Eureka Acceleration Tool: Algebra I
Module 4, Topic A

To become mathematically proficient, students **must** access on-grade-level content. This document aims to help teachers who use the Eureka curriculum to ensure readiness for students before and during on-grade-level work, creating opportunities for timely support directly connected to the new learning.

About this Topic

Focus Standards:

A1: A-SSE.A.1: Interpret expressions that represent a quantity in terms of its context.★

a. Interpret parts of an expression, such as terms, factors, and coefficients.

b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*

A1: A-SSE.A.2: Use the structure of an expression to identify ways to rewrite it. *For example, see x4 – y4 as (x2)2 – (y2)2, thus recognizing it as a difference of squares that can be factored as (x2 – y2)(x2 + y2), or see 2x2 + 8x as (2x)(x) + 2x(4), thus recognizing it as a polynomial whose terms are products of monomials and the polynomial can be factored as 2x(x+4).*

A1: A-SSE.B.3a: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★

a. Factor a quadratic expression to reveal the zeros of the function it defines.

A1: A-APR.A.1: Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

A1: A.CED.A.1: Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*★

A1: A.CED.A.2: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

A1: A-REI.B.4b: Solve quadratic equations in one variable.

b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b

A1: A-REI.D.11: Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, piecewise linear (to include absolute value), and exponential. ★

A1: F-IF.B.4: For linear, piecewise linear (to include absolute value), quadratic, and exponential functions that model a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.*

A1: F-IF.B.5: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.★

A1: F-IF.C.7a: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ★

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

Topic Overview per the Eureka Curriculum

Deep conceptual understanding of operations with polynomials is the focus of this topic. The emphasis is on using the properties of operations for multiplying and factoring quadratic trinomials, including the connections to numerical operations and rectangular geometry, rather than using common procedural gimmicks such as FOIL. In Topic A, students begin by using the distributive property to multiply monomials by polynomials. They relate binomial expressions to the side lengths of rectangles and find area by multiplying binomials, including those whose expanded form is the difference of squares and perfect squares. They analyze, interpret, and use the structure of polynomial expressions to factor, with the understanding that factoring is the reverse process of multiplication. There are two exploration lessons in Topic A. The first is Lesson 6, in which students explore all aspects of solving quadratic equations, including using the zero product property. The second is Lesson 8, where students explore the unique symmetric qualities of quadratic graphs. Both explorations are revisited and extended throughout this topic and the module.

In Lesson 3, students encounter quadratic expressions for which extracting the GCF is impossible (the leading coefficient, a, is not 1 and is not a common factor of the terms). They discover the importance of the product of the leading coefficient and the constant (ac) and become aware of its use when factoring expressions such as $6x^2 + 5x - 6$. In Lesson 4, students explore other factoring strategies strongly associated with the area model, such as using the area method or a table to determine the product-sum combinations. In Lesson 5, students discover the zero product property and solve for one variable by setting factored expressions equal to zero. In Lesson 6, they decontextualize word problems to create equations and inequalities that model authentic scenarios addressing area and perimeter.

Finally, students build on their prior experiences with linear and exponential functions and their graphs to include interpretation of quadratic functions and their graphs. Students explore and identify key features of quadratic functions and calculate and interpret the average rate of change from the graph of a function. Key features include x-intercepts (zeros of the function), y-intercepts, the vertex (minimum or maximum values of the function), end behavior, and intervals where the function is increasing or decreasing. It is important for students to use these features to understand how functions behave and to interpret a function in terms of its context.

A focus of this topic is to develop a deep understanding of the symmetric nature of a quadratic function. Students use factoring to reveal its zeros and then use these values and their understanding of quadratic function symmetry to determine the axis of symmetry and the coordinates of the vertex. Often, students are asked to use $x = -\frac{b}{2a}$ as an efficient way of finding the axis of symmetry or the vertex. (Note: Students learn to use this formula without understanding that this is a generalization for the average of the domain values for the x-intercepts.) Only after students develop an understanding of symmetry is $x = -\frac{b}{2a}$ explored as a general means of finding the axis of symmetry.

Throughout this topic, students use the notation of functions without naming it as such—they come to understand $f(n)$ as a "formula for the nth term of a sequence," expanding to use other letters such as $A(n)$ for Akelia's sequence and $B(n)$ for Ben's sequence. Their use of this same notation for functions is developed in Topic B.

This Eureka Acceleration Tool is considered a "living" document as we believe that teachers and other educators will find ways to improve the document as they use it. Please send feedback to STEM@la.gov so that we can use your input when updating this guide.

DEPARTMENT of EDUCATION
Louisiana Believes

Eureka Acceleration Tool: Algebra I
Module 4, Topic A

Overview

Eureka Acceleration Tools include:

1. a diagnostic assessment to help teachers determine the misunderstandings or gaps in mathematical knowledge related to a specific Topic in the Eureka curriculum
2. guidance for teachers to analyze student work on the diagnostic assessment
3. suggested materials for targeted remedial instruction

Note: The use of this guidance is not intended to delay students' engagement with on-grade-level learning. On-grade-level learning should be the focus of instructional time and be treated as an opportunity for students to "finish" learning previous skills and deepen conceptual understanding.

Diagnostic Assessment

The diagnostic assessment is designed to be administered to targeted students prior to beginning instruction on the given Topic. When appropriate, it is broken into parts (Part A, Part B, and so on); each part addresses a different prerequisite standard and contains three problems. If a student correctly answers at least 2 out of the 3 problems, it can be assumed that he/she is ready to engage with the new content of the Topic with little to no support needed prior to engaging with the Topic. The diagnostic assessment is designed in this way so that teachers can determine the "entry point" to remedial instruction and/or opportunities for unfinished learning within the context of the new learning. The entry points and opportunities for unfinished learning will vary between students.

Guidance for Acceleration

The Acceleration Guidance is designed for teacher use. It is also broken into parts (Part A, Part B, and so on) and correlates to the parts on the diagnostic assessment. Each part contains the following:

1. **The focus standard**: The focus standards are strategically chosen to address prerequisite skills and are purposefully arranged in the order that students typically master the skills and knowledge.
2. **Why this is important for current grade level work**: This section describes how the work of the prerequisite standard relates to the standard(s) addressed in the Topic of instruction.
3. **Using the diagnostic assessment to identify gaps**: This section identifies common errors students make on the diagnostic assessment items.
4. **Acceleration Resources for Targeted Instruction**: The resources pinpoint specific Eureka lessons and parts of lessons for teachers to use to address gaps in mathematical knowledge. Using Eureka materials to address acceleration ensures alignment to the standards, consistency in approach to learning, and similarities in strategies for solving problems.

Part A: 6.EE.A.3

1. Apply the distributive property to create an expression that is equivalent to:
   $5(y - 3 + 2z)$

2. Apply the distributive property to create an expression that is equivalent to:
   $\frac{2}{3}(9b - 15c)$

3. Apply the distributive property to create an expression that is equivalent to:
   $(5f - 4g + 6) \cdot 3$

Part B: 7.EE.A.1

4. Apply the distributive property to factor out the greatest common factor, resulting in an expression equivalent to:
   $35n - 77$

5. Apply the distributive property to factor out the greatest common factor, resulting in an expression equivalent to:
   $36m + 42t$

6. Apply the distributive property to factor out the greatest common factor, resulting in an expression equivalent to:
   $12 - 24r + 24d$

Part C: 8.EE.A.2

Solve each equation, using a root symbol to represent your solution. If your root solution equals an integer, evaluate it.

7. $x^2 = 16$

8. $x^2 = 225$

9. $x^2 = 24$

Solutions:

1. (sample) $5y - 15 + 10z$
2. (sample) $6b - 10c$
3. (sample) $15f - 12g + 18$
4. (sample) $7(5n - 11)$
5. (sample) $6(6m + 7t)$
6. (sample) $12(1 - 2r + 2d)$
7. $x = \pm\sqrt{16} = \pm 4$
8. $x = \pm\sqrt{225} = \pm 15$
9. $x = \pm\sqrt{24}$

Part A Focus: 6.EE.A.3: Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3(2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*

Why this is important for current grade level work:
The target Topic begins with a quick study of multiplying and factoring quadratic expression, skills that hinge on the understanding of and application of the distributive property. Students began working with the distributive property, though not by name, as early as Grade 3, but this foundational standard was their first exposure to using the distributive property on algebraic expressions. The item set here focuses solely on applying the distributive property to create an equivalent expression. Notice that the solutions are sample solutions as students may choose to not perform the actual multiplication but still show an accurate use of the distributive property. For example, for problem 1 a student may leave his/her answer as $\mathbf{5}(\boldsymbol{y}) - \mathbf{5}(\mathbf{3}) + \mathbf{5}(\mathbf{2}\boldsymbol{z})$. This should be considered acceptable as a sign of readiness for the target Topic. The most important look-fors here are that students understand distributing involves multiplication and that they distribute to every term.

Using the Diagnostic Assessment to identify gaps:

Problem 1:
Look for students who do not distribute to all three terms in the trinomial and/or do use multiplication when distributing as both mistakes show a gap in understanding of the distributive property.

Problem 2:
While it is more important to see if students correctly apply the distributive property to create an equivalent expression, it is also important to look for students who incorrectly multiply by $\frac{2}{3}$. Both coefficients are multiples of 3, so the multiplication should be relatively simple.

Problem 3:
Look for students who struggle to see how to use the distributive property since the expression is written "backwards." Encourage such students to think about the commutative property of multiplication to possibly reverse the order of the problem.

Acceleration Resources for Targeted Instruction:

6th Grade, Module 4, Topic D, Lesson(s) 10 – 12

Use the Classwork portion of each Lesson and a sampling of problems from the Problem Set focused on conceptual understanding and/or procedural skill and fluency.

Part B Focus: 7.EE.A.1: Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients to include multiple grouping symbols (e.g., parentheses, brackets, and braces).

Why this is important for current grade level work:
While Part A focuses on applying the distributive property to create equivalent expressions by multiplying, Part B focuses on applying the distributive property in reverse (i.e., factoring). While factoring out a common factor, in most cases the greatest common factor, is the beginning of students' work with factoring, Lessons 1 – 4 will extend this work to include new and advanced factoring techniques. Students ability to engage with the new factoring techniques will be directly impacted by their understanding, or lack thereof, of basic factoring of a common factor. The item set here will help determine which students have the basic understanding of factoring needed to be successful in the target Topic.

Using the Diagnostic Assessment to identify gaps:

Problem 4:
Students may choose to write their answer in one of two ways: the provided sample solution as well as the reverse of the provided sample solution, showing an application of the commutative property of multiplication. Both should be accepted as a sign of readiness for the target Topic.

Problem 5:
Look for students who think the two terms have no common factor since they are not "like terms." This mistake shows a gap in understanding of the differences between adding/subtracting expressions and expanding/factoring expressions.

Problem 6:
Look for students whose answer is a constant times a binomial, instead of a trinomial. Some students may think by factoring out the 12 that nothing is left in its place. Encourage such students to expand their answer to compare it to the original expression. If done correctly the expressions should be equivalent, helping them see their mistake of completely removing the 12 from the original expression.

Acceleration Resources for Targeted Instruction:

7th Grade, Module 3, Topic A, Lesson(s) 1 – 4

Use the Classwork portion of each Lesson and a sampling of problems from the Problem Set focused on conceptual understanding and/or procedural skill and fluency.

Part C Focus: 8.EE.A.2: Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that √2 is irrational.

Why this is important for current grade level work:
The target Topic extends students work with simple equations in the form $\boldsymbol{x^2 = p}$ to solving more complex quadratic equations in one variable. It is critical that students understand that taking the square root of both sides of the equation $\boldsymbol{x^2 = p}$ is the solution method by which you solve for $\boldsymbol{x}$. Some of the equations in the target Topic will either begin or end in this form; however, the focus of the target Topic is to extend this work to include new solution methods, leveraging the zero product property. Students will come to understand when it is acceptable and appropriate to take the square root of both sides and when another method must be used. The items here will help determine which students are ready to engage with Lesson 5-7 and beyond.

Using the Diagnostic Assessment to identify gaps:

Problems 7 – 8:
Look for students who divide both sides by 2 to find the solution as this shows a misunderstanding of the solution method for an equation of the form $\boldsymbol{x^2 = p}$, or, more generally, a misunderstanding of exponents. Both are problematic and will hinder students from engaging with the new learning of the target Topic. Also, look for students who exclude the negative answer, thinking the answer must be positive. Again, this shows a misunderstanding of exponents, specifically that, in all cases but 0 two numbers share the same square.

Problem 9:
Look for students who try to evaluate the square root of 24, not recognizing it is irrational. Students solve the equation correctly should be considered ready for the new learning of the target Topic, even if they misevaluate the radical.

Acceleration Resources for Targeted Instruction:

8th Grade, Module 7, Topic A, Lesson(s) 2 – 3, 5

Use the Classwork portion of each Lesson and a sampling of problems from the Problem Set focused on conceptual understanding and/or procedural skill and fluency.