Eureka Acceleration Tool: Grade 6
Module 2, Topic A

To become mathematically proficient, students **must** access on-grade-level content. This document aims to help teachers who use the Eureka curriculum to ensure readiness for students before and during on-grade-level work, creating opportunities for timely support directly connected to the new learning.

About this Topic

Focus Standards:

6.NS.A.1: Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4- cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*

Topic Overview per the Eureka Curriculum

In Topic A, students extend their previous understanding of multiplication and division to divide fractions by fractions. Students determine quotients through visual models, such as bar diagrams, tape diagrams, arrays, and number line diagrams. They construct division stories and solve word problems involving division of fractions (**6.NS.A.1**). Students understand and apply partitive division of fractions to determine how much is in each group. They explore real-life situations that require them to ask themselves, “How much is one share?” and “What part of the unit is that share?” Students use measurement to determine quotients of fractions. They are presented conceptual problems where they determine that the quotient represents how many of the divisor is in the dividend. Students look for and uncover patterns while modeling quotients of fractions to ultimately discover the relationship between multiplication and division. Later in the module, students understand and apply the direct correlation of division of fractions to division of decimals.

Eureka Acceleration Tool: Grade 6
Module 2, Topic A

Overview

Eureka Acceleration Tools include:

1. a diagnostic assessment to help teachers determine the misunderstandings or gaps in mathematical knowledge related to a specific Topic in the Eureka curriculum
2. guidance for teachers to analyze student work on the diagnostic assessment
3. suggested materials for targeted remedial instruction

Note: The use of this guidance is not intended to delay students' engagement with on-grade-level learning. On-grade-level learning should be the focus of instructional time and be treated as an opportunity for students to "finish" learning previous skills and deepen conceptual understanding.

Diagnostic Assessment

The diagnostic assessment is designed to be administered to targeted students prior to beginning instruction on the given Topic. When appropriate, it is broken into parts (Part A, Part B, and so on); each part addresses a different prerequisite standard and contains three problems. If a student correctly answers at least 2 out of the 3 problems, it can be assumed that he/she is ready to engage with the new content of the Topic with little to no support needed prior to engaging with the Topic. The diagnostic assessment is designed in this way so that teachers can determine the "entry point" to remedial instruction and/or opportunities for unfinished learning within the context of the new learning. The entry points and opportunities for unfinished learning will vary between students.

Guidance for Acceleration

The Acceleration Guidance is designed for teacher use. It is also broken into parts (Part A, Part B, and so on) and correlates to the parts on the diagnostic assessment. Each part contains the following:

1. **The focus standard**: The focus standards are strategically chosen to address prerequisite skills and are purposefully arranged in the order that students typically master the skills and knowledge.
2. **Why this is important for current grade level work**: This section describes how the work of the prerequisite standard relates to the standard(s) addressed in the Topic of instruction.
3. **Using the diagnostic assessment to identify gaps**: This section identifies common errors students make on the diagnostic assessment items.
4. **Acceleration Resources for Targeted Instruction**: The resources pinpoint specific Eureka lessons and parts of lessons for teachers to use to address gaps in mathematical knowledge. Using Eureka materials to address acceleration ensures alignment to the standards, consistency in approach to learning, and similarities in strategies for solving problems.

Part A: 3.OA.B.6

1. Write a multiplication sentence that can help you find the quotient for the following division problem: 56 ÷ 7.

2. Write a corresponding multiplication fact for the following number sentence: 12 ÷ 4 = 3.

3. Sarah's teacher has asked her to solve the problem, 63 ÷ 9, but Sarah does not know the answer. Explain how Sarah can use her multiplication facts to help her find the answer.

Part B: 5.NF.B.7a

4. Interpret the quotient, $\frac{1}{5} \div 2$.

5. Compute the quotient, $\frac{1}{4} \div 2$. Support your answer with a visual fraction model and write a multiplication sentence that supports your quotient.

6. Compute the quotient, $\frac{1}{9} \div 3$. Support your answer with a visual fraction model and write a multiplication sentence that supports your quotient.

Part C: 5.NF.B.7b

7. Interpret the quotient, $2 \div \frac{1}{4}$.

8. Compute the quotient, $5 \div \frac{1}{7}$. Support your answer with a visual fraction model and write a multiplication sentence that supports your quotient.

9. Compute the quotient, $6 \div \frac{1}{3}$. Support your answer with a visual fraction model and write a multiplication sentence that supports your quotient.

Part D: 5.NF.B.7c

10. Ms. Thomas has $\frac{1}{4}$ of a tub of ice cream to share equally among her 3 kids. What fraction of the tub of ice cream will each kid receive? Show and/or explain how you got your answer.

11. Desmond has 6 pounds of dog food. He feeds his dog $\frac{1}{2}$ cup of food per day. For how many days will the bag of dog food last? Show and/or explain how you got your answer.

12. Mackenzie bought 4 pounds of trail mix to divide equally into $\frac{1}{4}$ pound bags for school snacks. How many bags of trail mix can she make? Show and/or explain how you got your answer.

Solutions:

1. 8 x 7 = 56 or 7 x 8 = 56
2. 3 x 4 =12 or 4 x 3 = 12
3. If 7 x 9 = 63 and 9 x 7 =63, then 63 ÷ 9 = 7
4. Answers will vary.
5. $\frac{1}{8}$; $\frac{1}{8} \times 2 = \frac{1}{4}$
6. $\frac{1}{27}$; $\frac{1}{27} \times 9 = \frac{1}{3}$
7. Answers will vary.
8. 35; $35 \times \frac{1}{7} = 5$
9. 18; $18 \times \frac{1}{3} = 6$
10. $\frac{1}{12}$
11. 16

Acceleration Guidance: Grade 6
Eureka Module 2, Topic A

Part A Focus: 3.OA.B.6: Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*

<table>
<tr><td colspan="3">Why this is important for current grade level work:
Before students begin dividing fractions, it is important they have a foundational understanding of division as an unknown factor problem. Contextual fraction problems can be difficult for students to reason about conceptually if they do not understand the different types of division problems such as unknown groups (measurement division) or unknown size of groups (partitive division). If students can interpret a division problem as an unknown factor problem, then they can use multiplication to help them make sense of the problem.</td><td rowspan="2">Acceleration Resources for Targeted Instruction:
adapted from
3rd Grade, Module 1, Topic B, Lessons 4 - 6

Teacher Companion
Student Activity Sheet</td></tr>
<tr><td colspan="3">Using the Diagnostic Assessment to identify gaps:</td></tr>
<tr><td>Problem 1:
Students should determine that, since the two factors, 8 and 7, can be multiplied to attain the product of 56, then the inverse operation can be used to break down 56 into equal-sized groups of 8 or 7.</td><td>Problem 2:
Look for students who write another division problem instead of a multiplication problem. Such students may not understand the relationship between multiplication and division.</td><td>Problem 3:
Students should be able to articulate the relationship between division and multiplication, referencing division as an unknown factor problem. Specifically, they should explain that the solution to the given problem is the answer to, “what number makes 63 when multiplied by 9?”</td></tr>
</table>

Part B Focus: 5.NF.B.7a Apply and extend previous understandings of division to divide unit fraction by whole numbers and whole numbers by unit fractions. a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) x 4 = 1/3.*

Why this is important for current grade level work:
Students should understand that when a fraction is being divided by a whole number, this means that the unit fraction is the dividend, the number that is being divided. This is partitive division because the dividend is being split into a specified number of groups. If students have this foundational understanding, then they can conceptually imagine that if a number smaller than one is being split into equal groups, then those groups must be smaller than the fraction they had to begin with. It does not make sense that a number smaller than 1 could be split into group sizes of a number greater than 1. When students fully understand the different types of division, they can then reason using multiplication to solve the problem.

Using the Diagnostic Assessment to identify gaps:

Problem 4:
Look for students who incorrectly identify 2 as the dividend as this may show that a student thinks you cannot divide a fraction by a whole number.

Problem 5:
In the problem ¼ ÷ 2, ¼ is getting split into two equal groups; therefore, the group size must be half the size of ¼. The students can ask themselves, "2 groups of what is 1/4?" Since 2 x 1/8 = ¼, ¼ ÷ 2 must be 1/8.

Problem 6:
In the problem 1/9 ÷ 3, 1/9 is getting split into three equal groups; therefore, the group size must be one third the size of 1/9. The students can ask themselves, "3 groups of what is 1/9?" Since 3 x 1/27 = 1/9, 1/9 ÷ 3 must be 1/27. Look for students who try to "simplify" or "reduce" the 3 and the 9, leading to an answer of 1/3. Such students do not understand the problem and have likely misused a previously learned "trick."

Acceleration Resources for Targeted Instruction:
adapted from
5th Grade, Module 4, Topic G, Lesson 26

Teacher Companion
Student Activity Sheet

Part C Focus: 5.NF.B.7b Apply and extend previous understandings of division to divide unit fraction by whole numbers and whole numbers by unit fractions. b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 x (1/5) = 4.*

Why this is important for current grade level work:
It is important that students can determine the contextual meaning of a whole number being divided by a unit fraction for division of fractions to make sense conceptually. When a whole number is being divided by a unit fraction, this is called measurement division. In other words, the whole number is being split into groups the size of the divisor. It does not make sense conceptually for a whole number to be split equally into a fractional number of groups. Students can become very confused when dividing a fraction by a fraction if they do not first understand this concept. Once students become fluent in recognizing measurement division, they can use multiplication as an unknown factor to help determine the answer. For example, 4 ÷ ½ translates to 4 being split into 1/2 –sized groups, which would mean you can get 8 total groups. 8 groups of ½ is 4 wholes; therefore 8 x ½ = 4. Students may also ask themselves, "How many ½ - sized groups can fit into 4?"

Using the Diagnostic Assessment to identify gaps:

Problem 7:
Look to see if students use an if/then statement to relate the division of 2 by ¼ as how many fourths can go into 2 wholes. If there are 4 fourths in 1 whole, then there must be 8 fourths in 2 wholes.

Problem 8:
For students who struggle knowing where to start, encourage them to draw a visual model of 5 wholes, each split into 7 equal groups. Students should label/notate each whole as being equivalent to 7 sevenths which totals to 35 sevenths in 5 wholes; therefore, the answer is 35.

Problem 9:
Look for students who try to "simplify" or "reduce" the 6 and the 3, leading to an answer of 2. Such students do not understand the problem and have likely misused a previously learned "trick."

Acceleration Resources for Targeted Instruction:
adapted from
5th Grade, Module 4, Topic G, Lesson 25

Teacher Companion
Student Activity Sheet

Part D Focus: 5.NF.B.7c Apply and extend previous understandings of division to divide unit fraction by whole numbers and whole numbers by unit fractions. c. Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g. by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share ½ lb. of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?*

Why this is important for current grade level work:
Students need an in-depth, conceptual understanding of division of a whole number by a fraction and division of a fraction by a whole number within real-world contexts. Achieving flexibility in their ability to solve and make connections between multiplication and division in these types of problems will help students understand more complex situations, such as dividing a fraction by a fraction. Furthermore, it will prepare students for their discovery of a more efficient method in Lesson 7.

Using the Diagnostic Assessment to identify gaps:

Problem 10:
Look for students who write the division problem with 3 as the dividend rather than the divisor. A common misconception is that you must write the bigger number first in a division problem. These students do not have a conceptual understanding of partitive division.

Problem 11:
Look to see if students interpret this problem as a division problem. Some students may use repeated addition or repeated subtraction to solve, adding ½ until they have reached 6 wholes or subtracting ½ from 6 repeatedly, while other students may use reasoning to solve the problem rather than writing a division or multiplication fact.

Problem 12:
Take note as to whether students have the flexibility to answer this using reasoning rather than solving a multiplication or division fact. Students may be able to reason that because there are 4-fourths in one whole, there must be 16-fourths in 4-wholes. If students give the answer of 16/4, they do not have full understanding of measurement division, which answers the question, "how many groups?"

Acceleration Resources for Targeted Instruction:
adapted from
5th Grade, Module 4, Topic G, Lesson 27

Teacher Companion
Student Activity Sheet

Acceleration Guidance:
Grade 6 Eureka Module 2, Topic A

Appendix

Eureka Acceleration Tools

Prerequisite Support Lesson for Grade 6, Module 2, Topic A

Part A: Understand Division as an Unknown Factor Problem

Adapted from Eureka Lesson: Grade 3, Module 1, Topic B, Lesson 6

Use Prior to: Grade 6, Module 2, Topic A, Lesson 1

Standard: 3.OA.B.6: Understand division as an unknown-factor problem. For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.

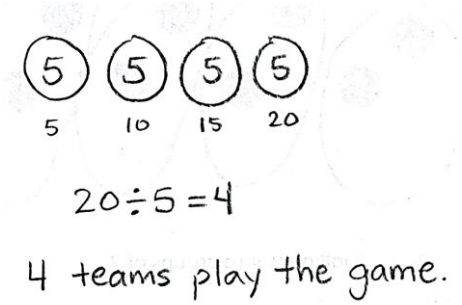
Purpose: Before students begin dividing fractions, it is important they have a foundational understanding of division as an unknown factor problem. Contextual fraction problems can be difficult for students to reason about conceptually if they do not understand the different types of division problems such as unknown groups (measurement division) or unknown size of groups (partitive division). If students can interpret a division problem as an unknown factor problem, then they can use multiplication to help them make sense of the problem.

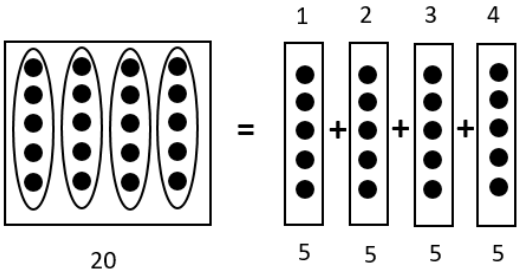
Note: Review Example Problem together, emphasizing the relationship between multiplication and division.

- Reinforce the connection between quotient and unknown factor.
- Depending on prior content knowledge, some students may benefit from the visual of an array in this problem. Encourage students to draw an array to show their thinking.
- Students should conceptualize the number of groups, the size of each group, and the total number in each problem.

Total Number — Size of each group — Number of groups

Example Problem: Twenty children play a game. There are 5 children on each team. How many teams play the game? Write a division sentence to represent the problem.





Relate Multiplication to Division: Think what number times 5 equals 20? ___ **x 5 = 20** **4 x 5 = 20**

Note: Students should complete Problems 1-3 with a partner and all of the parts with Problem 4 independently.

- Students should talk through the problems with each other.
- Encourage students to make sense of the problem by discussing what the numbers mean in the context.
- Visual models can be used to further develop the concept.
- When explaining the meaning of the unknown factor and quotient, explanations should include that finding the quotient of a division problem is the same as the unknown factor in a multiplication problem.

Problem Set

1. Deena makes 21 jars of tomato sauce. She puts 7 jars in each box to sell at the market. How many boxes does Deena need?

   21 ÷ 7 = __3__ __3__ × 7 = 21

   What is the meaning of the unknown factor and quotient? **The number of groups**

2. Nate solves the equation 5 x __**3**__ = 15 by writing and solving 15 ÷ 5 = __**3**__. Explain why Nate's method works.

Possible explanation: Nate's method works because in both problems there are 5 groups of 3 and a total of 15. The quotient in the division problem is like finding an unknown factor in a multiplication problem.

3. Judy washes 24 dishes. She then dries and stacks the dishes equally into 4 piles. How many dishes are in each pile?

24 ÷ 4 = __6__
4 × __6__ = 24

What is the meaning of the unknown factor and quotient? **The size of each group**

4. Relate multiplication to division and solve each equation below.

2 × 9 = 18 and 18 ÷ 2 = 9

5 × 9 = 45 and 45 ÷ 5 = 9

3 × 9 = 27 and 27 ÷ 9 = 3

4 × 7 = 28 and 28 ÷ 7 = 4

8 × 3 = 24 and 24 ÷ 3 = 8

8 × 6 = 48 and 48 ÷ 6 = 8

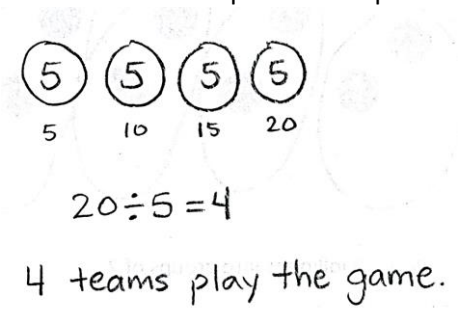
Name: _____

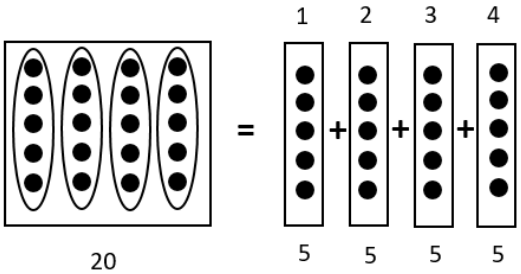
Eureka Acceleration Tools

Prerequisite Support Lesson for Module 2, Topic A

Part A: Understand Division as an Unknown Factor Problem

Example Problem: Twenty children play a game. There are 5 children on each team. How many teams play the game? Write a division sentence to represent the problem.





Relate Multiplication to Division: Think what number times 5 equals 20? **___ x 5 = 20** **4 x 5 = 20**

Problem Set

1. Deena makes 21 jars of tomato sauce. She puts 7 jars in each box to sell at the market. How many boxes does Deena need?

   21 ÷ 7 = _____ _____ × 7 = 21

   What is the meaning of the unknown factor and quotient? _____

2. Nate solves the equation 5 x_____ = 15 by writing and solving 15 ÷ 5 = ____. Explain why Nate's method works.

   _____
   _____
   _____

3. Judy washes 24 dishes. She then dries and stacks the dishes equally into 4 piles. How many dishes are in each pile?

   24 ÷ 4 = _____
   4 × _____ = 24

   What is the meaning of the unknown factor and quotient? _____

4. Relate multiplication to division and solve each equation below.

   2 × ____ = 18 and 18 ÷ 2 = ____ 5 × ____ = 45 and 45 ÷ 5 = ____

   ____ × 9 = 27 and 27 ÷ 9 = ____ ____ × 7 = 28 and 28 ÷ 7 = ____

   ____ × 3 = 24 and 24 ÷ 3 = ____ ____ × 6 = 48 and 48 ÷ 6 = ____

Eureka Acceleration Tools

Prerequisite Support Lesson for Grade 6, Module 2, Topic A

Part B: Divide Unit Fractions by Whole Numbers

Adapted from Eureka Lesson: Grade 5, Module 4, Topic G, Lesson 26

Use Prior to: Grade 6, Module 2, Topic A, Lesson 1

Standard: 5.NF.B.7a Apply and extend previous understandings of division to divide unit fraction by whole numbers and whole numbers by unit fractions.

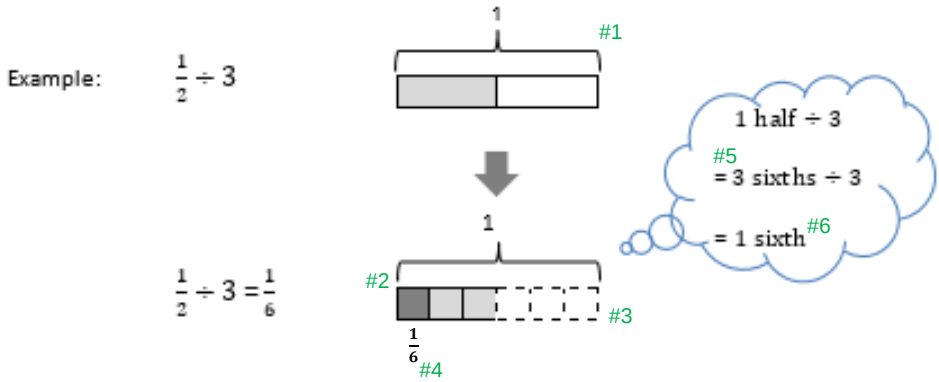
a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) x 4 = 1/3.

Purpose: Students should understand that when a fraction is being divided by a whole number, the unit fraction is the dividend (the number being divided). This is "partitive division" because the dividend is being split into a specified number of groups. Students should conceptually understand that if a number smaller than one is being split into equal groups, then those groups must be smaller than the fraction they had to begin with. Students can reason with multiplication to solve the problem.

Note: Review Example Problem together.

- By creating a visual model, students can conceptualize the problem.
- Model the problem below, asking the following questions along the way.
  - How can we represent ½ with a tape diagram? (Draw a bar and split it into 2 parts) #1
  - How can we represent splitting ½ into three parts? (Draw lines to show the 3 equal parts the half is divided into) #2
  - We have to show equal units, so what should we do to the other half? (Draw lines to show the three equal parts the other half is divided into) #3
  - How many equal parts did we cut the tape diagram into? (6) What does each section of the tape diagram represent? (1/6)
  - Label 1/6 underneath one part. #4
  - Now think, ½ is equal to how many sixths? (3 sixths) #5
  - So, what is 3 sixths ÷ 3? (1 sixth) #6
  - We could also ask, ½ is 3 of what number? ($\frac{1}{6}$) Or 3 of what number makes half? (Three 1 sixths makes half)

Example Problem $\frac{1}{2} \div 3 = \frac{1}{6}$

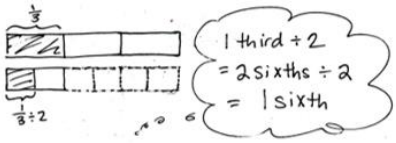


Note: Allow students to complete the Problem Set independently. Review answers together. Encourage students to explain their reasoning as they explain each problem.

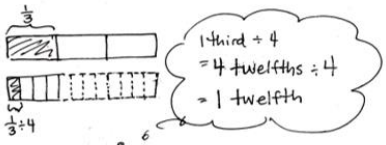
Problem Set

1. Draw a model or tape diagram to solve. Write your quotient in the blank.

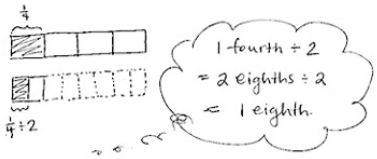
a. $\frac{1}{3} \div 2 = \frac{1}{6}$



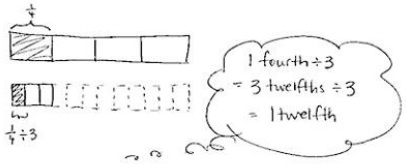
b. $\frac{1}{3} \div 4 = \frac{1}{12}$



c. $\frac{1}{4} \div 2 = \frac{1}{8}$



d. $\frac{1}{4} \div 3 = \frac{1}{12}$



2. Divide. Then, multiply to check.

a. $\frac{1}{2} \div 7 = \frac{1}{14}$	b. $\frac{1}{3} \div 6 = \frac{1}{18}$	c. $\frac{1}{4} \div 5 = \frac{1}{20}$	d. $\frac{1}{5} \div 4 = \frac{1}{20}$
e. $\frac{1}{5} \div 2 = \frac{1}{10}$	f. $\frac{1}{6} \div 3 = \frac{1}{18}$	g. $\frac{1}{8} \div 2 = \frac{1}{16}$	h. $\frac{1}{10} \div 10 = \frac{1}{100}$

a. $\frac{1}{2} \div 7$ $\frac{7}{14} \div 7 = \frac{1}{14}$ $\frac{1}{14} \times 7 = \frac{7}{14} = \frac{1}{2}$ ✓	b. $\frac{1}{3} \div 6$ $\frac{6}{18} \div 6 = \frac{1}{18}$ $\frac{1}{18} \times 6 = \frac{6}{18} = \frac{1}{3}$ ✓	c. $\frac{1}{4} \div 5$ $\frac{5}{20} \div 5 = \frac{1}{20}$ $\frac{1}{20} \times 5 = \frac{5}{20} = \frac{1}{4}$ ✓	d. $\frac{1}{5} \div 4$ $\frac{4}{20} \div 4 = \frac{1}{20}$ $\frac{1}{20} \times 4 = \frac{4}{20} = \frac{1}{5}$ ✓
e. $\frac{1}{5} \div 2$ $\frac{2}{10} \div 2 = \frac{1}{10}$ $\frac{1}{10} \times 2 = \frac{2}{10} = \frac{1}{5}$ ✓	f. $\frac{1}{6} \div 3$ $\frac{3}{18} \div 3 = \frac{1}{18}$ $\frac{1}{18} \times 3 = \frac{3}{18} = \frac{1}{6}$ ✓	g. $\frac{1}{8} \div 2$ $\frac{2}{16} \div 2 = \frac{1}{16}$ $\frac{1}{16} \times 2 = \frac{2}{16} = \frac{1}{8}$ ✓	h. $\frac{1}{10} \div 10$ $\frac{10}{100} \div 10 = \frac{1}{100}$ $\frac{1}{100} \times 10 = \frac{10}{100} = \frac{1}{10}$ ✓

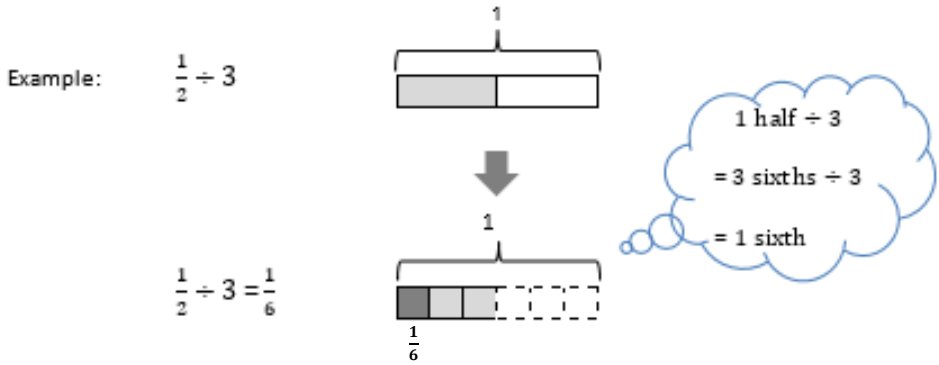
Name: _____

Eureka Acceleration Tools

Prerequisite Support Lesson for Module 2, Topic A

Part B: Divide Unit Fractions by Whole Numbers

Example Problem $\frac{1}{2} \div 3 = \frac{1}{6}$



Problem Set

1. Draw a model or tape diagram to solve. Write your quotient in the blank.

   a. $\frac{1}{3} \div 2 =$ _____

   b. $\frac{1}{3} \div 4 =$ _____

   c. $\frac{1}{4} \div 2 =$ _____

   d. $\frac{1}{4} \div 3 =$ _____

2. Divide. Then, multiply to check.

a. $\frac{1}{2} \div 7$	b. $\frac{1}{3} \div 6$	c. $\frac{1}{4} \div 5$	d. $\frac{1}{5} \div 4$
e. $\frac{1}{5} \div 2$	f. $\frac{1}{6} \div 3$	g. $\frac{1}{8} \div 2$	h. $\frac{1}{10} \div 10$

Eureka Acceleration Tools

Prerequisite Support Lesson for Grade 6, Module 2, Topic A

Part C: Divide Whole Numbers by Unit Fractions

Adapted from Eureka Lesson: Grade 5, Module 4, Topic G, Lesson 25

Use Prior to: Grade 6, Module 2, Topic A, Lesson 2

Standard: 5.NF.B.7 Apply and extend previous understandings of division to divide unit fraction by whole numbers and whole numbers by unit fractions.

> b. Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 x (1/5) = 4.

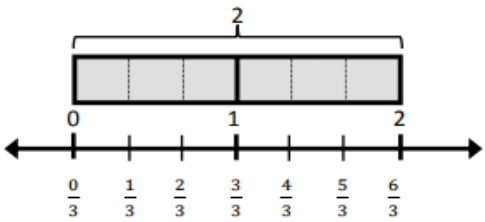
Purpose: Students should be able to determine the contextual meaning of a whole number being divided by a unit fraction in order for the problem to make sense. When a whole number is divided by a unit fraction, this is called "measurement division," meaning the whole number is being split into groups the size of the divisor. It does not make sense conceptually for a whole number to be split equally into a fractional number of groups. Students can use multiplication as an unknown factor to help determine the answer. For example, 4 ÷ 1⁄2 translates to 4 being split into 1/2 –sized groups, which would mean you can get 8 total groups. 8 groups of 1⁄2 is 4 wholes; therefore 8 x 1⁄2 = 4. Students may also ask themselves, "How many 1⁄2 - sized groups can fit into 4?"

Note: Students may use multiple forms of representation to solve each problem in the Problem set.

- Review the first Example Problem with students. This shows different representations of the division.
- Walk them through the next Example Problem.
- Allow students to complete the Problem Set independently and then review.

Example Problem $2 \div \frac{1}{3} = 6$

Example: $2 \div \frac{1}{3} =$ 6

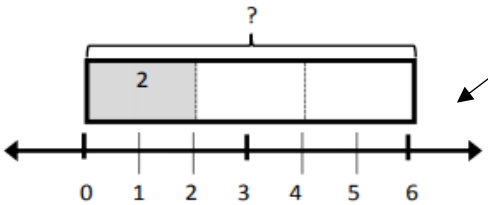


There are 3 thirds in 1 whole.

There are 6 thirds in 2 wholes.

Show students how to divide 2 wholes into thirds. Ask, how many thirds are in 1 whole? Then, how many thirds are in 2 wholes?

Relate division back to multiplication. If 2 is 1/3 of the whole, what is the whole? 1/3 x 6 = 2



If 2 is $\frac{1}{3}$, what is the whole? 6

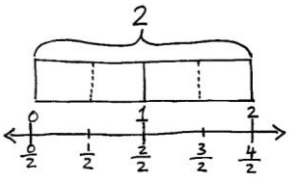
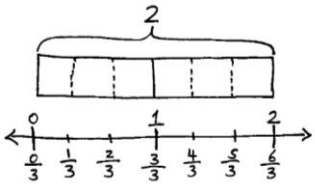
NOTES ON MULTIPLE MEANS OF REPRESENTATION:

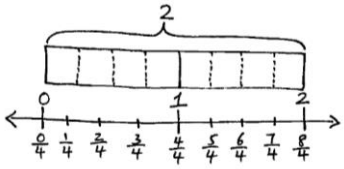
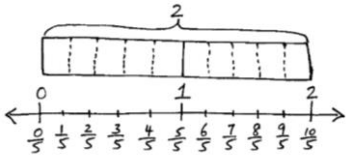
In addition to tape diagrams and area models, students can also use region models to represent the information in these problems. For example, students can draw circles to represent the pounds of pecans and divide the circles in half to represent halves.

Note: For this next example problem, encourage students to look for patterns as they divide 2 by smaller fraction units. Students should develop the following concepts:

- Students should discover that the 2 pounds are the same, but each time, the answer is being divided into smaller and smaller units. The answer is getting larger and larger.
- When the 2 pounds are divided into smaller units, then the answer is bigger. This is because when you cut a whole into smaller parts, then you get more parts.
- The more units you split from one whole, then the more parts you will have. That's why the quotient is becoming larger.

1. Jenny buys 2 pounds of pecans.

a. If Jenny puts 2 pounds in each bag, how many bags can she make? Say the division sentence (2 ÷ 2 = 1). How many bags can she make? (1 bag).	
b. If she puts 1 pound in each bag, how many bags can she make? Say the division sentence. (2 ÷ 1 = 2). How many bags can she make? (2 bags).	
c. If she puts ½ pound in each bag, how many bags can she make? What would the division sentence look like? (We still have 2 as the amount that's divided up, so it should still be $2 \div \frac{1}{2}$. We are putting pecans in half-pound groups, so 1 half will be our divisor, the size of the group. It is like asking how many halves are in 2). Will the answer be more or less than 2? (I looked at the other problems and saw a pattern. 2 ÷ 2 = 1, 2 ÷ 1 = 2, and now I think $2 \div \frac{1}{2}$ will be more than 2. It should be more because we're cutting each pound into halves so that will make more groups. I can visualize that each whole pound would have 2 halves, so there should be 4 half pounds in 2 pounds. Create a tape diagram to represent 2 wholes being divided by halves. How many halves are in 2 wholes? (4). How many ½-pound bags can she make? (She can make 4 bags). **Walk students through checking their answer. Use multiplication to check the division problem $4 \times \frac{1}{2} = 2$ or $\frac{1}{2} \times 4 = \frac{1 \times 4}{2}$; $\frac{4}{2} = 2$	 $2 \div \frac{1}{2} = 4$ She can make 4 bags.
d. If she puts $\frac{1}{3}$ pound in each bag, how many bags can she make? Division problem? ($2 \div \frac{1}{3}$). Create a tape diagram to represent the problem. How many thirds are in one whole? (3 thirds) How many thirds are in two wholes? (6 thirds) How many 1/3-pound can she make? (She can make 6 bags) Use multiplication to check the division problem.	 $2 \div \frac{1}{3} = 6$ She can make 6 bags.

e. If she puts ¼ pound in each bag, how many bags can she make? Division problem? ($2 \div \frac{1}{4}$). Create a tape diagram to represent the problem. How many fourths are in one whole? (4 fourths) How many fourths are in two wholes? (8 fourths) How many ¼-pound can she make? (She can make 8 bags) Use multiplication to check the division problem.	 $2 \div \frac{1}{4} = 8$ She can make 8 bags.
f. If she puts $\frac{1}{5}$ pound in each bag, how many bags can she make? **Considering the concepts developed and patterns discovered, see if students can figure out how many 1/5-pound bags Jenny can make. Have students create a tape diagram to explain their thinking.	 $2 \div \frac{1}{5} = 10$ She can make 10 bags.

Note: Allow students to complete the Problem Set independently. Upon review, have students explain their answers.

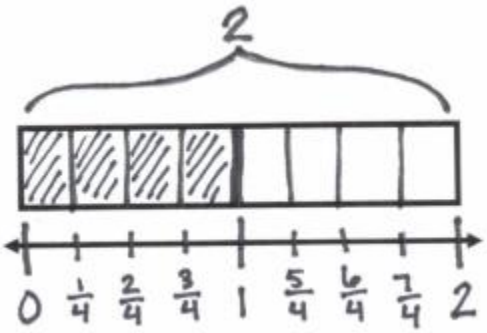
Problem Set

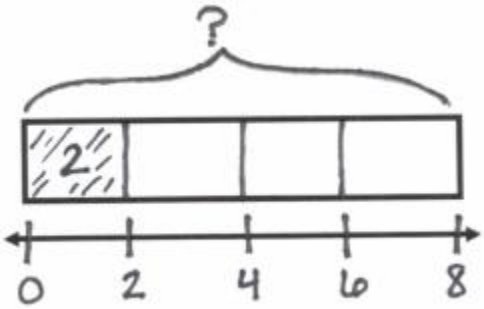
1. Draw a tape diagram and a number line to solve. You may draw the model that makes the most sense to you. Fill in the blanks that follow.

a. $2 \div \frac{1}{4} =$ 8

There are 4 fourths in 1 whole.
There are 8 fourths in 2 wholes.

If 2 is $\frac{1}{4}$, what is the whole? 8

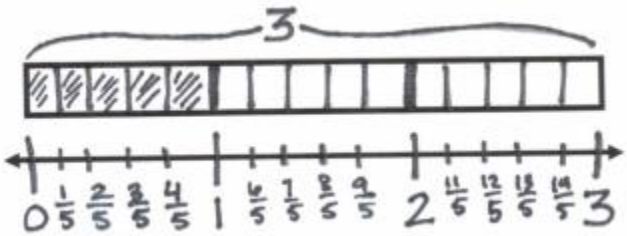


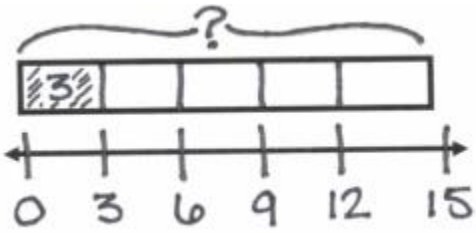


b. $3 \div \frac{1}{5} =$ 15

There are 5 fifths in 1 whole.
There are 15 fifths in 3 wholes.

If 3 is $\frac{1}{5}$, what is the whole? 15



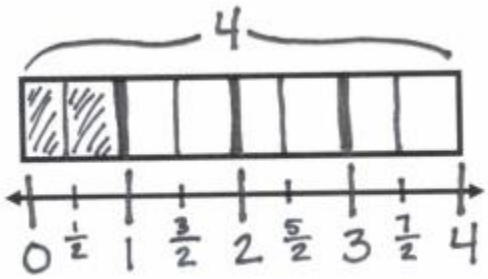


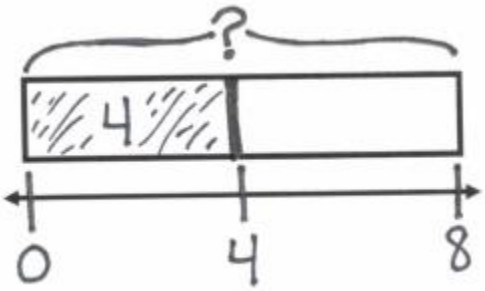
c. $4 \div \frac{1}{2} = 8$

There are 2 halves in 1 whole.

There are 8 halves in 4 wholes.

If 4 is $\frac{1}{2}$, what is the whole? 8



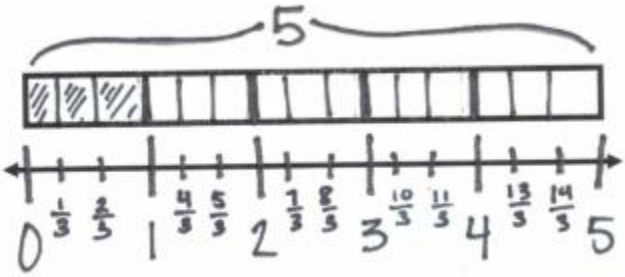


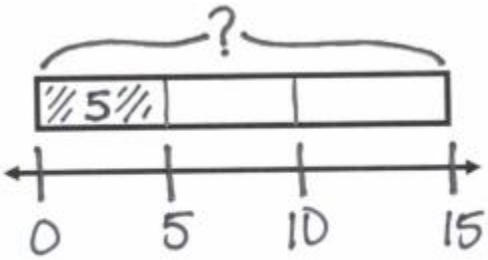
d. $5 \div \frac{1}{3} = 15$

There are 3 thirds in 1 whole.

There are 15 thirds in 5 wholes.

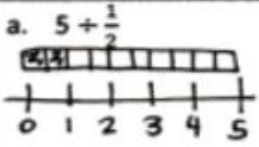
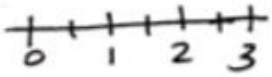
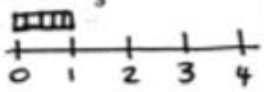
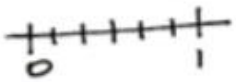
If 5 is $\frac{1}{3}$, what is the whole? 15





2. Divide. Then, multiply to check.

a. $5 \div \frac{1}{2} = 10$	b. $3 \div \frac{1}{2} = 6$	c. $4 \div \frac{1}{5} = 20$	d. $1 \div \frac{1}{6} = 6$
e. $2 \div \frac{1}{8} = 16$	f. $7 \div \frac{1}{6} == 42$	g. $8 \div \frac{1}{3} = 24$	h. $9 \div \frac{1}{4} = 36$

a. $5 \div \frac{1}{2}$  $5 \div \frac{1}{2} = 5 \times 2 = 10$ $\frac{1}{2} \times 10 = \frac{10}{2} = 5$ ✓	b. $3 \div \frac{1}{2}$  $3 \div \frac{1}{2} = 3 \times 2 = 6$ $\frac{1}{2} \times 6 = \frac{6}{2} = 3$ ✓	c. $4 \div \frac{1}{5}$  $4 \div \frac{1}{5} = 4 \times 5 = 20$ $\frac{1}{5} \times 20 = \frac{20}{5} = 4$ ✓	d. $1 \div \frac{1}{6}$  $1 \div \frac{1}{6} = 1 \times 6 = 6$ $\frac{1}{6} \times 6 = \frac{6}{6} = 1$ ✓
e. $2 \div \frac{1}{8}$ $2 \div \frac{1}{8} = 2 \times 8 = 16$ $\frac{1}{8} \times 16 = \frac{16}{8} = 2$ ✓	f. $7 \div \frac{1}{6}$ $7 \div \frac{1}{6} = 7 \times 6$ $= 42$ $\frac{1}{6} \times 42 = \frac{42}{6} = 7$ ✓	g. $8 \div \frac{1}{3}$ $8 \div \frac{1}{3} = 8 \times 3$ $= 24$ $\frac{1}{3} \times 24 = \frac{24}{3} = 8$ ✓	h. $9 \div \frac{1}{4}$ $9 \div \frac{1}{4} = 9 \times 4$ $= 36$ $\frac{1}{4} \times 36 = \frac{36}{4} = 9$ ✓

Name: _____

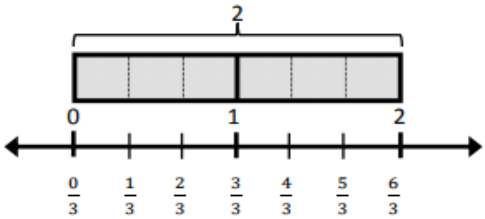
Eureka Acceleration Tools

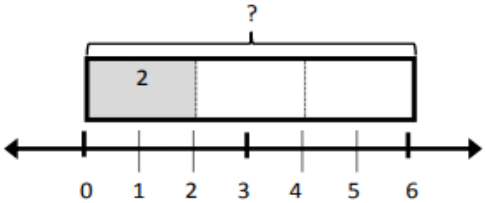
Prerequisite Support Lesson for Module 2, Topic A

Part C: Divide Whole Numbers by Unit Fractions

Example Problem $2 \div \frac{1}{3} = 6$

Example: $2 \div \frac{1}{3} =$ __6__





There are __3__ thirds in 1 whole.

There are __6__ thirds in 2 wholes.

If 2 is $\frac{1}{3}$, what is the whole? __6__

1. Jenny buys 2 pounds of pecans.

   a. If Jenny puts 2 pounds in each bag, how many bags can she make?

   b. If she puts 1 pound in each bag, how many bags can she make?

   c. If she puts ½ pound in each bag, how many bags can she make?

   d. If she puts $\frac{1}{3}$ pound in each bag, how many bags can she make?

   e. If she puts ¼ pound in each bag, how many bags can she make?

   f. If she puts $\frac{1}{5}$ pound in each bag, how many bags can she make?

Problem Set

1. Draw a tape diagram and a number line to solve. You may draw the model that makes the most sense to you. Fill in the blanks that follow.

a. $2 \div \frac{1}{4} =$ _____ There are ____ fourths in 1 whole. There are ____ fourths in 2 wholes. If 2 is $\frac{1}{4}$, what is the whole? _____

b. $3 \div \frac{1}{5} =$ _____ There are ____ fifths in 1 whole. There are ____ fifths in 3 wholes. If 3 is $\frac{1}{5}$, what is the whole? _____

c. $4 \div \frac{1}{2} =$ _____ There are ____ halves in 1 whole. There are ____ halves in 4 wholes. If 4 is $\frac{1}{2}$, what is the whole? _____

d. $5 \div \frac{1}{3} =$ _____ There are ____ thirds in 1 whole. There are ____ thirds in 5 wholes. If 5 is $\frac{1}{3}$, what is the whole? _____

2. Divide. Then, multiply to check.

a. $5 \div \frac{1}{2}$	b. $3 \div \frac{1}{2}$	c. $4 \div \frac{1}{5}$	d. $1 \div \frac{1}{6}$
e. $2 \div \frac{1}{8}$	f. $7 \div \frac{1}{6}$	g. $8 \div \frac{1}{3}$	h. $9 \div \frac{1}{4}$

Eureka Acceleration Tools

Prerequisite Support Lesson for Grade 6, Module 2, Topic A

Part D: Solve Word Problems Involving the Division of Fractions

Adapted from Eureka Lesson: Grade 5, Module 4, Topic G, Lesson 27

Use Prior to: Grade 6, Module 2, Topic A, Lesson 3

Standard: 5.NF.B.7 Apply and extend previous understandings of division to divide unit fraction by whole numbers and whole numbers by unit fractions.

> c. Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g. by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share ½ lb. of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?

Purpose: Students need an in-depth, conceptual understanding of division of a whole number by a fraction and division of a fraction by a whole number with real world context. Achieving flexibility in their ability to solve and make connections between multiplication and division in these types of problems will help students understand more complex situations, such as dividing a fraction by a fraction.

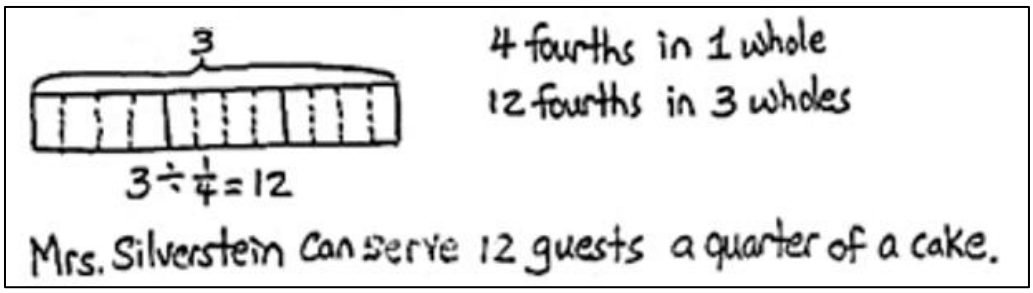
Note: Allow students to use what they have developed conceptually about the division of fractions and how division relates to multiplication to solve the Problem Set. Give students time to persevere through problems and then discuss answers together as a class/small group, clarifying any misconceptions. Encourage students to talk through their work, explaining their reasoning. Any of the Problems 3-5 would be a good opportunity to allow students to work in partners. The problems increase in complexity as students use measurement conversions in the last three problems.

Problem Set

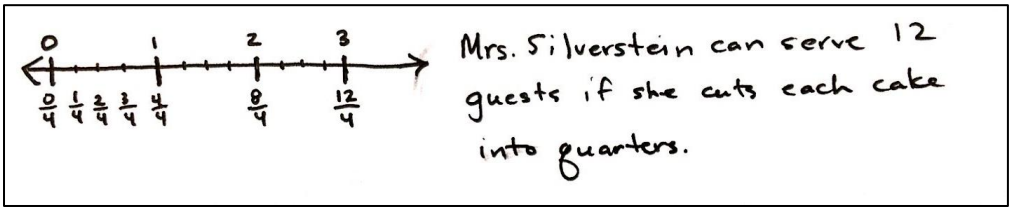
1. Mrs. Silverstein bought 3 mini cakes for a birthday party. She cuts each cake into quarters and plans to serve each guest 1 quarter of a cake. How many guests can she serve with all her cakes? Draw a model to support your response.

   Note: In this problem, students are asked to divide a whole number (3) by a unit fraction (1/4) and draw a model. A tape diagram or a number line would both be acceptable models to support their responses. The reference to the unit fraction as a quarter provides a bit of complexity.

   Method 1

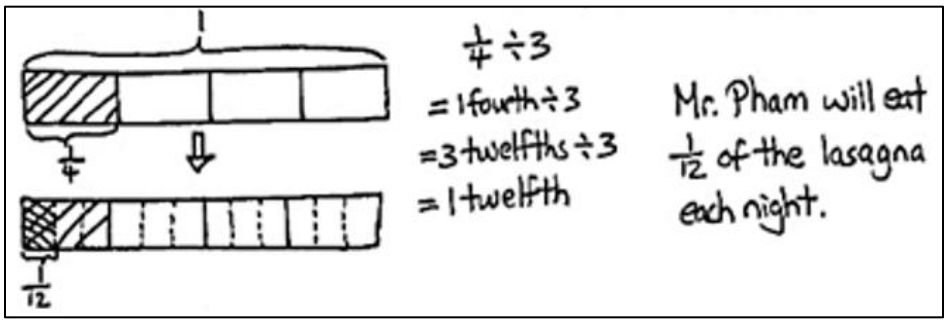


Method 2



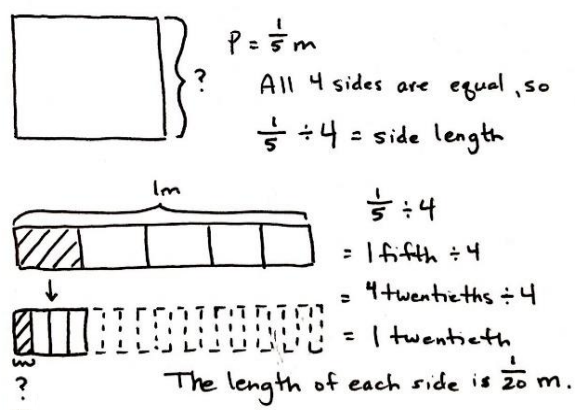
2. Mr. Pham has $\frac{1}{4}$ pan of lasagna left in the refrigerator. He wants to cut the lasagna into equal slices so he can have it for dinner for 3 nights. How much lasagna will he eat each night? Draw a picture to support your response.

Note: Problem 2 is intentionally similar to Problem 1. Although the numbers used in the problems are identical, careful reading reveals that 3 is now the divisor rather than the dividend. While drawing a supporting tape diagram, students should recognize that dividing a fourth into 3 equal parts creates a new unit—twelfths. The model shows that the fraction 1/4 is equal to 3/12; therefore, a division sentence using unit form (3 twelfths "÷" 3) is easy to solve. Facilitate a quick discussion about the similarities and differences of Problems 1 and 2. What do students notice about the division expressions and the solutions?



3. The perimeter of a square is $\frac{1}{5}$ meter.

   a. Find the length of each side in meters. Draw a picture to support your response.
   b. How long is each side in centimeters?

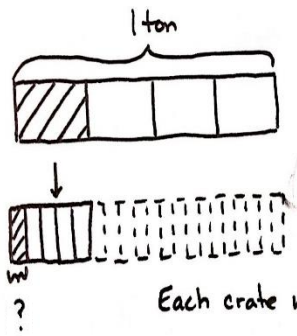
Note: Part (a) This problem requires that students recall their measurement work from Grades 3 and 4 involving perimeter. Students must know that all four side lengths of a square are equivalent; therefore, the unknown side length can be found by dividing the perimeter by 4 ($\frac{1}{5}$ meters ÷ 4 = side lengths). The tape diagram shows clearly that dividing a fifth into 4 equal parts creates a new unit, twentieths, and that $\frac{1}{5}$ is equal to $\frac{4}{20}$. Students may use a division expression using unit form (4 twentieths ÷ 4) to solve this problem very easily. This problem also provides an opportunity to point out a partitive division interpretation to students. While the model was drawn to depict 1 fifth divided into 4 equal parts, the question mark clearly asks "What is $\frac{1}{4}$ of $\frac{1}{5}$?" That is $\frac{1}{4} \times \frac{1}{5}$.	

Part (b) requires that students rename $\frac{1}{20}$ meter as centimeters. Since 1 meter is equal to 100 centimeters, students can multiply to find that $\frac{1}{20}$ m is equivalent to $\frac{100}{20}$ cm, or 5 cm.	b) $\frac{1}{20}$ m = ? cm Each side is 5 cm. ✓ $\frac{1}{20} \times 4 = \frac{4}{20} = \frac{1}{5}$ $\frac{1}{20}$ m $= \frac{1}{20} \times 1$m $= \frac{1}{20} \times 100$ cm $= \frac{1 \times \cancel{100}^{5}}{1\ \cancel{20}}$ cm $= 5$ cm

4. A pallet holding 5 identical crates weighs $\frac{1}{4}$ ton.

   a. How many tons does each crate weigh? Draw a picture to support your response.
   b. How many pounds does each crate weigh?

Note: Problem 2 presents another opportunity for students to interpret the division here as $\frac{1}{4} = 5 \times$____. Part (b) requires that students rename $\frac{1}{20}$ ton as pounds. Since 1 ton is equal to 2,000 pounds, students can multiply to find that $\frac{1}{20}$ ton is equivalent to $\frac{2{,}000}{20}$ pounds or 100 pounds.



$\frac{1}{4} \div 5$

$= 1$ fourth $\div 5$

$= 5$ twentieths $\div 5$

$= 1$ twentieth

Each crate weighs $\frac{1}{20}$ ton.

b) $\frac{1}{20}$ ton = ? pounds

Each crate weighs 100 pounds.

✓ $\frac{1}{20} \times 5 = \frac{5}{20} = \frac{1}{4}$

$\frac{1}{20}$ ton $= \frac{1}{20} \times 1$ ton

$= \frac{1}{20} \times 2{,}000$ pounds

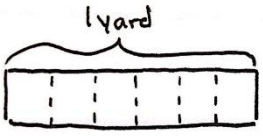
$= \frac{1 \times \cancel{2{,}000}^{100}}{1\ \cancel{20}}$ pounds

$= 100$ pounds

5. Faye has 5 pieces of ribbon, each 1 yard long. She cuts each ribbon into sixths.

   a. How many sixths will she have after cutting all the ribbons?
   b. How long will each of the sixths be in inches?

Note: In Problem 3, since Faye has 5 pieces of ribbon of equal length, students have the choice of drawing a tape diagram showing how many sixths are in 1 yard (and then multiplying that number by 5) or drawing a tape diagram showing all 5 yards to find 30 sixths in total. Part (b) requires that students rename $\frac{1}{6}$ yard as inches. Since 1 yard is equal to 36 inches, students can multiply to find that $\frac{1}{6}$ yd is equivalent to $\frac{36}{6}$ inches or 6 inches.



$1 \div \frac{1}{6} = 6$

6 sixths in 1 yard.

30 sixths in 5 yards.

Faye will have 30 sixths after cutting all the ribbons.

b) Each sixth is $\frac{1}{6}$ of a yard.

Each sixth is 6 inches long.

$\frac{1}{6}$ yd. = ? in.

$\frac{1}{6}$ yd $= \frac{1}{6} \times 1$ yd.

$= \frac{1}{6} \times 36$ in.

$= \frac{1 \times \cancel{36}^{6}}{1\ \cancel{6}}$ in.

$= 6$ in

Name: _____

Eureka Acceleration Tools

Prerequisite Support Lesson for Module 2, Topic A

Part D: Solve Word Problems Involving the Division of Fractions

Problem Set

1. Mrs. Silverstein bought 3 mini cakes for a birthday party. She cuts each cake into quarters and plans to serve each guest 1 quarter of a cake. How many guests can she serve with all her cakes? Draw a model to support your response.

2. Mr. Pham has $\frac{1}{4}$ pan of lasagna left in the refrigerator. He wants to cut the lasagna into equal slices so he can have it for dinner for 3 nights. How much lasagna will he eat each night? Draw a picture to support your response.

3. The perimeter of a square is $\frac{1}{5}$ meter.
   a. Find the length of each side in meters. Draw a picture to support your response.
   b. How long is each side in centimeters?

4. A pallet holding 5 identical crates weighs $\frac{1}{4}$ ton.
   a. How many tons does each crate weigh? Draw a picture to support your response.
   b. How many pounds does each crate weigh?

5. Faye has 5 pieces of ribbon, each 1 yard long. She cuts each ribbon into sixths.
   a. How many sixths will she have after cutting all the ribbons?
   b. How long will each of the sixths be in inches?