Eureka Acceleration Tool: Algebra I
Module 3, Topic A

To become mathematically proficient, students **must** access on-grade-level content. This document aims to help teachers who use the Eureka curriculum to ensure readiness for students before and during on-grade-level work, creating opportunities for timely support directly connected to the new learning.

About this Topic

Focus Standards:

A1: F-IF.A.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

A1: F-IF.A.2: Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

A1: F-IF.A.3: Recognize that sequences are functions whose domain is a subset of the integers. Relate arithmetic sequences to linear functions and geometric sequences to exponential functions.

A1: F-IF.B.6: Calculate and interpret the average rate of change of a linear, quadratic, piecewise linear (to include absolute value), and exponential function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★

Topic Overview per the Eureka Curriculum

In Lesson 1 of Topic A, students challenge the idea that patterns can be defined by merely seeing the first few numbers of the pattern. They learn that a sequence is an ordered list of elements and that it is sometimes intuitive to number the elements in a sequence beginning with 0 rather than 1. In Lessons 2 and 3, students learn to define sequences explicitly and recursively and begin their study of arithmetic and geometric sequences that continues through Lessons 4–7 as students explore applications of geometric sequences. In the final lesson, students compare arithmetic and geometric sequences as they compare growth rates.

A1: F-BF.A.1: Write a linear, quadratic, or exponential function that describes a relationship between two quantities.★

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

A1: F-LE.A.1: Distinguish between situations that can be modeled with linear functions and with exponential functions.★

a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

A1: F-LE.A.2: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).★

A1: F-LE.A.3: Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.★

Eureka Acceleration Tool: Algebra I
Module 3, Topic A

Overview

Eureka Acceleration Tools include:

1. a diagnostic assessment to help teachers determine the misunderstandings or gaps in mathematical knowledge related to a specific Topic in the Eureka curriculum
2. guidance for teachers to analyze student work on the diagnostic assessment
3. suggested materials for targeted remedial instruction

Note: The use of this guidance is not intended to delay students' engagement with on-grade-level learning. On-grade-level learning should be the focus of instructional time and be treated as an opportunity for students to "finish" learning previous skills and deepen conceptual understanding.

Diagnostic Assessment

The diagnostic assessment is designed to be administered to targeted students prior to beginning instruction on the given Topic. When appropriate, it is broken into parts (Part A, Part B, and so on); each part addresses a different prerequisite standard and contains three problems. If a student correctly answers at least 2 out of the 3 problems, it can be assumed that he/she is ready to engage with the new content of the Topic with little to no support needed prior to engaging with the Topic. The diagnostic assessment is designed in this way so that teachers can determine the "entry point" to remedial instruction and/or opportunities for unfinished learning within the context of the new learning. The entry points and opportunities for unfinished learning will vary between students.

Guidance for Acceleration

The Acceleration Guidance is designed for teacher use. It is also broken into parts (Part A, Part B, and so on) and correlates to the parts on the diagnostic assessment. Each part contains the following:

1. **The focus standard**: The focus standards are strategically chosen to address prerequisite skills and are purposefully arranged in the order that students typically master the skills and knowledge.
2. **Why this is important for current grade level work**: This section describes how the work of the prerequisite standard relates to the standard(s) addressed in the Topic of instruction.
3. **Using the diagnostic assessment to identify gaps**: This section identifies common errors students make on the diagnostic assessment items.
4. **Acceleration Resources for Targeted Instruction**: The resources pinpoint specific Eureka lessons and parts of lessons for teachers to use to address gaps in mathematical knowledge. Using Eureka materials to address acceleration ensures alignment to the standards, consistency in approach to learning, and similarities in strategies for solving problems.

Part A: 6.EE.A.2c

1. Evaluate $\frac{r}{4} + 8$ for $r = 12$. Show your work.

2. Evaluate $3t - 2$ for $t = 6$. Show your work.

3. While considering expanding their living room and carpeting the new space, the Jones family created the expression $(18 + n)^2 + 64$, where n represents the number of feet to be added to each side of the main portion of their living room, to represent the number of square feet that will need to be carpeted. Determine the total number of square feet that will need to be carpeted if $n = 7$. Show your work.

Part B: 8.F.A.3

4. Does the equation $y = 2^x + 3$ define a linear function? Explain.

5. Does the equation $y = 3 + 2x$ define a linear function? Explain.

6. Does the selection of input-output pairs shown in the table below represent a linear function? Explain.

x	***y***
-2	-5
0	1
2	7
5	16

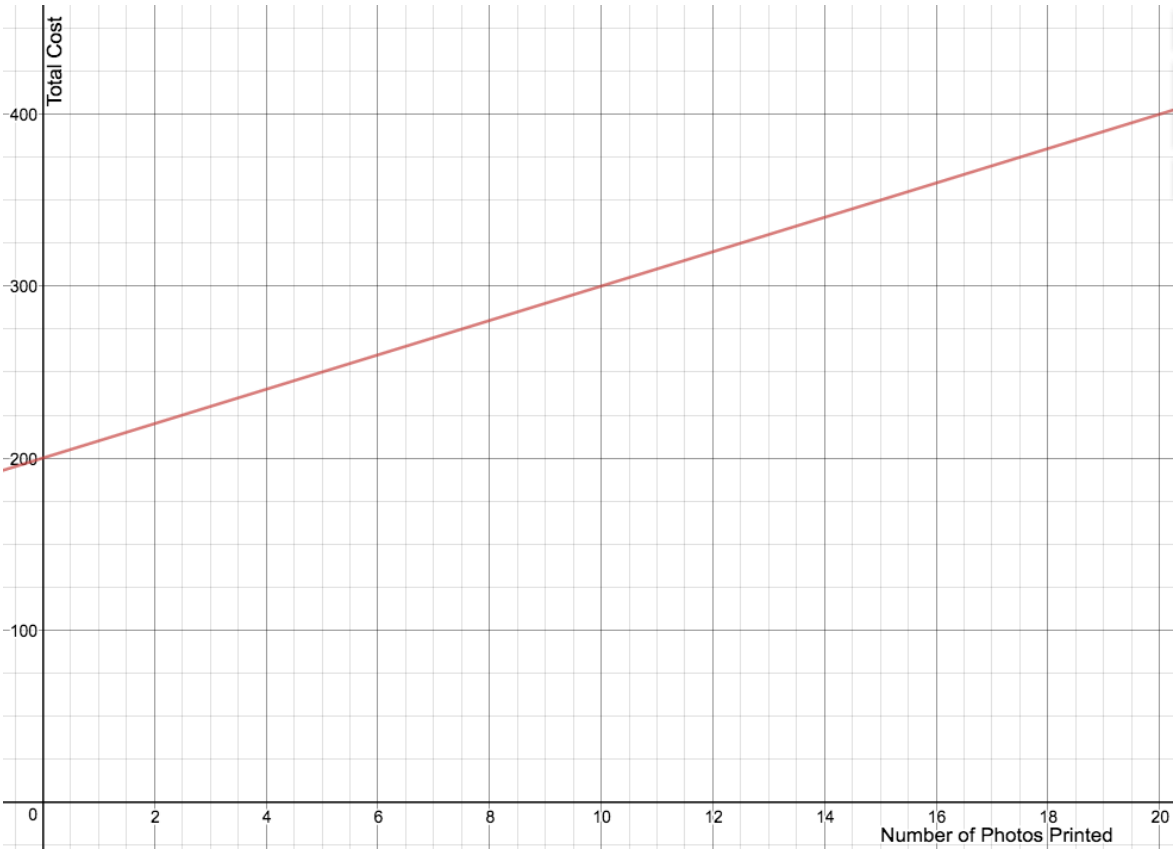
Part C: 8.F.B.4

7. Susan has a business helping people to set up their home computer systems. She advertises the following rates for a home visit, including a consultation.

Hours	***Total Cost***
0	$50
0.5	$65
1	$80
2	$110

How much does Susan charge per hour of service? Explain.

8. Anne Marie owns a photography studio. She charges a set fee for an outdoor session, and clients pay for each photo that is ordered and printed. She uses the graph below to show her rates.



In Anne Marie's graph, what is the y-intercept and what does it mean in the context of the problem? What is the slope and what does it mean in the context of the problem?

9. Vance's dad gives him \$50 to start a savings account. Vance plans to save an additional \$25 each week from his after-school job. Can the relationship between Vance's savings account balance, b, and the number of weeks, w, be modeled by a linear function? If so, create an equation to model the relationship.

Solutions:

1. 11
2. 16
3. 689
4. (sample) No, if graphed, the points would not fall on a straight line.
5. (sample) Yes, the equation shows a constant rate of change.
6. (sample) Yes, the function could be modeled by the equation, $y = 3x + 1$.
7. (sample) Susan charges $30 per hour of service as shown by finding the difference in cost between 0 and 1 hour and again between 1 and 2 hours.
8. (sample) The y-intercept is located at (0, 200), which indicates a set fee of $200. The slope is 10, which indicates a cost $10 per photo ordered and printed.
9. (Sample) The relationship is linear as it has a constant rate of change. It can be modeled by the equation, $b = 50 + 25m$.

Part A Focus: 6.EE.A.2c: Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).

Why this is important for current grade level work:
Topic A focuses on seeing sequences or patterns as functions whose domain consists of positive natural numbers. In Lesson 1 students are given a formula for the nth term and are asked to evaluate it for a given term number. To do so correctly, students must have the fundamental skill of evaluating expressions using the correct order of operations. Students then, in Lessons 2 and 3, define sequences explicitly and recursively as they write expressions for the nth term of a sequence, or a_n. In order to both create correct expressions and use them to generate subsequent terms, students must have a firm grasp on evaluating expressions for a given value, as well as the order of operations. For the rest of Topic A, students experience sequences arising from real-life contexts, continuing to evaluate various expressions for given values. The problems scaffold in difficulty.

Using the Diagnostic Assessment to identify gaps:

Problem 1:
Students should recognize that the fraction bar means to divide and that this operation should come first followed by adding. Look for students who do not recognize that evaluate means to substitute the given value in for the variable and then simplify the resulting numerical expression using the order of operations.

Problem 2:
Some students may not realize that a coefficient on a variable implies multiplication, even without a symbol. Look for students who substitute in and do not multiply (who write 36 for $3\ x\ 6$) or who make an error in the order of simplifying the expression.

Problem 3:
Students should be able to evaluate Look for students who square one or both addends prior to finding the value of the addition sentence within the parenthesis. Also, look for students who fail to add the additional 64 in the expression. Support should not focus on a pneumonic or list of tricks; rather, should focus on conceptual understanding and repeated opportunities for practice.

Acceleration Resources for Targeted Instruction:

6th Grade, Module 4, Topic F, Lesson(s) 18 – 22

Use the Classwork portion of each Lesson, depending on the operations, and a sampling of problems from the Problem Set focused on conceptual understanding and/or procedural skill and fluency.

Part B Focus: 8.F.A.3: Interpret the equation *y = mx + b* as defining a linear function, whose graph is a straight line; categorize functions as linear or nonlinear when given equations, graphs, or tables. *For example, the function A = s^2 giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*

Why this is important for current grade level work:
As students analyze sequences and come to understand them as functions, they should make connections between what they have previously understood as an arithmetic pattern and what they are studying as linear functions. That is, a pattern that has a constant difference between terms is arithmetic and, when graphed, will make a linear pattern of points. Students should begin to recognize patterns or sequences that are arithmetic in the same way they recognized sets of ordered pairs as linear. In particular, in Lesson 3 students differentiate between and use algebraic and geometric sequences, writing formulas for each. Students who do not recognize the familiar $\boldsymbol{y = mx + b}$ equation as a formula for a linear relationship, having a constant rate of change (m) with an initial value of b (when x = 0), will struggle to write formulas for arithmetic sequences which follow a similar pattern. Students who recall that the rate of change in a linear relationship is the coefficient of the input variable are naturally prepared to write formulas for arithmetic sequences and differentiate them from geometric sequences.

Using the Diagnostic Assessment to identify gaps:

Problem 4:
Look for students who think $\boldsymbol{2^x}$ is acceptable for $\boldsymbol{y = mx + b}$, not recognizing that the general equation of a line does not allow for the input to be the exponent. Push such students to create a set of input-output pairs and graph them to see if they fall on a straight line.

Problem 5:
While it is sufficient for students to simply recognize the structure of $\boldsymbol{y = mx + b}$, it is encouraged that students support their choice with further evidence, either referencing the constant rate of change or graphing.

Problem 6:
Look for students who think the selection of input-output pairs does not come from a linear function, using the fact that the difference in subsequent x and y terms is not constant (e.g., 0 to 2 then 2 to 5). This shows a lack of understanding of rate of change associated with a function.

Acceleration Resources for Targeted Instruction:

8th Grade, Module 5, Topic A, Lesson(s) 3, 5 – 6, 8

Use the Classwork portion of each Lesson and a sampling of problems from the Problem Set focused on conceptual understanding.

Part C Focus: 8.F.B.4: Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (*x, y*) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

Why this is important for current grade level work:
Students do in-depth work with linear equations in eighth grade. In the target Topic they continue this work and strengthen their understanding of sequences as functions, some of which are linear. Students will connect the common difference of an arithmetic sequence to their understanding of slope in linear equations. Likewise, the first term of a sequence, a_1, can be seen as the first term. Using the common difference, students can work backwards to find an a_0 term, which is equivalent to the y-intercept in a linear equation. Drawing from their extensive work with linear equations in 8th grade will help students in Algebra I make strong connections, as well as identify real-life situations that are linear. Students should have a strong sense of an initial value and its meaning versus a rate of change and its meaning in tables, graphs, and real-world scenarios.

Using the Diagnostic Assessment to identify gaps:

Problem 7:
Look for students who neglect to find the constant rate of change, instead just finding the difference between the first two outputs. Students should always be encouraged to look first for a constant rate of change.

Problem 8:
Look for students who struggle to interpret the y-intercept as the initial value and/or the slope as the rate of change (i.e., price per photo). Students who can identify and/or calculate values without being able to interpret them in the larger context will struggle with the new learning of the target Topic.

Problem 9:
Look for students who think the initial value is 75, implying that Vance begins saving the same week his dad gives him $50. This should be considered as an acceptable response and a sign of readiness for the new learning of the target Topic.

Acceleration Resources for Targeted Instruction:

8th Grade, Module 6, Topic A, Lesson(s) 1 – 3

Use the Classwork portion of each Lesson and a sampling of problems from the Problem Set focused on conceptual understanding.