***Note: Students may also choose to use the recursive formula here, which would require them to find all of the values for 6 – 16 books of stamps.*

2. A customer wants to hang a poster on Jerome's bulletin board. Jerome notices that the poster is too tall to fit in the space that he has available, so he decides to use his copier to reduce the height of the poster. Jerome knows that he will have to reduce the poster more than one time to make it fit in the space that he has available. The original height of the poster is 20 inches. For each reduction, the copier will reduce the poster's height to 64% of the previous height.

   a. Is this sequence arithmetic, geometric, or neither? Show or explain your thinking.

      Sample response:

      This sequence is geometric. Each time, the height is going to be multiplied by .64 to obtain the next height. Since each term is multiplied by a common factor to obtain the next term, this is a geometric sequence.

   b. Write the recursive and explicit formulas for this sequence.

      Sample response:

      Explicit:

$$a_n = (a_1)(r)^{n-1}$$

$$a_n = (12.8)(.64)^{n-1}$$

      Recursive:

$$\begin{cases} a_1 = start \\ a_n = (a_{n-1})(r) \end{cases}$$

$$\begin{cases} a_1 = 12.8 \\ a_n = (a_{n-1})(.64) \end{cases}$$

   c. Use either of the formulas that you created in the step above to find the height of the photograph after three reductions. Show your work.

$$a_n = (12.8)(.64)^{n-1}$$

$$a_n = (12.8)(.64)^{3-1}$$

$$a_n = (12.8)(.64)^2$$

$$a_n = (12.8)(.4096)$$

$$a_n = 5.24288$$

      The height of the poster after three reductions is about 5.3 inches.

      ***Note: Students may also choose to use the recursive formula here, which would require them to find all of the values for all three reductions.*

3. Last year, Jerome's town has a population of approximately 5,000 people. The population this year is 1.01 times the population last year.

   a. Assuming that the population continues to grow at this rate, write a formula for this sequence.

$$a_n = (5050)(1.01)^{n-1}$$

   b. Find the population on the third year of the sequence. Show your work.

$$a_n = (5050)(1.01)^{n-1}$$
$$a_n = (5050)(1.01)^{3-1}$$
$$a_n = (5050)(1.01)^2$$
$$a_n = (5050)(1.0201)$$
$$a_n = 5151.505$$

   The population on the third year will be approximately 5,151 people.

   c. If each person uses approximately 20 stamps per year, how many stamps should Jerome order for the third year in the sequence? Show your work.

$$5151 \times 20 = 103020$$

   Jerome needs to order 102,030 stamps.

Student Well-Being (ECR)

Overview

Students will use sample survey data to calculate population mean, margin of error, and sample size.

Standard

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

HSS-IC.B.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are prerequisites for student success with this task's standards.

Grade Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSS-IC.B.4	• HSS-IC.A.2 • HSS-IC.A.3	1. Suppose that the Gallup organization's latest poll sampled 1,000 people from the United States, and the results show that 520 people (52%) think the president is doing a good job, compared to 48% who don't think so. Find the margin of error that corresponds to a 95% confidence level for this poll. a. .0310 or 3.1% **2.** http://www.illustrativemathematics.org/illustrations/1411	• http://www.illustrativemathematics.org/illustrations/244 • http://www.illustrativemathematics.org/illustrations/125 • http://www.illustrativemathematics.org/illustrations/1099 • http://www.illustrativemathematics.org/illustrations/1029 • http://learnzillion.com/lessonsets/531-use-survey-data-to-estimate-means-and-proportions-develop-a-margin-of-error-through-simulation-models-and-evaluate-reports-based-on-data • http://learnzillion.com/lessonsets/401-use-data-from-a-sample-survey-and-evaluate-reports-based-on-data

Real-World Preparation: The following questions will prepare students for some of the real-world components of this task:

- ***What are some key factors that drive student success?*** According to the Gallup Poll, the following were relevant to student success:
    - Ideas and energy students have for the future (hopeful)
    - Involvement in and enthusiasm for school (engagement)
    - How students think about and experience their lives (well-being)
- ***How was the general well-being of students in grades 5-12 measured?***
    - The Gallup Student Poll is a 20-question survey that measures the hope, engagement, and well-being of students in grades 5-12.
    - The primary application was to measure noncognitive metrics that predict student success in academic and other youth development settings.
- ***What is meant by* noncognitive*?***
    - *Noncognitive* means related to personality or preferences rather than intelligence.

Source: http://www.gallupstudentpoll.com/121688/Online-Learning-Webinars.aspx

After the Task

Students may struggle with trying to remember how to find the margin of error. Provide students with additional practice finding the margin of error for various confidence intervals. For problem 2, students need to select only one of the 6 items. Be sure to check the accuracy of their work and statements. When working problem 3, students may overlook the fact that they are to assume there is no information about the population mean. They must use a constant value of 0.25 when finding the sample size. Students who overlook the given information for problem 3 will likely struggle with problem 4.

Student Extended Constructed Response

In 2013, a Gallup Poll was administered to 589,997 students in grades 5-12. The purpose of this study was to determine the overall well-being of students. It is important to measure such noncognitive areas because they serve as a predictor of student success in academic and other youth development settings. The following chart includes the results of 6 items included on the survey.

Well-Being Survey Items	% of Students Responding "Yes"
Were you treated with respect all day yesterday?	**69%**
Did you smile or laugh a lot yesterday?	**85%**
Did you learn or do something interesting yesterday?	**76%**
Did you have enough energy to get things done yesterday?	**75%**
Do you have health problems that keep you from doing any of the things other people your age normally do?	**16%**
If you are in trouble, do you have family or friends you can count on to help whenever you need them?	**93%**

Source Data: http://www.gallupstudentpoll.com/166037/2013-gallup-student-poll-overall-report.aspx

1. Assuming that the sample is a simple random sample, find the margin of error that corresponds to a 95% confidence level for each item.

2. Select 1 of the 6 items and use the calculated margin of error to define the confidence interval and write a statement that could be used to report this data.

3. A member of your community would like to complete a similar study. How many students in grades 5-12 must be surveyed in order to be 95% confident that the sample percentage is in error by no more than four percentage points? Assume that we have no prior information of the population mean.

4. The mean “yes” response rate across all 6 items is 69%. Based on this new piece of data, how many students in grades 5-12 must be surveyed in order to be 95% confident that the sample percentage is in error by no more than four percentage points?

Extended Constructed Response Exemplar Response

In 2013, a Gallup Poll was administered to 589,997 students in grades 5-12. The purpose of this study was to determine the overall well-being of students. It is important to measure such noncognitive areas because they serve as a predictor of student success in academic and other youth development settings. The following chart includes the results of 6 items included on the survey.

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Were you treated with respect all day yesterday?	**69%**
Did you smile or laugh a lot yesterday?	**85%**
Did you learn or do something interesting yesterday?	**76%**
Did you have enough energy to get things done yesterday?	**75%**
Do you have health problems that keep you from doing any of the things other people your age normally do?	**16%**
If you are in trouble, do you have family or friends you can count on to help whenever you need them?	**93%**

Source Data: http://www.gallupstudentpoll.com/166037/2013-gallup-student-poll-overall-report.aspx

1. Assuming that the sample is a simple random sample, find the margin of error that corresponds to a 95% confidence level for each item.

Formula for calculating the margin of error: $E = z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$

Item 1. $E = 1.96\sqrt{\frac{(.69)(.31)}{589{,}997}}$ = .0012 Margin of error: 0.12%

Item 2. $E = 1.96\sqrt{\frac{(.85)(.15)}{589{,}997}}$ = .00091 Margin of error: 0.09%

Item 3. $E = 1.96\sqrt{\frac{(.76)(.24)}{589{,}997}}$ = .0011 Margin of error: 0.11%

Item 4. $E = 1.96\sqrt{\frac{(.75)(.25)}{589{,}997}}$ = .0011 Margin of error: 0.11%

Item 5. $E = 1.96\sqrt{\frac{(.16)(.84)}{589{,}997}}$ = .00094 Margin of error: 0.10%

Item 6. $E = 1.96\sqrt{\frac{(.93)(.07)}{589{,}997}}$ = .00065 Margin of error: 0.07%

2. Select 1 of the 6 items and use the calculated margin of error to define the confidence interval and write a statement that could be used to report this data.

   (Note: Exemplar response is provided for Item 3. Responses for all items will be similar.)

   Item 3. Exemplar: $\hat{p} - E < p < \hat{p} + E$

$$.76 - .0011 < p < .76 + .0011$$

$$.75891 < p < .76109$$

$$75.9\% < p < 76.1\%$$

   It is estimated that 76% of students in grades 5-12 indicated they learned or did something interesting yesterday, with a margin of error of plus or minus 0.109%.

3. A member of your community would like to complete a similar study. How many students in grades 5-12 must be surveyed in order to be 95% confident that the sample percentage is in error by no more than four percentage points? Assume that we have no prior information of the population mean.

$$n = \frac{\left[z_{\alpha/2}\right]^2 \cdot 0.25}{E^2}$$

$$n = \frac{(1.96)^2 \cdot 0.25}{(.04)^2}$$

$$n = 600.25$$

   In order to be in error by no more than 4% and 95% confident in our results, 601 students in grades 5-12 must be surveyed.

4. The mean "yes" response rate across all 6 items is 69%. Based on this new piece of data, how many students in grades 5-12 must be surveyed in order to be 95% confident that the sample percentage is in error by no more than four percentage points?

$$n = \frac{\left[z_{\alpha/2}\right]^2 \hat{p}\hat{q}}{E^2}$$

$$n = \frac{(1.96)^2(.69)(.31)}{(.04)^2}$$

$$n = 513.574$$

   Using the known population mean of 69%, we will need a sample size of 514 students in grades 5-12 in order to be 95% confident with an error of no more than 4%.

Radical Table (ECR)

Overview

Students will use the properties of exponents to simplify expressions while demonstrating an understanding of the notations for radicals in terms of rational exponents.

Standard

Extend the properties of exponents to rational exponents.

HSN-RN.A.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are prerequisites for student success with this task's standards.

Grade Level Standard	The Following Standard Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSN-RN.A.2	• HSN-RN.A.1	1. Rewrite the following in rational notation: $\sqrt[5]{x^2y^3}$ a. $x^{\frac{2}{5}}y^{\frac{3}{5}}$ 2. Simplify: $\sqrt{\frac{16x^5}{9y^3}}$ a. $\frac{4x^2\sqrt{x}}{3y\sqrt{y}} = \frac{4x^2\sqrt{xy}}{3y^2}$ 3. http://www.illustrativemathematics.org/illustrations/1220 4. http://www.illustrativemathematics.org/illustrations/1842	• http://www.illustrativemathematics.org/illustrations/1823 • http://learnzillion.com/lessonsets/646-rewrite-expressions-involving-radicals-and-rational-exponents

After the Task

Students who struggle with simplifying the expressions with rational exponents in this task may need additional practice applying the properties of integer exponents for scaffolding purposes.

For item 4, students may leave the radical in the denominator, which is acceptable depending on the situation. If students do leave the radical expression in the denominator, discuss with them the process of rationalizing the denominator and in which situations that process would be helpful.

It may be helpful to have students discuss as a whole class the different strategies they used to simplify some of the expressions in the task in order to help struggling students understand the different procedures they can use.

Student Extended Constructed Response

Part A. Complete the following table to so that each item is represented both in radical and rational forms. Simplify when possible while assuming all variables represent positive real numbers.

Item #	Radical Notation	Rational (Fraction) Notation	Simplified Radical Form
1.	$\sqrt[4]{x^3}$		
2.		$z^{-\frac{1}{3}}$	
3.	$\sqrt[4]{128x^5}$		
4.		$\left(\frac{27}{z^2}\right)^{\frac{1}{3}}$	
5.		$\frac{x^{\frac{3}{4}}}{x^{\frac{1}{4}}}$	
6.	$\frac{\sqrt[3]{\frac{1}{z}}}{\sqrt[3]{z^2}}$		
7.		$\left(y \cdot y^{\frac{1}{4}}\right)^{\frac{4}{3}}$	
8.	$\sqrt{\frac{243a^{10} \cdot b^3}{3b}}$		

Extended Constructed Response Exemplar Response

Part A. Complete the following table to so that each item is represented both in radical and rational forms. Include the simplest form while assuming all variables represent positive real numbers.

(Note: Red text indicates item filled in by students.)

Item #	Radical Notation	Rational (Fraction) Notation	Simplified Radical Form
1.	$\sqrt[4]{x^3}$	$x^{\frac{3}{4}}$	$\sqrt[4]{x^3}$
2.	$\sqrt[3]{\frac{1}{z}}$	$z^{-\frac{1}{3}}$	$\sqrt[3]{\frac{1}{z}}$ Or equivalent form
3.	$\sqrt[4]{128x^5}$	$(128x^5)^{\frac{1}{4}}$	$2x\sqrt[4]{8x}$
4.	$\sqrt[3]{\frac{27}{z^2}}$	$\left(\frac{27}{z^2}\right)^{\frac{1}{3}}$	$\frac{3\sqrt[3]{z}}{z}$
5.	$\sqrt[4]{\frac{x^3}{x}}$ or $\frac{\sqrt[4]{x^3}}{\sqrt[4]{x}}$	$\frac{x^{\frac{3}{4}}}{x^{\frac{1}{4}}}$	$\sqrt{x}$ or $x^{\frac{1}{2}}$
6.	$\frac{\sqrt[3]{\frac{1}{z}}}{\sqrt[3]{z^2}}$	$\frac{z^{-\frac{1}{3}}}{z^{\frac{2}{3}}}$	$\frac{1}{z}$ or z^{-1}
7.	$(\sqrt[3]{y^4})\cdot(\sqrt[3]{y})$	$\left(y \cdot y^{\frac{1}{4}}\right)^{\frac{4}{3}}$	$y\sqrt[3]{y^2}$ or $y^{\frac{5}{3}}$
8.	$\sqrt{\frac{243a^{10}\cdot b^3}{3b}}$	$\left(\frac{243a^{10}b^3}{3b}\right)^{\frac{1}{2}}$	$9a^5b$