Eureka Acceleration Tool: Grade 6
Module 4, Topic D

To become mathematically proficient, students **must** access on-grade-level content. This document aims to help teachers who use the Eureka curriculum to ensure readiness for students before and during on-grade-level work, creating opportunities for timely support directly connected to the new learning.

About this Topic

Focus Standards:

6.EE.A.2: Write, read, and evaluate expressions in which letters stand for numbers.

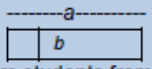
a. Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 – y.*

b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.*

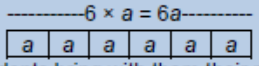
6.EE.A.3: Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3(2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.* 6.EE.A.4: Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*

Topic Overview per the Eureka Curriculum

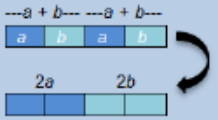
In Topic D, students formally utilize their understanding of expressions in order to expand, factor, and distribute. In Lesson 9, students write expressions that record addition and subtraction operations with numbers through the use of models. With a bar diagram, students understand that any number aa plus any number bb is the same as adding the numbers $bb + aa$. Students also use bar diagrams to differentiate between the mathematical terms subtract and subtract from. For instance, when subtracting bb from aa, they know they must first represent aa in order to take away bb, leading to an understanding that the expression must be written $a - b$.



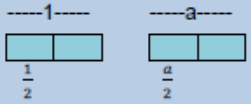
This concept deters students from writing the incorrect expression $bb - aa$, which is a common misconception because the number bb is heard first in the expression "subtract bb from aa." Students continue to write expressions by combining operations with the use of parentheses. In Lesson 10, students identify parts of an expression using mathematical terms for multiplication. They view one or more parts of an expression as a single entity. They determine that through the use of models, when aa is represented 6 times, the expression is written as one entity: $6 \times a$, $6 \cdot a$, or $6a$.



In Lesson 11, students bring with them their previous knowledge of GCF and the distributive property from Module 2 to model and write expressions using the distributive property. They move from a factored form to an expanded form of an expression, while in Lesson 12, they move from an expanded form to a factored form. In Lesson 11, students are capable of moving from the expression $2a + 2b$ to $a + b$ written twice as $(a + b) + (a + b)$ and conclude that $2a + 2b = 2(a + b)$. Conversely, students determine in Lesson 12 that $(a + b) + (a + b) = 2a + 2b$ through the following model:



Finally, in Lessons 13 and 14, students write division expressions in two forms: dividend ÷ divisor and dividend divisor , noting the relationship between the two. They determine from the model below that $1 \div 2$ is the same as $\frac{1}{2}$. They make an intuitive connection to expressions with letters and also determine that $a \div 2$ is the same as $\frac{a}{2}$.



This Eureka Acceleration Tool is considered a "living" document as we believe that teachers and other educators will find ways to improve the document as they use it. Please send feedback to STEM@la.gov so that we can use your input when updating this guide.

DEPARTMENT of EDUCATION
Louisiana Believes

Eureka Acceleration Tool: Grade 6
Module 4, Topic D

Overview

Eureka Acceleration Tools include:

1. a diagnostic assessment to help teachers determine the misunderstandings or gaps in mathematical knowledge related to a specific Topic in the Eureka curriculum
2. guidance for teachers to analyze student work on the diagnostic assessment
3. suggested materials for targeted remedial instruction

Note: The use of this guidance is not intended to delay students' engagement with on-grade-level learning. On-grade-level learning should be the focus of instructional time and be treated as an opportunity for students to "finish" learning previous skills and deepen conceptual understanding.

Diagnostic Assessment

The diagnostic assessment is designed to be administered to targeted students prior to beginning instruction on the given Topic. When appropriate, it is broken into parts (Part A, Part B, and so on); each part addresses a different prerequisite standard and contains three problems. If a student correctly answers at least 2 out of the 3 problems, it can be assumed that he/she is ready to engage with the new content of the Topic with little to no support needed prior to engaging with the Topic. The diagnostic assessment is designed in this way so that teachers can determine the "entry point" to remedial instruction and/or opportunities for unfinished learning within the context of the new learning. The entry points and opportunities for unfinished learning will vary between students.

Guidance for Acceleration

The Acceleration Guidance is designed for teacher use. It is also broken into parts (Part A, Part B, and so on) and correlates to the parts on the diagnostic assessment. Each part contains the following:

1. **The focus standard**: The focus standards are strategically chosen to address prerequisite skills and are purposefully arranged in the order that students typically master the skills and knowledge.
2. **Why this is important for current grade level work**: This section describes how the work of the prerequisite standard relates to the standard(s) addressed in the Topic of instruction.
3. **Using the diagnostic assessment to identify gaps**: This section identifies common errors students make on the diagnostic assessment items.
4. **Acceleration Resources for Targeted Instruction**: The resources pinpoint specific Eureka lessons and parts of lessons for teachers to use to address gaps in mathematical knowledge. Using Eureka materials to address acceleration ensures alignment to the standards, consistency in approach to learning, and similarities in strategies for solving problems.

Part A: 5.OA.A.2

Write a numerical expression to represent each of the following statements.

1. The difference of 11 and 4

2. The sum of 5 and 7, divided by 2.

3. Three times the quantity of 5 subtracted from 10.

Part B: 5.OA.A.2

4. Describe how the following two expressions are related:
   $5 \times (13{,}385 + 2{,}963)$ AND $13{,}385 + 2{,}963$

5. Describe how the following two expressions are related:
   $13{,}385 - 2{,}963$ AND $(13{,}385 - 2{,}963) \div 3$

6. Describe how the following two expressions are related:
   $\frac{3}{2} \times (13{,}385 + 2{,}963)$ AND $13{,}385 + 2{,}963$

Part C: 3.OA.B.5

Apply the properties of operation to complete each sentence by filling in the blanks.

7. 8 x 7 = 8 x (_____ + 2) = (_____ x _____) + (8 x 2) = _____ + 16 = _____.

8. 22 x 14 = 22 x (4 + _____) = (_____ x 4) + (_____ x _____) = _____ + _____ = 308.

9. 37 x 19 = _____ x (_____ – 1) = (_____ x _____) – (_____ x 1) = _____ – 37 = _____.

Solutions:

1. $11 - 4$
2. $(5 + 7) \div 2$
3. $3 \times (10 - 5)$
4. (Sample) The value of the first expression is five times as large the value of the second expression.
5. (Sample) The value of the second expression is one third the value of the first expression.
6. (Sample) The value of the first expression is one and a half times as large the value of the second expression.
7. 8 x 7 = 8 x (5 + 2) = (8 x 5) + (8 x 2) = 40 + 16 = 56.
8. 22 x 14 = 22 x (4 + 10) = (22 x 4) + (22 x 10) = 88 + 220 = 308.
9. 37 x 19 = 37 x (20 – 1) = (37 x 20) – (37 x 1) = 740 – 37 = 703.

Part A Focus: 5.OA.A.2: Write simple expressions that record calculations with whole numbers, fractions, decimals, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 x (8+7). Recognize that 3 x (18,932 + 9.21) is three times as large as 18,932 + 9.21, without having to calculate the indicated sum or product.*

Why this is important for current grade level work: In Topic D, student will expand their prior knowledge of writing simple numerical expressions to writing addition/subtraction and multiplication/division expressions in which a variable stands for an unknown number. Furthermore, students will learn new notation for multiplication and division such as 2 x b = 2b and 6 ÷ a = 6/a. Understanding how write numerical expressions from a verbal expression is foundational to being able to write analogous algebraic expressions.			**Acceleration Resources for Targeted Instruction:** Adapted from: 5th Grade, Module 2, Topic B, Lesson(s) 3 - 4 Teacher Companion Student Activity Sheet
Using the Diagnostic Assessment to identify gaps:			
Problem 1: Look to see if students use an equal sign and solve. If so, they may not understand the difference between an expression and an equation.	**Problem 2:** Students should use parentheses around 5 and 7 to ensure the order of operations would be followed when the expression is evaluated.	**Problem 3:** Look for students who write 5 – 10 instead of 10 – 5 as this is a sign they do not understand the difference between the term "subtracted" and the phrase "subtracted from." Students may also be confused with the phrase "quantity of" and not include parenthesis around 10 – 5.	

Part B Focus: 5.OA.A.2: Write simple expressions that record calculations with whole numbers, fractions, decimals, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 x (8+7). Recognize that 3 x (18,932 + 9.21) is three times as large as 18,932 + 9.21, without having to calculate the indicated sum or product.*

Why this is important for current grade level work: In Topic D students will build on their ability to interpret numerical expressions without evaluating them by learning to expand, factor, and distribute expressions. For example, in grade 5 students analyze the expressions 6 x (4 +3) and 4 + 3, to determine that the first expression is 6 times larger than the second expression; likewise, 42 is 6 times larger than 7. In 6th grade students will learn how to use the distributive property to expand and factor expressions, and, having this base understanding of interpreting expressions, students will be equipped to know whether or not their expansion by distributing is reasonable.	**Acceleration Resources for Targeted Instruction:** Adapted from: 5th Grade, Module 2, Topic B, Lesson(s) 3 - 4 Teacher Companion Student Activity Sheet
Using the Diagnostic Assessment to identify gaps: **Problems 4-6:** Look for students who evaluate the expressions without describing how one is related to another. This likely shows a gap in understanding how to analyze expressions that could prove problematic as students begin to work with algebraic expressions.	

Part C Focus: 3.OA.B.5: Apply properties of operations as strategies to multiply and divide. *Examples: If 6 x 4 = 24 is known, then 4 x 6 = 24 is also known. (Commutative property of multiplication.) 3 x 5 x 2 can be found by 3 x 5 = 15 then 15 x 2 = 30 or 5 x 2 = 10 then 10 x 3 =30. (Associative property of multiplication.) Knowing that 8 x 5 = 40 and 8 x 2 = 16, one can find 8 x 7 as 8 x (5 + 2) = (8 x 5) + (8 x 2) = 40 + 16 = 56 (Distributive Property)*

Why this is important for current grade level work: Lesson 12 is solely devoted to using the distributive property to generate equivalent expressions. These problems will help determine which students have a fundamental understanding of the distributive property (combined with some general number sense). Students whose understanding of the distributive property is limited to a set of steps or a simple procedure may struggle with these items; however, having a surface level understanding of the distributive property will prove problematic as students begin engaging with algebraic expressions. The problems scaffold in difficulty.			**Acceleration Resources for Targeted Instruction:** Adapted from: 3rd Grade, Module 3, Lesson(s) 2, 6, and 10 Teacher Companion Student Activity Sheet
Using the Diagnostic Assessment to identify gaps:			
Problem 7: Look for students who simply fill in the final blank (i.e., know that 8 x 7 = 56), leaving all other blanks unfilled. Such students likely do not understand the distributive property and will need support prior to engaging in Lesson 12.	**Problem 8:** Since this problem gives the product, look for students who incorrectly fill the blanks, not using 22 in both sums. Such students likely do not understand the distributive property and will need support prior to engaging in Lesson 12.	**Problem 9:** This is the most challenging problem as it includes subtraction, forcing students to think of 19 as 20 – 1. Some students may believe that you can only distribute over addition and, as such, may not know where to start with this problem.	

Acceleration Guidance:
Grade 6 Eureka Module 4, Topic D

Appendix

Eureka Acceleration Tools

Prerequisite Support Lesson for Grade 6, Module 4, Topic D

Part A: Writing Numerical and Verbal Expressions

Adapted from Eureka Lesson: Grade 5, Module 2, Topic B, Lessons 3-4
Use Prior to: 6th Grade, Module 4, Topic D, Lesson 9

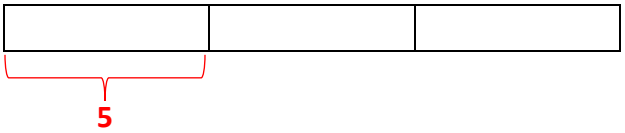
Standard: 5.OA.A.2
Write simple expressions that record calculations with whole numbers, fractions, decimals, and interpret numerical expressions without evaluating them. For example, express the calculation "add 8 and 7, then multiply by 2" as 2 x (8+7). Recognize that 3 x (18,932 + 9.21) is three times as large as 18,932 + 9.21, without having to calculate the indicated sum or product.

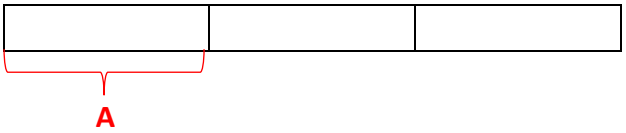
Purpose: In Topic D, student will expand their prior knowledge of writing simple numerical expressions to writing addition/subtraction and multiplication/division expressions in which a variable stands for an unknown number. Furthermore, students will learn new notation for multiplication and division such as 2 x b = 2b and 6 ÷ a = 6⁄a. Understanding how write numerical expressions from a verbal expression is foundational to being able to write analogous algebraic expressions.

Note: The focus of this mini lesson is to write numerical expressions and verbal expressions and to compare expressions. The lesson begins with using visual models so help students visualize the math. A review of terms may be necessary, such as sum, product, difference, and quotient. Work the Example Problems with the students, and then allow the students to complete the **Problem Set** independently as you check for understanding.

Example Problems

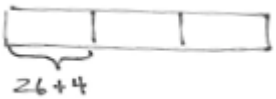
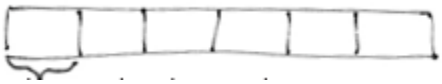
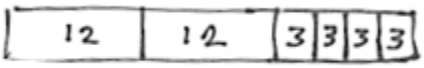
Use the tape diagrams to help students visualize 3 x 5. Draw the first tape diagram on the board, and write a 5 under the first block to show that each unit is equal to 5. Ask students to do the same and ask, "What expression describes the total value of these three equal units? (3 x 5). Now ask, "How about 3 times an unknown number called A?" (3 x A). See if the students can label the second diagram to show 3 x A.





1. Write the numerical expressions. Use models if necessary.
   This next set extends the previous examples to include expressions that are more complex.

a) 3 times the sum of 26 and 4	b) 6 times the difference between 60 and 51
As you work through the problem, asks students why the parentheses around 26 and 4 are necessary. (You have to add the 26 and 6 before multiplying). Consider showing what the problem would express if the parentheses were not there.	For this problem, ask the students if there is a difference in the two expressions: 6 x (60 - 51) or (60 - 51) x 6 (Yes, the factors are just reversed). What is the name of that property? (commutative property)

3 x (26 + 4)  	6 x (60 - 51) or (60 - 51) x 6  60 - 51
c) the sum of 2 twelves and 4 threes (2 x 12) + (4 x 3)  	d) 5 times the sum of 16 and 14 5 x (16 + 14) 16 + 14 | | | |

2. Write the numerical expressions in words.

Walk students through the first example, ensuring they understand the correct order of operations as intended by the expression. Write the expression on the board and ask students to read the expression. They are likely to read the expression as "eight times 43 minus 13." Use the prompt below to explain and then repeat the process with b –d.

- T: Let me write down what I hear you saying. (Write 8 × 43 – 13.) It sounds like you are saying that we should multiply 8 and 43 and then subtract 13. Is that what you meant? Is this second expression equivalent to the one I wrote at first? Why or why not?
- S: No. It's not the same. → You didn't write any parentheses. Without them, you will get a different answer because you won't subtract first. → We are supposed to subtract 13 from 43 and then multiply by 8.
- T: Why can't we simply read every expression left to right and translate it?
- S: We need to use words that tell which operation we should do first.
- T: Let's name the two factors we are multiplying. Turn and talk.
- S: 8 and the answer to 43 – 13. → We need to multiply the answer to the expression inside the parentheses by 8.
- T: Since one of the factors is the answer to *this part* (make a circular motion around 43 – 13), what could we say to make sure we are talking about the answer to this subtraction problem? What do we call the answer to a subtraction problem?
- S: The difference between 43 and 13.
- T: What is happening to the difference of 43 and 13?
- S: It's being multiplied by 8.
- T: We can say and write, "8 times the difference of 43 and 13." Compare these words to the ones we said at first. Do they make sure we are multiplying the right numbers together? What other ways are there to say it?
- S: Yes, the words clearly tell us what to multiply. → The product of 8 and the difference between 43 and 13. → 8 times as much as the difference between 43 and 13. → The difference of 43 and 13 multiplied 8 times.

a) $8 \times (43 - 13)$ 8 times the difference of 43 and 13	b) $(16 + 9) \times 4$ the sum of 16 and 9 times 4 (explain why 16 plus 9 times 4 is incorrect if needed)
c) $(20 \times 3) + (5 \times 3)$ the sum of 20 threes and 5 threes or the sum of 3 twenties and 3 fives, or the product of 20 and 3 plus the product of 5 and 3	d) $(2 \times 15) + (2 \times 20)$ the sum of 2 fifteens and 2 twenties or the sum of 15 twos and 20 twos, or the product of 2 and 15 plus the product of 2 and 20

3. Describe how the two expressions are related.

The use of tape diagrams can be used here to show the comparison of the two expressions. Encourage students to not only compare, but, if one is larger, to say how many times larger. If they are equal, have students explain why.

a) 9×13 **and** 8 thirteens

9×13 **>** 8 thirteens

9×13 **is one thirteen greater than** 8 thirteens

13	13	13	13	13	13	13	13	13

13	13	13	13	13	13	13	13

c) the sum of 10 and 9, doubled **and** $(2 \times 10) + (2 \times 9)$

the sum of 10 and 9, doubled = $(2 \times 10) + (2 \times 9)$

Doubling the sum of 10 and 9 is the same thing as doubling the 10 and doubling the 9 and then adding them together

10	9	10	9

10	10	9	9

d) 13 fifteens minus 1 fifteen **and** 12×15

13 fifteens minus 1 fifteen = 12×15

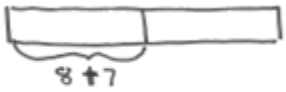
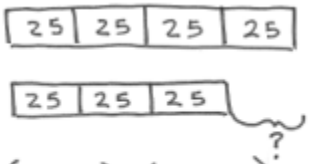
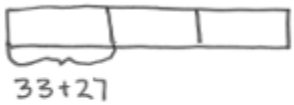
13 fifteens minus 1 fifteen equals 12 fifteens, and 12 fifteens is the same as 12 x 15

15	15	15	15	15	15	15	15	15	15	15	15	✗

15	15	15	15	15	15	15	15	15	15	15	15

Problem Set

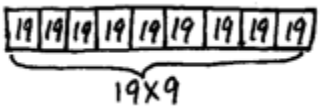
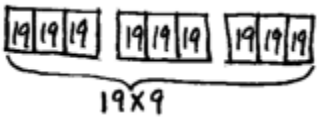
1. Write the numerical expressions. Use models if necessary.

a) The sum of 8 and 7, doubled  $(8+7)\times 2$	b) 4 times the sum of 14 and 26 $4\times(14+26)$
c) The difference between 4 twenty-fives and 3 twenty-fives  $(4\times 25)-(3\times 25)$	d) Triple the sum of 33 and 27  $3\times(33+27)$

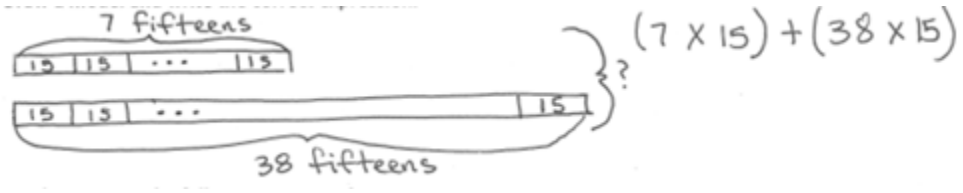
2. Write the numerical expressions in words.

Expression	Words
a. $12\times(5+25)$	12 times the sum of 5 and 25
b. $(62-12)\times 11$	11 times the difference between 62 and 12
c. $(45+55)\times 23$	23 times the sum of 45 and 55
d. $(30\times 2)+(8\times 2)$	The sum of 30 twos and 8 twos

3. Compare the two expressions using >, <, or =.

a)	$24\times(20+5)$	>	$(20+5)\times 12$
b)	18×27	<	20 twenty sevens minus 1 twenty-seven
c)	19×9  	=	3 nineteens, tripled If students struggle here, have them create tape diagram to see how they are equal. 

4. Mr. Huynh wrote the sum of 7 fifteens and 38 fifteens on the board.
   a. Write a numerical expression.
      (7 x 15) + (38 x 15)
      Tape Diagram if needed:



   b. Two students wrote the following numerical expressions:
      Angeline: $(7 \times 15) \times (38 \times 15)$ MeiLing: $15 \times (7 + 38)$.

      Are the students' expressions equivalent to your answer in Part a? Explain your answer.

      MeiLing's answer is equivalent. She found the sum (added) of the 2 bars together and then multiplied by 15.
      Angeline's answer is not equivalent. She multiplied the value of the two bars.

Name: _____

Eureka Acceleration Tools

Prerequisite Support Lesson for Grade 6, Module 4, Topic D

Part A: Writing Numerical and Verbal Expressions

Example Problems

1. Write the numerical expressions. Use models if necessary.

a) 3 times the sum of 26 and 4	b) 6 times the difference between 60 and 51
c) the sum of 2 twelves and 4 threes	d) 5 times the sum of 16 and 14

2. Write the numerical expressions in words.

a) $8 \times (43 - 13)$	b) $(16 + 9) \times 4$
c) $(20 \times 3) + (5 \times 3)$	d) $(2 \times 15) + (2 \times 20)$

3. Describe how the two expressions are related.

a) 9×13 **and** 8 thirteens
c) the sum of 10 and 9, doubled **and** $(2 \times 10) + (2 \times 9)$
d) 13 fifteens minus 1 fifteen **and** 12×15

Problem Set

1. Write the numerical expressions. Use models if necessary.

a) The sum of 8 and 7, doubled	b) 4 times the sum of 14 and 26
c) The difference between 4 twenty-fives and 3 twenty-fives	d) Triple the sum of 33 and 27

2. Write the numerical expressions in words.

Expression	Words
a. $12 \times (5 + 25)$	
b. $(62 - 12) \times 11$	
c. $(45 + 55) \times 23$	
d. $(30 \times 2) + (8 \times 2)$	

3. Compare the two expressions using >, <, or =.

a)	$24 \times (20 + 5)$		$(20 + 5) \times 12$
b)	18×27		20 twenty sevens minus 1 twenty-seven
c)	19×9		3 nineteens, tripled

4. Mr. Huynh wrote the sum of 7 fifteens and 38 fifteens on the board.
   a. Write a numerical expression.

   b. Two students wrote the following numerical expressions:
      Angeline: $(7 \times 15) \times (38 \times 15)$ MeiLing: $15 \times (7 + 38)$.

      Are the students' expressions equivalent to your answer in Part a? Explain your answer.

Eureka Acceleration Tools

Prerequisite Support Lesson for Grade 6, Module 4, Topic D

Part B: Convert Expressions into Unit Form for Multi-digit Multiplication

Adapted from Eureka Lesson: Grade 5, Module 2, Topic B, Lessons 3-4
Use Prior to: 6th Grade, Module 4, Topic D, Lesson 11

Standard: 5.OA.A.2
Write simple expressions that record calculations with whole numbers, fractions, decimals, and interpret numerical expressions without evaluating them. For example, express the calculation "add 8 and 7, then multiply by 2" as 2 x (8+7). Recognize that 3 x (18,932 + 9.21) is three times as large as 18,932 + 9.21, without having to calculate the indicated sum or product.

Purpose: In Topic D students will build on their ability to interpret numerical expressions without evaluating them by learning to expand, factor, and distribute expressions. For example, in grade 5 students analyze the expressions 6 x (4 +3) and 4 + 3, to determine that the first expression is 6 times larger than the second expression; likewise, 42 is 6 times larger than 7. In 6th grade students will learn how to use the distributive property to expand and factor expressions, and, having this base understanding of interpreting expressions, students will be equipped to know whether or not their expansion by distributing is reasonable.

Note: When students convert expressions into unit form, it can be easier for the students to decompose numbers. In doing so, students can manipulate the numbers and mentally multiply with multi-digit numbers. Walk through the Example Problems with the students, and then allow the students to complete the Problem Set independently, as you check for understanding.

Example Problems

Circle **all** of the expressions that are equivalent to the expression: 14 x 59

Let students attempt this problem on their own and then discuss the answers. Encourage students to explain their reasoning.

a. 14 x 59

59 fourteens 14 x (60 – 1) (14 – 1) x 59 50 fourteens + 9 fourteens

59 fourteens - 59 fourteens is the same as multiplying fourteen 59 times. It could also be 14 fifty-nines.

14 x (60 – 1) - When you subtract 1 from 60, that equals 59, so you are still multiplying 14 by 59.

(14 – 1) x 59 – Subtracting 1 from 14 equals 13, so it is not equivalent to 14 x 59.

50 fourteens + 9 fourteens – 50 fourteens plus 9 fourteens equals 59 fourteens, which is equivalent to 14 x 59 (if students struggle here, replace "fourteens" with an object, such as apples. They can think of 50 apples plus 9 apples equals 59 apples.)

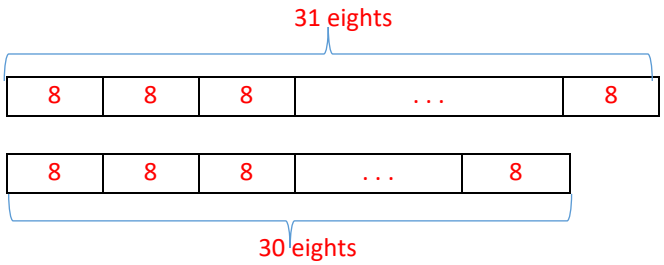
Solve the following problems by decomposing numbers.

a. 8 x 31

What does this means in terms of addition? (add thirty-one 8 times, or add eight 31 times)

When changing the order of the factors, does it change the product? What property is that? (no, commutative property)

Let's use 8 as the unit and draw out two diagrams, 8 x 31 and 8 x 30.



Notice that 31 eights is the same as 30 eights plus 1 eight.

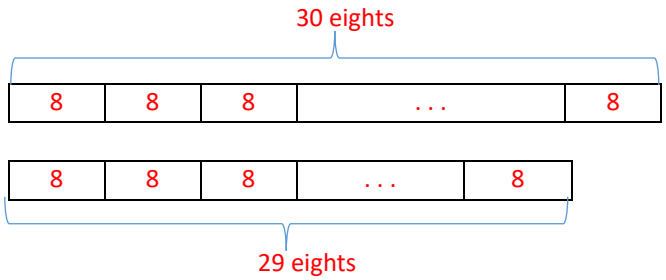
How can this help me solve 8 x 31? (30 eights is equal to 240 and adding one more eight equals 248)

If we record our thinking, it looks like this:

(Write → 31 eights = 30 eights + 1 eight; so 31 x 8 = (30 x 8) + (1 x 8); so 240 + 8 = 248)

b. 8 x 29

Now let's think about 8 x 29. How can we use this same approach to find 8 x 29?



(If 30 eights is equal to 240, then we can subtract 1 eight to equal 232)

Let's record our thinking,

(Write → 29 x 8 = (30 x 8) – (1 x 8) ; so 240 – 8 = 232

c. 49 x 20 = 980

49 twenties = 50 twenties – 1 twenty

49 x 20 = (50 x 20) - (1 x 20) → 1,000 – 20 = 980

d. 20 x 51 = 1,020

51 twenties = 50 twenties + 1 twenty

51 x 20 = (50 x 20) + (1 x 20) → 1,000 + 20 = 1,020

e. 101 x 12

101 twelves = 100 twelves + 1 twelve

101 x 12 = (100 x 12) + (1 x 12) → 1,200 + 12 = 1,212

f. 12 x 98

98 twelves = 100 twelves – 2 twelves

98 x 12 = (100 x 12) – (2 x 12) → 1,200 – 24 = 1,176

Problem Set

1. Circle each expression that is **not equivalent** to the expression in **bold**.

   a. **16 × 29**

   29 sixteens 16 × (30 – 1) (15 – 1) × 29 (10 × 29) – (6 × 29)

   b. **38 × 45**

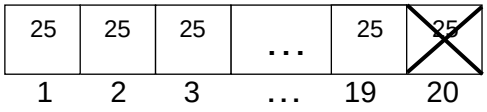
   (38 + 40) × (38 + 5) (38 × 40) + (38 × 5) 45 × (40 + 2) 45 thirty-eights

   c. **74 × 59**

   74 × (50 + 9) 74 × (60 – 1) (74 × 5) + (74 × 9) 59 seventy-fours

2. Solve using mental math. Draw a tape diagram and fill in the blanks to show your thinking. The first one is partially done for you.

a. 19 × 25 = 19 twenty-fives



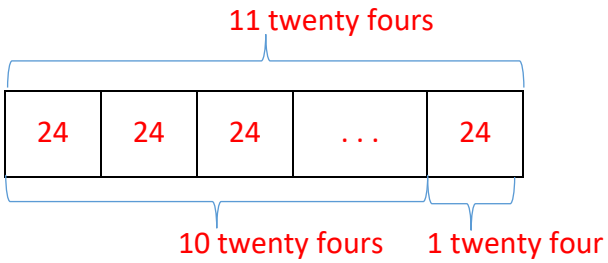
Think: 20 twenty-fives – 1 twenty-five.

= (20 × 25) – (1 × 25)

= 500 – 25

= 475

b. 24 × 11 = 11 twenty-fours



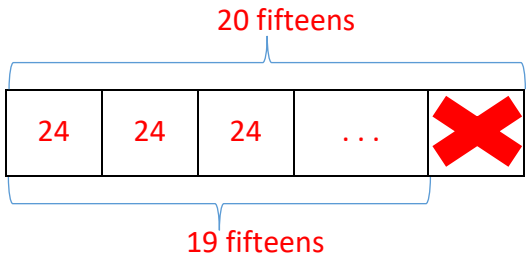
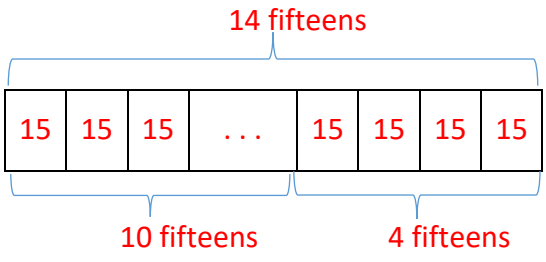
Think: 10 twenty fours + 1 twenty four

= (10 × 24) + (1 × 24)

= 240 + 24

= 264

3. Define the unit in word form and complete the sequence of problems as was done in the lesson.

a. 19 × 15 = 19 fifteens  Think: 20 fifteens – 1 fifteen = (20 × 15) – (1 × 15) = 300 – 15 = 285	b. 14 × 15 = 14 fifteens  Think: 10 fifteens + 4 fifteens = (10 × 15) + (4 × 15) = 150 + 60 = 210

4. How can 14 × 50 help you find 14 × 49?

Once I find 50 fourteens, I can subtract 1 fourteen and it will give me 49 fourteens.
(10 x 50) + (4 x 50) = 500 + 200 → 500 + 200 = 700 → 700 – 14 = 686

5. Solve mentally.

a. 101 × 15 = 1,115
If needed → (100 x 15) + (1 x 15) = 1,500 + 15 → 1,500 + 15 = 1, 515

b. 18 × 99 = 1,782
If needed → (100 x 18) – (1 x 18) = 1,800 – 18 → 1,800 – 18 = 1,782

Name: _____

Eureka Acceleration Tools

Prerequisite Support Lesson for Grade 6, Module 4, Topic D

Part B: Convert Expressions into Unit Form for Multi-Digit Multiplication

Example Problems

Circle **all** of the expressions that are equivalent to the expression: 14 x 59

a. 14 x 59

59 fourteens 14 x (60 – 1) (14 – 1) x 59 50 fourteens + 9 fourteens

Solve the following problems by decomposing numbers.

a. 8 x 31

b. 8 x 29

c. 49 x 20

d. 20 x 51

e. 101 x 12

f. 12 x 98

Problem Set

1. Circle each expression that is **not equivalent** to the expression in **bold**.

   a. **16 × 29**

   29 sixteens 16 × (30 – 1) (15 – 1) × 29 (10 × 29) – (6 × 29)

   b. **38 × 45**

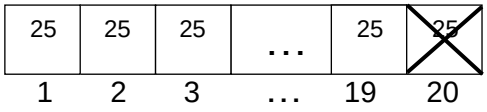
   (38 + 40) × (38 + 5) (38 × 40) + (38 × 5) 45 × (40 + 2) 45 thirty-eights

   c. **74 × 59**

   74 × (50 + 9) 74 × (60 – 1) (74 × 5) + (74 × 9) 59 seventy-fours

2. Solve using mental math. Draw a tape diagram and fill in the blanks to show your thinking. The first one is partially done for you.

a. 19 × 25 = _____ twenty-fives



Think: 20 twenty-fives – 1 twenty-five.

= (_____ × 25) – (_____ × 25)

= _____ – _____

= _____

b. 24 × 11 = _____ twenty-fours

Think: _____ twenty fours + _____ twenty four

= (_____ × 24) + (_____ × 24)

= _____ + _____

= _____

3. Define the unit in word form and complete the sequence of problems as was done in the lesson.

a. 19 × 15 = 19 _____ Think: 20 _____ – 1 _____ = (20 × _____) – (1 × _____) = _____ – _____ = _____	b. 14 × 15 = 14 _____ Think: 10 _____ + 4 _____ = (10 × _____) + (4 × _____) = _____ + _____ = _____

4. How can 14 × 50 help you find 14 × 49?

5. Solve mentally.

   a. 101 × 15 = _____

   b. 18 × 99 = _____

Eureka Acceleration Tools

Prerequisite Support Lesson for Grade 6, Module 4, Topic D

Part C: Apply the Properties of Operations

Adapted from Eureka Lesson: Grade 3, Module 3, Topic B, Lessons 2, 6, 10
Use Prior to: 6th Grade, Module 4, Topic D, Lesson 12

Standard: 3.OA.B.5
Apply properties of operations as strategies to multiply and divide. Examples: If 6 x 4 = 24 is known, then 4 x 6 = 24 is also known. (Commutative property of multiplication.) 3 x 5 x 2 can be found by 3 x 5 = 15 then 15 x 2 = 30 or 5 x 2 = 10 then 10 x 3 =30. (Associative property of multiplication.) Knowing that 8 x 5 = 40 and 8 x 2 = 16, one can find 8 x 7 as 8 x (5 + 2) = (8 x 5) + (8 x 2) = 40 + 16 = 56 (Distributive Property)

Purpose: Lesson 12 is solely devoted to using the distributive property to generate equivalent expressions. These problems will help determine which students have a fundamental understanding of the distributive property (combined with some general number sense). Students whose understanding of the distributive property is limited to a set of steps or a simple procedure may struggle with these items; however, having a surface level understanding of the distributive property will prove problematic as students begin engaging with algebraic expressions.

Note: Students will use the properties of operations to solve multiplication and division problems. Several example problems are provided to demonstrate use of the properties while solving multiplication and division problems. Work through each of the Example Problems with the students, and then allow the students to complete the Problem Set independently, as you check for understanding.

Example Problem 1

In this first example, remind students of the commutative property as they multiply. Tell students that they can use facts that they know to find the product of larger factors. Knowing 5 times 7 can help them solve 6 times 7.

7	7	7	7	7

Each unit has a value of 7.

Write the unit form: 5 sevens

Write equivalent expressions: 5 x 7 =7 x 5

We know this is true because of the commutative property.

What if one more unit of 7 is added?

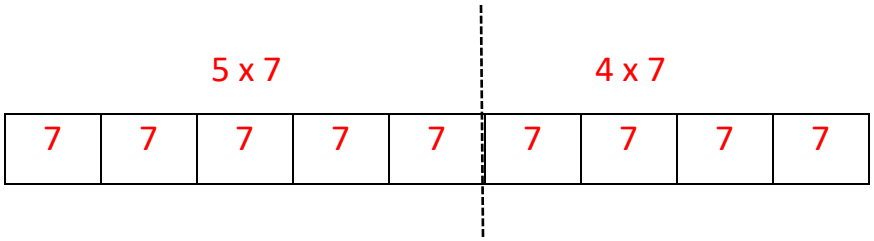
Write the unit form: 5 sevens + 1 seven

Write the expression: 35 + 7 = 42

Write equivalent expressions: 6 x 7 =7 x 6

Example Problem 2

A similar approach will be used to multiply 7 x 9. This time, the 7 will be distributed to two parts.



Each unit has a value of 7. This tape diagram is a model of 9 x 7. Label each unit with a 7.

Break the larger fact up into 2 smaller facts: 9 x 7 = (5 x 7) + (4 x 7)

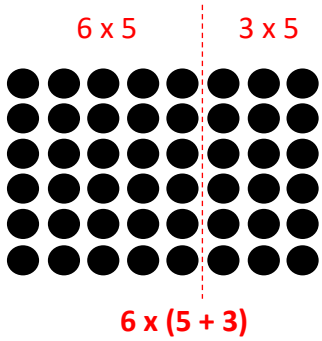
Represent the two smaller facts with the tape diagram. Label each part.

We can distribute the 7 to the two parts because of the distributive property.

Solve 9 x 7: (5 x 7) + (4 x 7) = 35 + 28 35 + 28 = 63

Example Problem 3

Use the distributive property to solve 6 x 8. Use the array to help your explain your thinking.



We will break up the number of columns. How many columns are there? (8)
What can we break 8 into? (Various answers, 5 and 3 will be used)
Write 6 x (5 + 3) under the array.
Draw a line to show how we broke up the 8 columns.
Label each part of the array with (6 x 3) and (6 x 5).
Now solve the problem.
6 x 3 = 18 → 6 x 5 = 30 → 18 + 30 = 48

Example Problem 4

Use the following prompt to solve 48 ÷ 6:

T: We need to break apart 48 ÷ 6 into two smaller division expressions. Why would 30 make a good breaking point?

S: 30 divided by 6 is an easy fives fact.

T: Write 30 ÷ 6 on the number bond. What division expression do we need to write for the other part? How do you know?

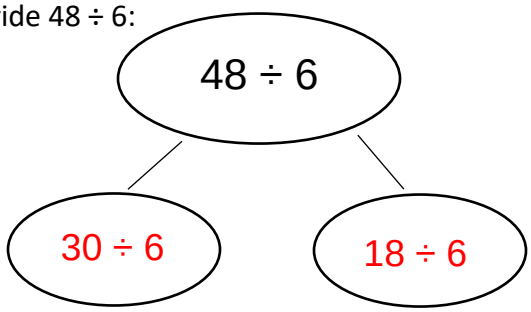
S: 18 ÷ 6. 30 plus 18 equals 48. We used 30, and we need 18 more to get to 48.

T: Write 18 ÷ 6 on the other part of the number bond. Let's show that work with an equation (shown below). Be sure to put parentheses around the two expressions to show that we solve these two division facts first. How can we use the quotients of these two facts to find of the quotient of 48 ÷ 6?

S: Add the quotient of 30 ÷6 and the quotient of 18 ÷ 6. (shown below)

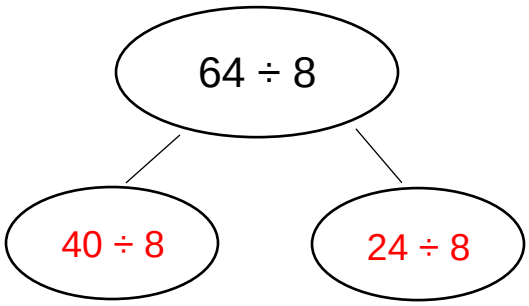
Repeat the procedure with 64 ÷ 8.

Use the Distributive Property to divide 48 ÷ 6:



Solve: 48 ÷ 6 = (30 ÷ 6) + (18 ÷6) → 5 + 3 = 8

Use the Distributive Property to divide 64 ÷ 8:



Solve: 64 ÷ 8 = (40 ÷8) + (24 ÷ 8) → 5 + 3 = 8

Problem Set

1. Each dot has a value of 8.

Unit form: 5 eights

Facts: 5 × 8 = 8 × 5

Total = 40

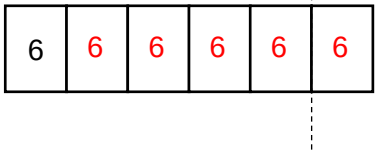
Use the fact above to find 8 x 6. Show your work.

If 5 x 8 = 40, then I can add one more 8, which equals 48.

2. Label the tape diagrams. Then, fill in the blanks below to make the statements true.

a. **6 × 6** = 36

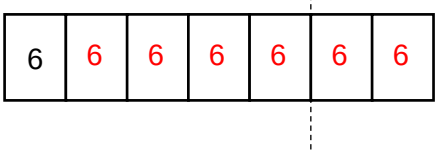
(5 × 6) = 30 (1× 6) = 6



(6 × 6) = (5 + 1) × 6
= (5 × 6) + (1 × 6)
= 30 + 6
= 36

b. **7 × 6** = 42

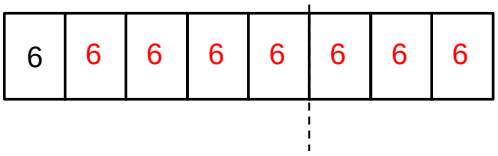
(5 × 6) = 30 (2 × 6) = 12



(7 × 6) = (5 + 2) × 6
= (5 × 6) + (2 × 6)
= 30 + 12
= 42

c. **8 × 6** = **48**

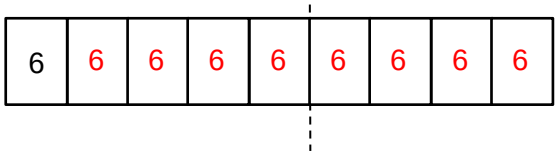
(5 × 6) = 30 (3× 6) = 18



8 × 6 = (5 + 3) × 6
= (5 × 6) + (3 × 6)
= 30 + 18
= 48

d. **9 × 6** = **54**

(5 × 6) = 30 (4 × 6) = 24

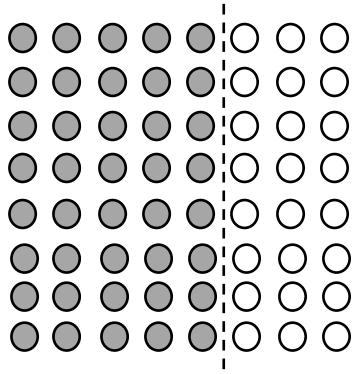


9 × 6 = (5 + 4) × 6
= (5 × 6) + (4 × 6)
= 30 + 24
= 54

3. Label the arrays. Then, fill in the blanks below to make the statements true.

a. **8 × 8 = 64**

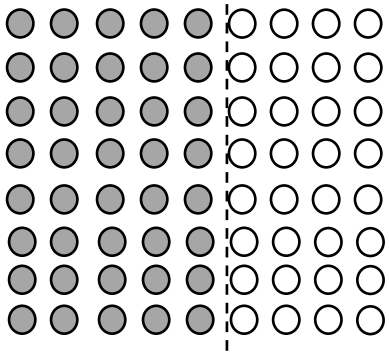
(8 × 5) = 40 (8 × 3) = 24



8 × 8 = 8 × (5 + 3)
= (8 × 5) + (8 × 3)
= 40 + 24
= 64

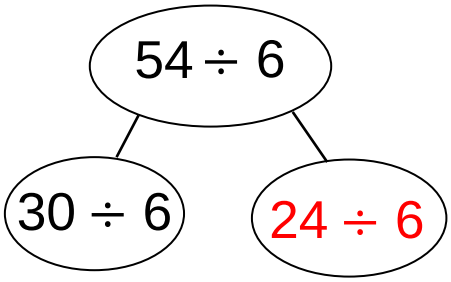
b. **8 × 9 = 72**

(8 × 5) = 40 (8 × 4) = 32

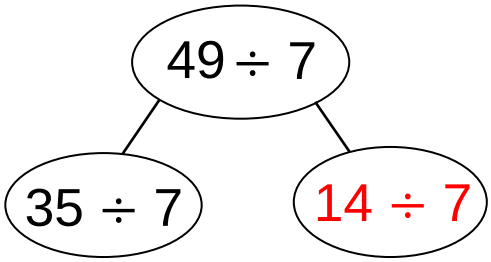


9 × 8 = 8 × (5 + 4)
= (8 × 5) + (8 × 4)
= 40 + 32
= 72

4. Break apart the dividend to solve.



54 ÷ 6 = (30 ÷ 6) + (24 ÷ 6)
= 5 + 4
= 9



49 ÷ 7 = (35 ÷ 7) + (14 ÷ 7)
= 5 + 2
= 7

Name: _____

Eureka Acceleration Tools

Prerequisite Support Lesson for Grade 6, Module 4, Topic D

Part C: Apply the Properties of Operations

Example Problem 1

Each unit has a value of 7.

Write the unit form: _____

Write equivalent expressions: _____ x _____ =_____ x _____

We know this is true because of the _____ property.

What if one more unit of 7 is added?

Write the unit form: _____ sevens + _____ seven

Write the expression: _____ + _____ = _____

Write equivalent expressions: _____ x _____ =_____ x _____

Example Problem 2

Each unit has a value of 7. This tape diagram is a model of 9 x 7. Label each unit with a 7.

Break the bigger fact up into 2 smaller facts: 9 x 7 = (_____ x _____) + (_____ x _____)

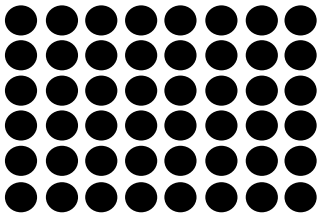
Represent the two smaller facts with the tape diagram. Label each part.

We can distribute the 7 to the two parts because of the _____ property.

Solve 9 x 7: (_____ x _____) + (_____ x _____) = _____ + _____ _____ + _____= _____

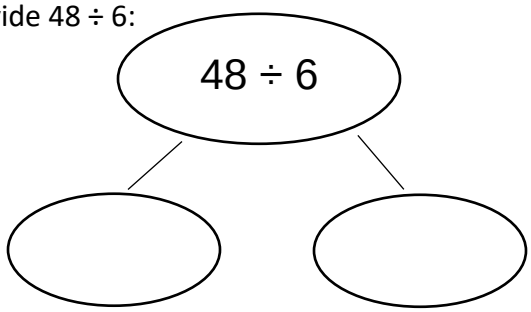
Example Problem 3

Use the distributive property to solve 6 x 8. Use the array to help your explain your thinking.



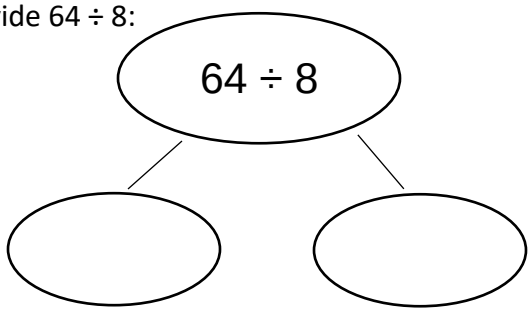
Example Problem 4

Use the Distributive Property to divide 48 ÷ 6:



Solve: 48 ÷ 6 = (_____÷_____) + (_____÷_____) → _____ + _____= _____

Use the Distributive Property to divide 64 ÷ 8:



Solve: 64 ÷ 8 = (_____÷_____) + (_____÷_____) → _____ + _____= _____

Problem Set

1. Each dot has a value of 8.

Unit form: 5 _____

Facts: 5 × _____ = _____× 5

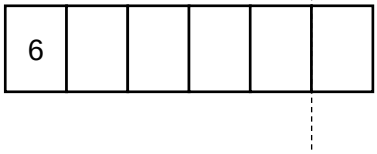
Total = _____

Use the fact above to find 8 x 6. Show your work.

2. Label the tape diagrams. Then, fill in the blanks below to make the statements true.

a. **6 × 6** = _____

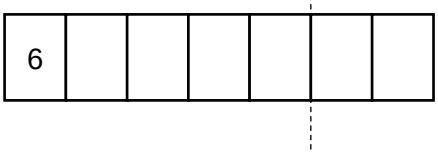
(5 × 6) = _____ (_____× 6) = _____



(6 × 6) = (5 + 1) × 6
= (5 × 6) + (1 × 6)
= __30__+
= _____

b. **7 × 6** = _____

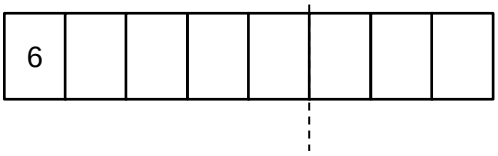
(5 × 6) = _____ (____ × 6) = ____



(7 × 6) = (5 + 2) × 6
= (5 × 6) + (2 × 6)
= __30__+ _____
= _____

c. **8 × 6** = _____

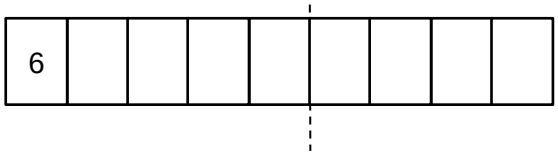
(5 × 6) = _____ (_____× 6) = ____



8 × 6 = (5 + _____) × 6
= (5 × 6) + (____ × 6)
= __30__+ _____
= _____

d. **9 × 6** = _____

(5 × 6) = _____ (____ × 6) = ____

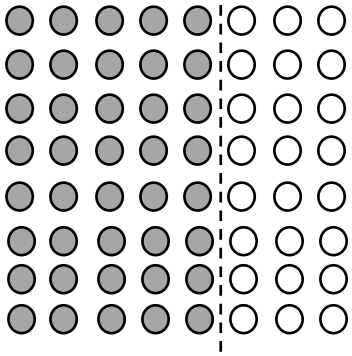


9 × 6 = (5 + _____) × 6
= (5 × 6) + (____ × 6)
= __30__+ _____
= _____

3. Label the arrays. Then, fill in the blanks below to make the statements true.

a. **8 × 8 =** _____

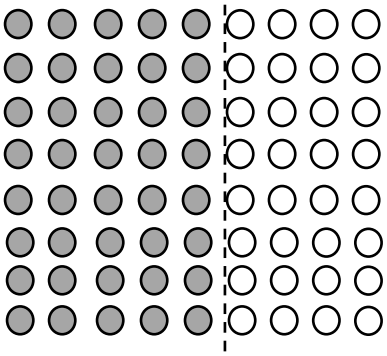
(8 × 5) = _____ (8 × _____) = _____



8 × 8 = 8 × (5 + _____)
= (8 × 5) + (8 × _____)
= __40__ + _____
= _____

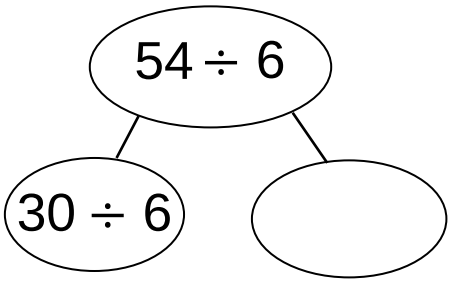
b. **8 × 9 =** _____

(8 × 5) = _____ (8 × _____) = _____

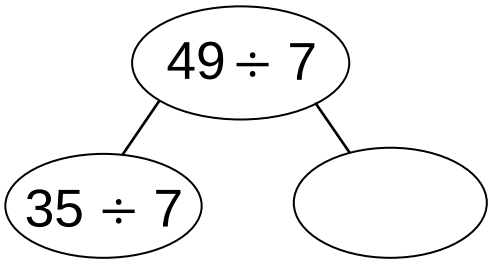


9 × 8 = 8 × (5 + _____)
= (8 × 5) + (8 × _____)
= __40__ + _____
= _____

4. Break apart the dividend to solve.



54 ÷ 6 = (30 ÷ 6) + (_____ ÷ 6)
= 5 + _____
= _____



49 ÷ 7 = (35 ÷ 7) + (_____ ÷ 7)
= 5 + _____
= _____