Eureka Acceleration Tool: Algebra I
Module 1, Topic C

To become mathematically proficient, students **must** access on-grade-level content. This document aims to help teachers who use the Eureka curriculum to ensure readiness for students before and during on-grade-level work, creating opportunities for timely support directly connected to the new learning.

About this Topic

Focus Standards:

A1: A-CED.A.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

A1: A-REI.C.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

A1: A-REI.C.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

A1: A-REI.D.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

A1: A-REI.D.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Topic Overview per the Eureka Curriculum

Teaching the process of how to solve an equation is fraught with well-meaning models and procedures suggested by textbook curricula (balance scales, algebra tiles, equivalent equations, etc.) that are often incompatible with what it actually means “to solve.” An equation with variables can be viewed as a question asking for which values of the variables (the solution set) will result in true number sentences when those values are substituted into the equation. Equations are manifestly about numbers and understanding true and false number sentences. In Algebra I, the application of this idea expands to include solutions to compound statements such as equations or inequalities joined by “and” or “or,” including simultaneous systems of equations or inequalities.

The Common Core Learning Standards rightfully downplay the notion of equivalent equations and instead place a heavy emphasis on students studying the solution sets to equations. In Lessons 12–14 of this topic, students formalize descriptions of what they learned before (true/false equations, solution sets, identities, properties of equality, etc.) and learn how to explain the steps of solving equations to construct viable arguments to justify their solution methods. They then learn methods for solving inequalities, again by focusing on ways to preserve the (now infinite) solution sets. With these methods now on firm footing, students investigate in Lessons 15–18 solution sets of equations joined by “and” or “or” and investigate ways to change an equation such as squaring both sides, which changes the solution set in a controlled (and often useful) way. In Lesson 19, students learn to use these same skills as they rearrange formulas to define one quantity in terms of another. Finally, in Lessons 20–24, students apply all of these new skills and understandings as they work through solving equations and inequalities with two variables including systems of such equations and inequalities.

Eureka Acceleration Tool: Algebra I
Module 1, Topic C

Overview

Eureka Acceleration Tools include:

1. a diagnostic assessment to help teachers determine the misunderstandings or gaps in mathematical knowledge related to a specific Topic in the Eureka curriculum
2. guidance for teachers to analyze student work on the diagnostic assessment
3. suggested materials for targeted remedial instruction

Note: The use of this guidance is not intended to delay students' engagement with on-grade-level learning. On-grade-level learning should be the focus of instructional time and be treated as an opportunity for students to "finish" learning previous skills and deepen conceptual understanding.

Diagnostic Assessment

The diagnostic assessment is designed to be administered to targeted students prior to beginning instruction on the given Topic. When appropriate, it is broken into parts (Part A, Part B, and so on); each part addresses a different prerequisite standard and contains three problems. If a student correctly answers at least 2 out of the 3 problems, it can be assumed that he/she is ready to engage with the new content of the Topic with little to no support needed prior to engaging with the Topic. The diagnostic assessment is designed in this way so that teachers can determine the "entry point" to remedial instruction and/or opportunities for unfinished learning within the context of the new learning. The entry points and opportunities for unfinished learning will vary between students.

Guidance for Acceleration

The Acceleration Guidance is designed for teacher use. It is also broken into parts (Part A, Part B, and so on) and correlates to the parts on the diagnostic assessment. Each part contains the following:

1. **The focus standard**: The focus standards are strategically chosen to address prerequisite skills and are purposefully arranged in the order that students typically master the skills and knowledge.
2. **Why this is important for current grade level work**: This section describes how the work of the prerequisite standard relates to the standard(s) addressed in the Topic of instruction.
3. **Using the diagnostic assessment to identify gaps**: This section identifies common errors students make on the diagnostic assessment items.
4. **Acceleration Resources for Targeted Instruction**: The resources pinpoint specific Eureka lessons and parts of lessons for teachers to use to address gaps in mathematical knowledge. Using Eureka materials to address acceleration ensures alignment to the standards, consistency in approach to learning, and similarities in strategies for solving problems.

Part A: 6.EE.B.7

For each problem below, write and solve an equation to answer the question.

1. 21.9 less than y is 35.2. What is the value of y?

2. 5 is the same as n divided by 7. What is the value of n?

3. Two-thirds the value of p is equal to $\frac{4}{5}$. What is the value of p?

Part B: 7.EE.B.4a

Solve each equation. Be sure to show your steps.

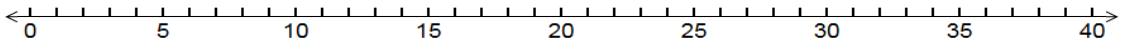
4. $6n - 5 = 37$

5. $2\left(m - \frac{3}{2}\right) = 7$

6. $\frac{1}{3}(x + 6) = 12$

Part C: 7.EE.B.4b

7. Donovan currently weighs 180 pounds, but he wants to weigh at least 200 pounds. He commits to gaining 2 pounds per week. Write and solve an inequality to determine the number of weeks Donovan needs to reach his goal. Graph your solution set on the number line.



8. As a salesperson you are paid \$50 per week plus \$30 per sale. This week you want your pay to be at least \$100. Write and solve an inequality to determine the number of sales needed to reach your goal.

9. Judah makes and sells beignets by the dozen. He made 240 beignets to sell one morning, and he wants to have no more than 20 beignets left over by the end of the morning. Write and solve an inequality to determine the number of batches Judah needs to sell.

Part D: 8.EE.C.7a

For each equation below, determine if the equation has one solution, no solution, or infinitely many solutions and provide evidence to support your choice.

10. $-4(c+1) = -6c + 10$

11. $5z + 9 + 2z = 9z + 8 - 2z$

12. $2(3x - 5) = 6x - 10$

Part E: 8.EE.C.7b

Solve each equation and show the steps you took to find each solution.

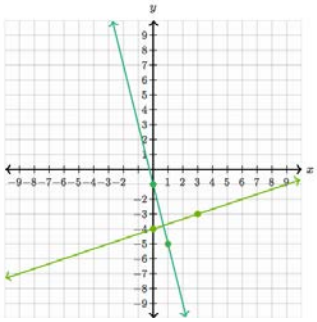
13. $5 - 2(8 + 3n) = 1$

14. $\frac{2}{3}y + 5 = -3 - 2y$

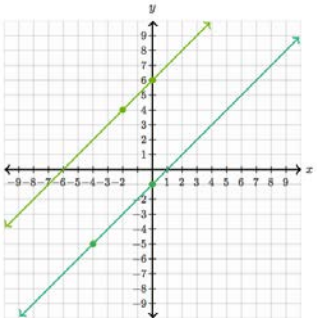
15. $2b + 4(2b - 4) - 4 = 15b$

Part F: 8.EE.C.8a

16. How many solutions will the system graphed below have? Explain your answer.



17. How many solutions will the system graphed below have? Explain your answer.



18. Is $(-2,7)$ a solution to the system? Explain your answer.
$y = -2x + 3$

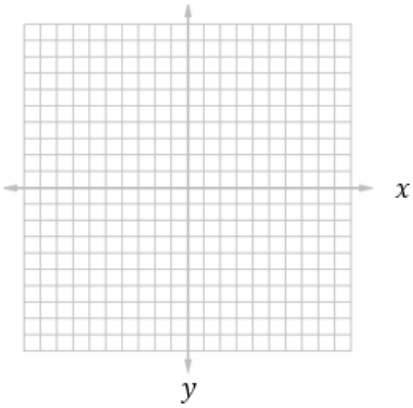
$3x - y = -13$

Part E: 8.EE.C.8b

19. Approximate the solution to the following equation by graphing the system on the provided coordinate grid.

$y = -\frac{1}{2}x + 5$

$y = \frac{3}{2}x - 1$



20. Solve the system of equations by inspection and explain your choice.

$-2x + 15y = -24$

$-4x + 30y = -48$

21. Solve the system of equations algebraically. Show your work.

$6x - 5y = 15$

$y = x + 3$

Part E: 8.EE.C.8c

For each problem create a system of equations and use the system to solve the problem.

22. a and b are both integers. The sum of a and b is -2. If you take b from a, the value is 12. What are the values of a and b?

23. Kristy works as a freelance writer and an editor. She earns $20 per hour as a writer and $25 per hour as an editor. One week, Kristy worked a total of 30 hours and earned a total of $690 between both jobs. How long did Kristy work as a writer last week and how long did she work as an editor?

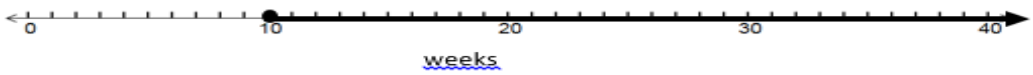
24. Paul and Charles are making a number puzzle about their ages. Together, their ages total 21 years. Four times Paul's age is 24 more than Charles's age. What are their ages?

25.

Solutions:

1. $y - 21.9 = 35.2; y = 57.1$
2. $5 = \frac{n}{7}; n = 35$
3. $\frac{2}{3}p = \frac{4}{5}; p = \frac{6}{5}$
4. $n = 8$
5. $m = 5$
6. $x = 30$
7. (Sample) Let w represent the number of weeks.

   $180 + 2w \geq 200$; It will take Donovan a minimum of 10 weeks to reach his goal.



8. (Sample) Let s represent the number of sales.

   $50 + 30s \geq 100$; I must make at least 2 sales to reach my goal.
9. (Sample) Let b represent the number of batches of beignets.

   $240 - 12b \leq 20$; Judah must sell at least 19 batches of beignets.
10. (Sample) The equation has a single solution, because I can use properties of equality to transform the equation to $c = 7$.
11. (Sample) The equation has no solution. I transformed the equation to be $7z + 9 = 7z + 8$. This cannot be true as there is no number that, when adding 8 and 9, equals the same amount. Therefore, this is a false equation.
12. (Sample) The equation has infinitely many solutions. I transformed the equation to be $6x - 10 = 6x - 10$. Since the expressions on both sides of the equals sign are equivalent, the equation defined by those expressions has infinitely many solutions.
13. $n = -2$
14. $y = -3$
15. $b = -4$
16. (Sample) The system has one solution, because the graphs of each equation intersect at a single point.
17. (Sample) The system has no solutions, because the graphs of the equations are parallel and will never intersect.
18. (Sample) Yes, the point satisfies both equations making it a solution to the system of equations.
19. (Sample) $\left(4\frac{3}{4}, 2\frac{1}{2}\right)$
20. (Sample) The solution has infinitely many solutions, because the second equation is a multiple of the first.

21. $(30, 33)$

22. $a + b = -2$ and $a - b = 12$; $a = 5$, and $b = -7$.

23. Let w = the number of hours worked as a writer and e be the number of hours worked as an editor.

    $w + e = 30$ and $20w + 25e = 690$; Kristy worked 12 hours as a writer and 18 hours as an editor.

24. Let p be Paul's age and c be Charles' age.

    $p + e = 21$ and $4p = c + 24$; Paul is 9, and Charles is 12.

Part A Focus: 6.EE.B.7: Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.

Why this is important for current grade level work:
As the target Topic focuses on solving single variable equations, leading to systems of equations in two variables, students need to have a strong understanding of the language used and the understanding of what it means to solve an equation. In sixth grade, students learn to solve one-variable equations with addition or subtraction and multiplication and division, laying the groundwork for an understanding of using inverse operations to isolate the variable. Students should understand that they are using properties of equality to rewrite the equation, forming an equivalent, simpler equation that reveals the solution. Also included in these problems is arithmetic with positive rational numbers in various forms. While the focus is on creating and solving the equations, the problems will give the teacher a sense of whether or not the students are fluent in arithmetic with positive rational numbers.

Using the Diagnostic Assessment to identify gaps:

Problem 1:
Look for students who confuse "less than" to mean the problem is represented by an inequality instead of an equation. Such students will need additional supports to understand the language used in equations and inequalities.

Problem 2:
Look for students who can interpret the problem and know the answer must be 35 without being able to create the appropriate equation. While this shows a high level of understanding that will prove beneficial for the students long term, students must be able to model a written sentence with a mathematical sentence.

Problem 3:
Look for students create the correct equation yet struggle to solve the equation, not knowing how to appropriately handle the fractions. While accurately creating the equation is important, students should recognize that the same solution method works for fractions as it does for whole numbers.

Acceleration Resources for Targeted Instruction:

6th Grade, Module 4, Topic G, Lesson(s) 26 – 27

Use the Classwork portion of each Lesson and a sampling of problems from the Problem Set that focus on conceptual understanding.

Part B Focus: 7.EE.B.4a: Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*

Why this is important for current grade level work:
Extending from understanding and solving one-step equations in Grade 6, this standard pushes students to solve two-step equations. Since most of the work with one-variable equations in the target Topic focus on procedural skill and fluency, these problems are limited to fluency. Similar to Part A, these problems do include rational numbers beyond whole numbers, allowing the teachers to see how well students can compute with fractions. The most important look-fors here are the accuracy of the answers and the solution methods. While some students might be able to solve a two-step equation without showing all of their steps, it is important for the purpose of diagnosing readiness to see all of the solution method for each problem.

Using the Diagnostic Assessment to identify gaps:

Problem 4:
Look for students who fail to use inverse operations to isolate the variable, most likely subtracting 5 from 37 instead of adding 5 to 37. Such students lack the understanding needed to become fluent in solving equations and will struggle to engage with the complex equations of the target Topic.

Problem 5:
Look for students who distribute first as this shows a lack of understanding of how the equation was built, resulting in a lack of understanding of the most appropriate solution method for such a problem. Such students would benefit from studying the way equations are built prior to engaging with the target Topic.

Problem 6:
Similar to problem 5, look for students who distribute first, or, look for students who take one third of 12 in their solution method. Challenging the student to check his/her answer should help prove the solution method was inaccurate. Moreover, being able the accurately check solutions is a skill that will prove beneficial for students long term.

Acceleration Resources for Targeted Instruction:

7th Grade, Module 3, Topic B, Lesson(s) 7 – 9

Use the Classwork portion of the Lesson and a sampling of problems from the Problem Set that focus on conceptual understanding and procedural skill and fluency.

Part C Focus: 7.EE.B.4b: Solve word problems leading to inequalities of the form px + q > r or px + q < r, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*

Why this is important for current grade level work:
While most of the target Topic focuses on developing skill, the Topic ends by extending systems of equations and inequalities to solving word problems. These problems will help identify which students are comfortable with the language of inequalities. Furthermore, problem 7 includes graphing the solution set on a number line. It is especially important to look for the accuracy of the graph as this understanding will extend to linear inequalities in two-variables, beginning in Lesson 21. Problems 8 and 9 will force students to reason about their algebraic solution in terms of the context of the problem, which will help determine which students are ready for Lesson 24. The problems scaffold in difficulty.

Using the Diagnostic Assessment to identify gaps:

Problem 7:
Look for students who use an incorrect inequality symbol when modeling the problem as this shows a lack of understanding of the language used with inequalities. Also, look for students whose graph does not include the value 10 as this shows a misunderstanding of modeling inequalities with a graph. Both misunderstandings will impact students' ability to engage with the grade-level content.

Problem 8:
Look for students who are unable to use their algebraic solution to answer the problem, thinking a sale of 1 and 2/3 is a possible answer in the context of the problem. While having the algebraic skills needed to solve a linear inequality will prove advantageous for students, they also need to be able to interpret solutions in the context of word problems.

Problem 9:
Look for students struggle to accurately model the situation with an inequality, not seeing the decreasing nature of the situation. Also, look for students who do not reverse the inequality symbol when dividing both sides by a negative integer, leading them to the answer is 18 or fewer. Challenge such students to think about their answer in the context of the problem to determine if it is reasonable.

Acceleration Resources for Targeted Instruction:

7th Grade, Module 3, Topic B, Lesson(s) 12 – 15

Use the Classwork portion of each Lesson and a sampling of problems from the Problem Set focused on conceptual understanding.

Part D Focus: 8.EE.C.7a: Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form x = a, a = a, or a = b results (where a and b are different numbers).

Why this is important for current grade level work:
Work with equations in Algebra I focuses on preserving and justifying the solution set through a series of moves based on the properties of equality. In 8th grade, students begin to work with equations that have no solution or infinitely many solutions and learn that these possibilities can be anticipated once an equation is transformed into simpler forms. Students must understand that the process of solving is really a process of revealing the solution set, starting with the assumption that the initial equation is true. Furthermore, the foundation for solving a system of equation in two-variables is grounded in the ability to identify an equation in one variable with no solution or infinitely many solutions without having to transform the equation to its simplest form.

Using the Diagnostic Assessment to identify gaps:

Problem 10:
Look for students who transform the equation to its simplest form before realizing it has a single solution. Push such students to analyze the equation at earlier points to determine if they can recognize when a single solution should be evident.

Problem 11:
Look for students who do not simplify each side of the equation first, making it more difficult to determine the number of solutions. Also, look for students who, in thinking the variable must be present, think the equation has a single solution (e.g., either 1 or -1) as this shows a lack of understanding around the possible solutions to a single-variable equation. This might be viewed as an opportunity for unfinished learning, cleared up for students when they study a system of equations that has no solutions.

Problem 12:
Look for students who, either by distributing or dividing both sides by 2, fail to recognize the equivalent expressions on each side of the equation and continue to transform the equation to a simpler form. While this is not incorrect, we want students to think at each step to see if they have enough information to determine the number of solutions.

Acceleration Resources for Targeted Instruction:

8th Grade, Module 4, Topic A, Lesson(s) 6 – 7

Use the Classwork portion of each Lesson and a sampling of problems from the Problem Set focused on conceptual understanding.

Part E Focus: 8.EE.C.7b: Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

Why this is important for current grade level work:
The single-variable equations included in the target Topic are very complex and, in most cases, extend beyond the expectations of Grade 8; thus, it is important to determine how well students can engage with the Grade 8 equations prior to engaging with the new grade-level content. Students need to have mastered solving multi-step equations in middle school to engage in the work of justifying solution sets in Algebra I. Students should know how to distribute, combine like terms, and move variable terms to one side by the end of 8th Grade. Fluency with these skills is also a prerequisite for manipulating literal equations, an important topic in Algebra I (A1: A-CED.A.4), as well as for science. Students who have previously been able to guess and check to solve equations must now master the steps used in rewriting equations, particularly those with variables on both sides. The most important look-fors here are the accuracy of the answers and the solution methods. The problems scaffold in difficulty.

Using the Diagnostic Assessment to identify gaps:

Problem 13:
Look for students who subtract the 2 from 5 as the first step in their solution method as this shows a gap in understanding of how the equation was built, leading to a misunderstanding of how to solve the equation. Push such students to identify how the equation was built prior to designing their solution method.

Problem 14:
Look for students who either combine variable and non-variable terms (e.g., creating $-5y$ on the right side of the equation) or who combine like terms from opposite sides of the equals sign without using inverse operations to transform the equation (i.e., add $\frac{2}{3}y$ and $-2y$). Both show a gap in understanding that will greatly impact students' ability to engage with the work of the target Topic.

Problem 15:
Look for students who distribute incorrectly or who combine like terms on the left side of the equation without considering the structure of the expression defining the left side of the equation (e.g., combining the variable terms without first distributing the 4). Such students will need support in generating equivalent expressions in order to solve complex equations.

Acceleration Resources for Targeted Instruction:

8th Grade, Module 4, Topic A, Lesson(s) 4 and 6

Use the Classwork portion of each Lesson and a sampling of problems from the Problem Set focused on procedural skill and fluency.

Part F Focus: 8.EE.C.8a: Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

Why this is important for current grade level work:
In Lesson 22 the Topic shifts its focus from single-variable equations and inequalities to systems of equations and inequalities in two variables. Students were introduced to systems of equations in Grade 8, and the curriculum assumes students not only mastered the Grade 8 work around systems of equations but also retained that knowledge. As such the target Topic does not allow for much time focused on developing the understanding of systems before it moves towards procedural skill and application of systems. Therefore, students need to have a strong understanding of systems prior to engaging with Lesson 22. This particular section will determine which students understand that the solution to a system of equations is the point of intersection, which has a pair of values that satisfy both equations. The only scenario not captured in these three items is a system with infinitely many solutions.

Using the Diagnostic Assessment to identify gaps:

Problem 16:
The most important look-for here is students connecting the single point of intersection to a single solution. It is not necessary in this problem for students to name the point of intersection to be considered ready for the target Topic.

Problem 17:
Look for students who do not recognize that parallel lines model a system with no solution. This understanding is essential prior to engaging with the new grade-level work of the target Topic.

Problem 18:
Look for students who do not test the point in both equations, thinking that satisfying one equation is enough evidence for the system. A student who checks the ordered pair by graphing the system should be considered ready for the target Topic.

Acceleration Resources for Targeted Instruction:

8th Grade, Module 4, Topic D, Lesson(s) 24 – 27

Use the Classwork portion of each Lesson and a sampling of problems from the Problem Set focused on conceptual understanding.

Part G Focus: 8.EE.C.8b: Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection.

Why this is important for current grade level work:
The target standards concerning systems of equations primarily focus on solving systems algebraically, building from the fundamental understanding of what it means to be a solution to a system of two linear equations. In Grade 8 students learned both to approximate solutions by graphing and to solve algebraically (primarily by means of substitution), distinguishing algebraic methods as producing exact solutions. The three problems here will assess both approximating and solving algebraically, and, whereas a system with infinitely many solutions was not included in Part F, one is included here. These problems should help determine if students have the fundamental skills to solve a system, allowing for better engagement with Lessons 22 and 23.

Using the Diagnostic Assessment to identify gaps:

Problem 19:
Look for students who can recognize that the system will have one solution simply by analyzing the slope of each line from the given equations. This understanding will prove beneficial for students long term, but this understanding is not enough. Students must be able to accurately graph a system of linear equations prior to graphing a system of linear inequalities, which is part of the new learning in the target Topic.

Problem 20:
Look for students who rely on procedural skill to solve this system, not recognizing the relationship between the two equations. This shows a lack of understanding that will hinder students from progressing through the target Topic. Furthermore, solving by linear combination (i.e., elimination) is not an explicit expectation of Grade 8, so do not expect many, if any, students to solve by this method.

Problem 21:
Look for students who do not recognize the second equation gives you an expression to be substituted for y in the first equation. If students were taught a single method for solving by which they engage with every problem, this will greatly limit their ability to master the new grade-level content. Students should be flexible in their methods for solving and should always first use their understanding to analyze the system prior to attempting to solve the system algebraically.

Acceleration Resources for Targeted Instruction:

8th Grade, Module 4, Topic D, Lesson(s) 27 – 28

Use the Classwork portion of each Lesson and a sampling of problems from the Problem Set focused on conceptual understanding and/or procedural skill and fluency.

Part H Focus: 8.EE.C.8c: Solve real-world and mathematical problems leading to two linear equations in two variables.

Why this is important for current grade level work:
Lesson 24 focuses entirely on using systems of equations (and inequalities) to solve real-world problems. As there is some overlap between the Grade 8 expectations around applications of systems and the Algebra I expectations, this standard should not be considered a true prerequisite for the grade-level work. Rather, these problems should be used to identify the students who will need additional supports to engage with Lesson 24. The most important look-fors here are the ability to accurately define the two unknowns and the ability to accurately model the situation with a system of two linear equations. If students can do these two things, even without being able to accurately finish the problem and answer the question correctly, they should be considered ready for the grade-level work. The problems scaffold in difficulty.

Using the Diagnostic Assessment to identify gaps:

Problem 22:
Look for students who misinterpret the statement "If you take a $\boldsymbol{b}$ from $\boldsymbol{a}$" to mean $\boldsymbol{b} - \boldsymbol{a}$ as this shows a lack of language development with academic vocabulary. While such students may be proficient in their ability to solve a system, the language gap will prove problematic when engaging with the target Topic.

Problem 23:
Look for students who struggle to solve the system, since the most appropriate model of the situation is not set up for direct substitution. If a student accurately models the situation but struggles to solve the system, he/she can be considered ready for the target Topic but will likely need additional supports while engaging with the grade-level content.

Problem 24:
Look for students who make an algebraic or arithmetic mistake when solving the system, thus creating an incorrect solution. Challenge such students to check their answers in the original context of the problem to see if their answer is reasonable.

Acceleration Resources for Targeted Instruction:

8th Grade, Module 4, Topic D, Lesson 29

Use the Classwork portion of the Lesson and a sampling of problems from the Problem Set focused on application.