Eureka Acceleration Tool: Algebra I
Module 3, Topic B

To become mathematically proficient, students **must** access on-grade-level content. This document aims to help teachers who use the Eureka curriculum to ensure readiness for students before and during on-grade-level work, creating opportunities for timely support directly connected to the new learning.

About this Topic

Focus Standards:

A1: F-IF.A.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

A1: F-IF.A.2: Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

A1: F-IF.A.3: Recognize that sequences are functions whose domain is a subset of the integers. Relate arithmetic sequences to linear functions and geometric sequences to exponential functions.

A1: F-IF.B.4: For linear, piecewise linear (to include absolute value), quadratic, and exponential functions that model a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.★

A1: F-IF.B.5: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.★

A1: F-IF.C.7a: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ★

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

Topic Overview per the Eureka Curriculum

In Lesson 8, students consider that the notation they have been using to write explicit formulas for sequences can be applied to situations where the inputs are not whole numbers**.** In Lessons 9 and 10, they revisit the notion of function that was introduced in Grade 8. They are now prepared to use function notation as they write functions, interpret statements about functions, and evaluate functions for inputs in their domains. They formalize their understanding of a function as a correspondence between two sets, X and Y, in which each element of X is matched (or assigned) to one and only one element of Y, and add the understanding that the set X is called the *domain*, and the set Y is called the *range.*

Students study the graphs of functions in Lessons 11–14 of this topic. In Lesson 11, students learn the meaning of the graph of a function, f, as the set of all points $(x, f(x))$ in the Cartesian plane, such that x is in the domain of f and $f(x)$ is the value assigned to x by the correspondence of the function. Students use plain English to write the instructions needed to plot the graph of a function. The instructions are written in a way similar to writing computer "pseudocode"—before actually writing the computer programs. In Lesson 12, students learn that the graph of $y = f(x)$ is the set of all points (x, y) in the plane that satisfy the equation $y = f(x)$ and conclude that it is the same as the graph of the function explored in Lesson 11. In Lesson 13, students use a graphic of the planned landing sequence Mars Curiosity Rover to create graphs of specific aspects of the landing sequence—altitude over time and velocity over time—and use the graphs to examine the meaning of increasing and decreasing functions. Finally, Lesson 14 capitalizes on students' new knowledge of functions and their graphs to contrast linear and exponential functions and the growth rates which they model.

Eureka Acceleration Tool: Algebra I
Module 3, Topic B

Overview

Eureka Acceleration Tools include:

1. a diagnostic assessment to help teachers determine the misunderstandings or gaps in mathematical knowledge related to a specific Topic in the Eureka curriculum
2. guidance for teachers to analyze student work on the diagnostic assessment
3. suggested materials for targeted remedial instruction

Note: The use of this guidance is not intended to delay students' engagement with on-grade-level learning. On-grade-level learning should be the focus of instructional time and be treated as an opportunity for students to "finish" learning previous skills and deepen conceptual understanding.

Diagnostic Assessment

The diagnostic assessment is designed to be administered to targeted students prior to beginning instruction on the given Topic. When appropriate, it is broken into parts (Part A, Part B, and so on); each part addresses a different prerequisite standard and contains three problems. If a student correctly answers at least 2 out of the 3 problems, it can be assumed that he/she is ready to engage with the new content of the Topic with little to no support needed prior to engaging with the Topic. The diagnostic assessment is designed in this way so that teachers can determine the "entry point" to remedial instruction and/or opportunities for unfinished learning within the context of the new learning. The entry points and opportunities for unfinished learning will vary between students.

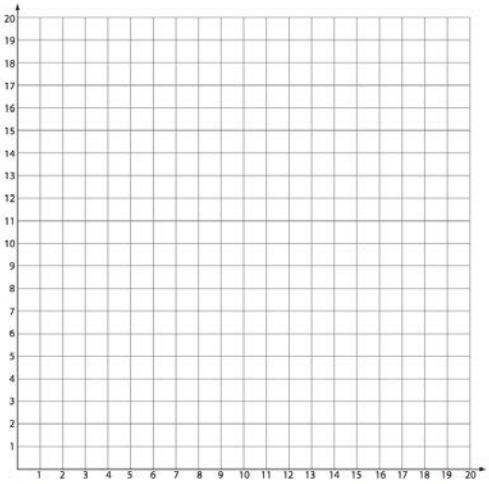
Guidance for Acceleration

The Acceleration Guidance is designed for teacher use. It is also broken into parts (Part A, Part B, and so on) and correlates to the parts on the diagnostic assessment. Each part contains the following:

1. **The focus standard**: The focus standards are strategically chosen to address prerequisite skills and are purposefully arranged in the order that students typically master the skills and knowledge.
2. **Why this is important for current grade level work**: This section describes how the work of the prerequisite standard relates to the standard(s) addressed in the Topic of instruction.
3. **Using the diagnostic assessment to identify gaps**: This section identifies common errors students make on the diagnostic assessment items.
4. **Acceleration Resources for Targeted Instruction**: The resources pinpoint specific Eureka lessons and parts of lessons for teachers to use to address gaps in mathematical knowledge. Using Eureka materials to address acceleration ensures alignment to the standards, consistency in approach to learning, and similarities in strategies for solving problems.

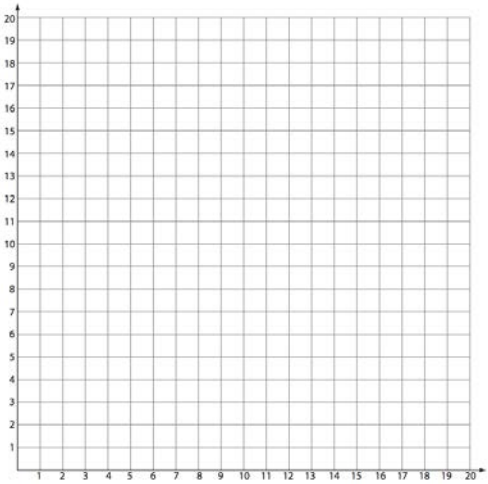
Part A: 8.EE.B.5

1. Joseph's teacher created a review game where she assigned each correct response the same number of points. The table below shows some examples of correct responses and associated point totals. Graph the relationship between correct responses and total points on the provided coordinate grid. Be sure to label your axes.

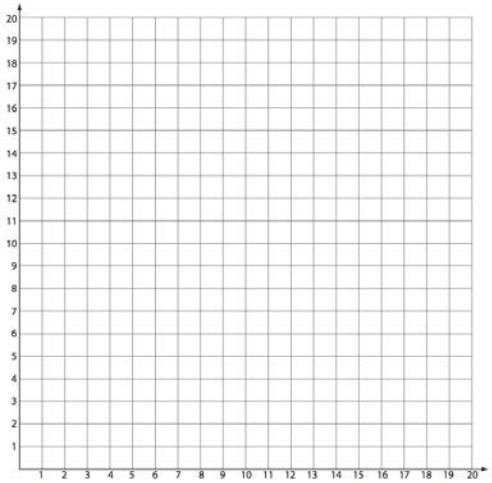


Correct Responses	Total Points
0	0
1	4
2	8
5	20

2. Paul is using a recipe that calls for $\frac{2}{3}$ of a cup of drink mix for every 1 cup of water. The relationship is represented by the equation $y = \frac{2}{3}x$, where x represents the number of cups of water and y represents the number of cups of drink mix. Graph the equation on the provided coordinate grid. Be sure to label your axes.



3. Evan created the equation $c = 6f$ to help him calculate the total cost, c, to take f friends to the movies at the matinee price. Graph the equation on the provided coordinate grid. Be sure to label your axes.

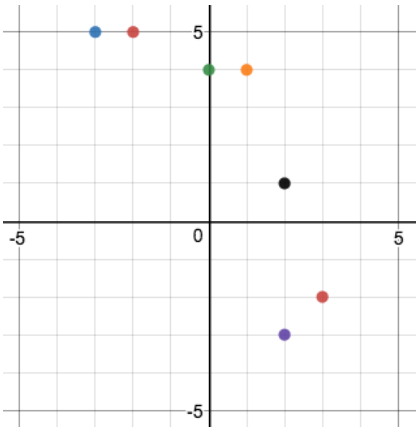


Part B: 8.F.A.1

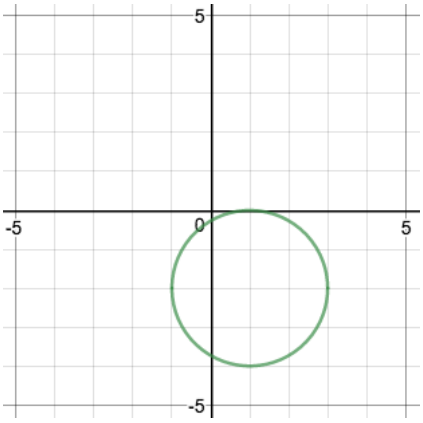
4. Determine if the following relation is a function. Explain your answer.

x	***y***
0	12
1	9
5	2
2	7
-1	2
3	8

5. Determine if the following relation is a function. Explain your answer.



6. Determine if the following relation is a function. Explain your answer.



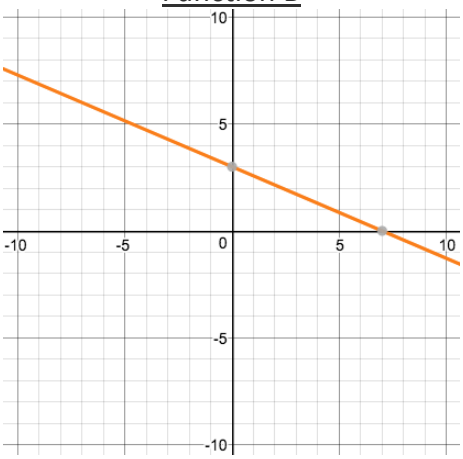
Part C: 8.F.A.2

7. Which function has a greater y-intercept? Explain your choice and/or show your work.

Function A

$$y = -\frac{2}{5}x + 6$$

Function B



8. Which function has a greater rate of change? Explain your choice and/or show your work.

Function A

x	-6	-3	0	3	6
y	2	4	6	8	10

Function B

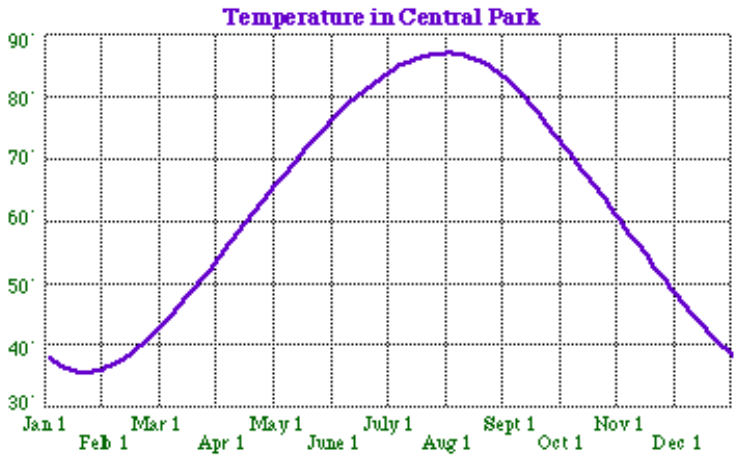
$$y = \frac{5}{4}x - 10$$

9. Which function has a greater initial value? Explain your choice and/or show your work.

Function A	Function B
Elizabeth charges a $250 fee for a home visit to do interior design. She then charges $45 per hour that she works with a client.	*Kay uses the equation* $C(h) = 200 + 50h$ *to calculate how much a job costs for her to help a client redesign part of a home.*

Part D: 8.F.B.5

10. The graph below shows the (approximate) average daily high temperature in New York's Central Park.



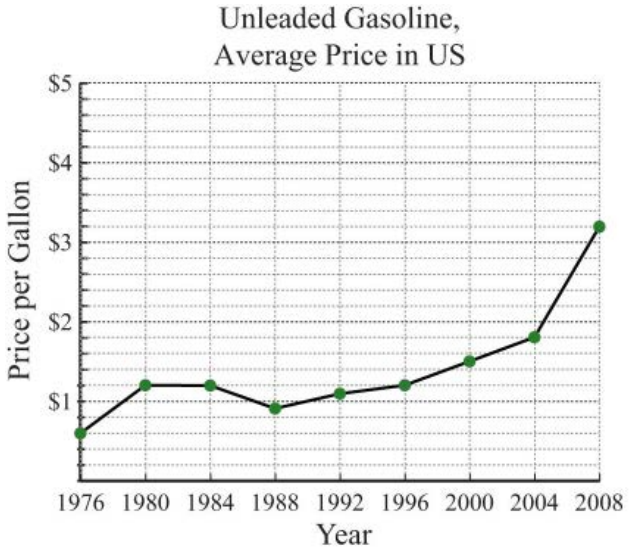
Source: National Weather Service/*The New York Times*, January 7, 1996, p. 36.

a. Between which months is the function increasing?

b. Between which months is the function decreasing?

c. Does the function ever appear to be linear?

11. The graph below shows average price of a gallon of regular unleaded gasoline in US cities.



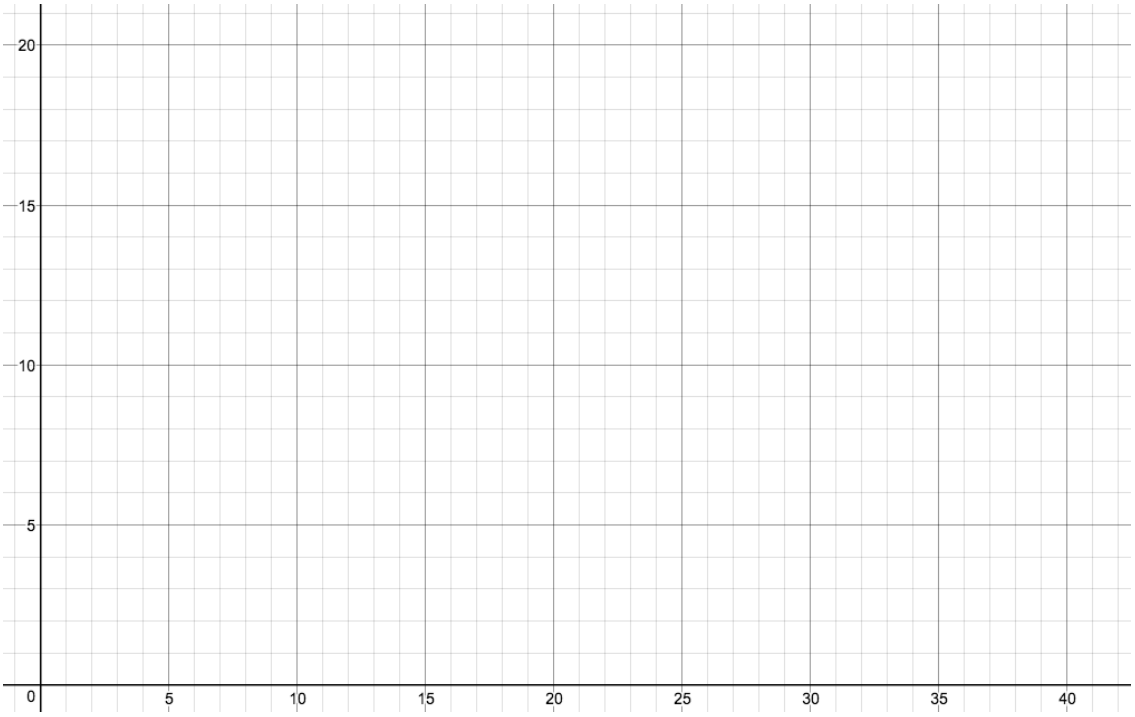
Source: **Bureau of Labor Statistics**

a. Between which years is the function increasing?

b. Between which years is the function decreasing?

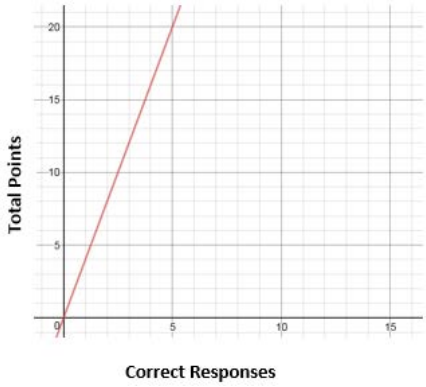
c. Does the function ever appear to be linear?

12. Mary started running a race and ran steadily for 20 minutes, completing 5 laps. She stopped to get water for five minutes then continued running at a faster, constant rate for the next 15 minutes, completing another 5 laps. Sketch a graph of Mary's race on the provided coordinate grid. Be sure to label your axes.



Solutions:

1.



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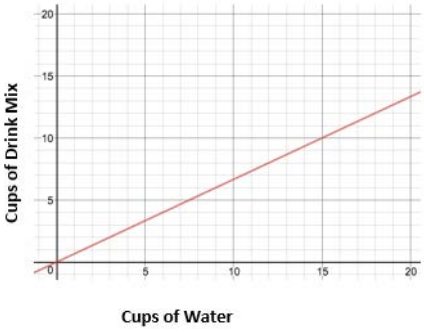

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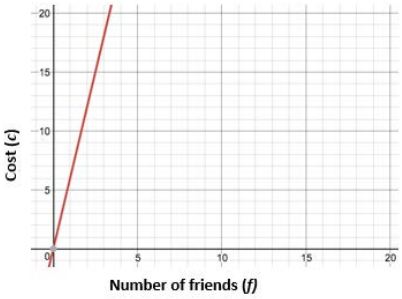
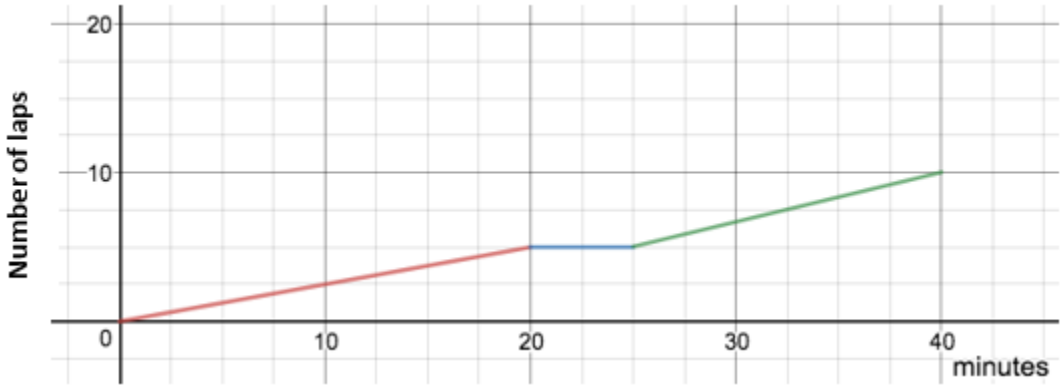


4. The relation is a function as each input only has one output.
5. The relation is not a function as the input 2 is paired with multiple outputs.
6. The relation is not a function as there exists multiple inputs that have two outputs.
7. Function A. Explanations will vary.
8. Function B. Explanations will vary.
9. Function A. Explanations will vary.
10. a. February to August
    b. January to February and August to December
    c. (sample) Yes, between March and July and between September and December
11. a. 1976 to 1980 and 1988 to 2008
    b. 1984 to 1988
    c. (sample) Yes, the function appears to be linear on every individual 4-year interval.
12.



Part A Focus: 8.EE.B.5: Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance - time graph to a distance - time equation to determine which of two moving objects has greater speed.*

Why this is important for current grade level work:
Students graphed linear equations throughout their work in 8th grade and will build on these skills, as well as their understanding of linear equations, as they work with the graphs of functions in Algebra I. Students should be able to fluently generate and graph ordered pairs (points) using a linear equation, beginning with simple proportional equations or relationships. It is not always intuitive for students that the way to graph an equation is to use it to generate ordered pairs. The simplest example of this is when students graph an equation representing a proportional relationship. Students who do not know what to do when presented with an equation will struggle to make the leap to graphing other functions. In addition, students should have already connected that the rate of change in the proportional relationship is the slope of the graph, leading to a more efficient way to graph linear equations in Algebra I. In Algebra I, students will solidify their understanding that the graph of any function is the set of points $(\boldsymbol{x}, \boldsymbol{f}(\boldsymbol{x}))$ that satisfy the equation $\boldsymbol{y} = \boldsymbol{f}(\boldsymbol{x})$. This work begins formally in Lesson 11. The graph can be generated by substituting values from the domain of the function into the equation and finding corresponding values for $\boldsymbol{y}$ or $\boldsymbol{f}(\boldsymbol{x})$. Students will also realize that values for *x*, or input values, need not be whole numbers and can often be any value unless constrained by a real-life situation.

Using the Diagnostic Assessment to identify gaps:

Problem 1:
Students should be able to simply graph the given ordered pairs. Look for students who graph pairs backwards (y, x) or who do not have a firm grasp on graphing points in general. Students should recognize that this table of values contains (0, 0) and will form a line since it is proportional.

Problem 2:
From 8th Grade, students should be familiar with proportional relationships as a subset of linear relationships whose initial value or y-intercept is (0, 0). Look for students who incorrectly graph two points using the 2/3 not as a slope, but as two numbers. Students may be confused since there is no y-intercept value shown in the equation.

Problem 3:
Students should be able to use their understanding of the situation to produce points and a graph. Look for students who graph points backwards, based on the context (e.g., graphing (6, 1) instead of (1, 6)). Students should connect that the graphed line has a slope of 6, because the rate of change is $6 per ticket.

Acceleration Resources for Targeted Instruction:

8th Grade, Module 4, Topic B, Lesson(s) 10 – 11

Use the Classwork portion of each Lesson and a sampling of problems from the Problem Set focused on conceptual understanding and/or procedural skill and fluency.

Part B Focus: 8.F.A.1: Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (Function notation is not required in this grade level.)

Why this is important for current grade level work:
While not the first Topic in Algebra I that addresses the concept of a function, this Topic digs deeper into the idea of the graph of a function, extending students knowledge of the graph of a function from Grade 8. Students should have a clear understanding of inputs and their associated outputs, both from a table and a graph. These items will help determine which students are still struggling with the idea of a function.

Using the Diagnostic Assessment to identify gaps:

Problem 4:
Look for students who think the relation is not a function due to a repeating output value as this shows a clear misunderstanding of the definition of a function.

Problem 5:
Look for students who neglect to see that an input of 2 produces two different outputs as this likely shows a misunderstanding of the definition of a function.

Problem 6:
Look for students who struggle to identify that nearly every input has two outputs since the graph is not discrete points. This likely shows a misunderstanding of what the graph of a relation or function represents.

Acceleration Resources for Targeted Instruction:

8th Grade, Module 5, Topic A, Lesson(s) 2 and 5

Use the Classwork portion of each Lesson and a sampling of problems from the Problem Set focused on conceptual understanding.

Part C Focus: 8.F.A.2: Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*

Why this is important for current grade level work:
The target Topic culminates with a lesson devoted to comparing growth rates: linear, learned in Grade 8, and exponential, new learning of Algebra I. Not only will students need to be able to compare functions presented in multiple ways (e.g., algebraically, graphically, etc.), they will also need a solid understanding of rate of change (i.e., slope). Students should be able to calculate the rate of change for a function as well as readily identify the rate of change when given an equation or graph of the function. Furthermore, students need to have a solid understanding that linear functions have a constant rate of change, a fact that serves as the crux of the new learning of exponential functions.

Using the Diagnostic Assessment to identify gaps:

Problem 7:
Look for students who cannot readily identify the y-intercept of Function A, possibly substituting zero for x to calculate the y-intercept. This shows a lack of understanding of the general form for an equation of a line and will prove detrimental for students as they move their study of functions beyond linear functions.

Problem 8:
Look for students who fail to connect rate of change to slope and/or who calculate the slope of Function A to be $\frac{3}{2}$, the reciprocal of the actual slope. Such students should be encouraged to graph the function, connecting the graph to the table.

Problem 9:
Look for students who fail t recognize the fee in Function A as a constant value, the initial value. Much of the target Topic is grounded in context, requiring students to pull pertinent information from a context and connect that information to the concept of a function.

Acceleration Resources for Targeted Instruction:

8th Grade, Module 5, Topic A, Lesson 7

Use the Classwork portion of the Lesson and a sampling of problems from the Problem Set focused on conceptual understanding.

Part D Focus: 8.F.B.5: Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

<table>
<tr><td colspan="2">Why this is important for current grade level work:
Lessons 11 – 14 all focus on the graph of a function with the latter two lessons focused on interpreting graphs of functions, helping students distinguish what they know about linear functions from what they are learning about exponential functions. Students need a solid understanding of the graph of a linear function to be able to distinguish it from the new learning. Students without a solid understanding of the graph of a linear function will struggle to engage with the new learning of the target Topic.</td><td rowspan="2">Acceleration Resources for Targeted Instruction:

<u>8th Grade, Module 6, Topic A, Lesson(s) 4 – 5</u>

Use the Classwork portion of each Lesson and a sampling of problems from the Problem Set focused on conceptual understanding.</td></tr>
<tr><td colspan="2">Using the Diagnostic Assessment to identify gaps:</td></tr>
<tr><td>Problems 10 – 11:
Look for students who confuse the ideas of increasing and decreasing as this may be a sign that students are reading the graph from right to left, instead of left to right. For such students, connect the idea of reading a sentence, left to right, to reading a graph, also left to right.</td><td>Problem 12:
Look for students who do not end their graph at 10 laps, neglecting that information from the context. It is not enough for students to pull out some useful information; they must be able to pull out all pertinent information from a given context.</td></tr>
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