Eureka Acceleration Tool: Grade 6
Module 2, Topic D

To become mathematically proficient, students **must** access on-grade-level content. This document aims to help teachers who use the Eureka curriculum to ensure readiness for students before and during on-grade-level work, creating opportunities for timely support directly connected to the new learning.

About this Topic

Focus Standards:

6.NS.B.4: Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4* (*9 + 2*).

Topic Overview per the Eureka Curriculum

Students have previously developed facility with multiplication and division. They now begin to reason logically about them in Topic D. Students apply odd and even number properties and divisibility rules to find factors and multiples. They extend this application to consider common factors and multiples and to find greatest common factors and least common multiples. Students explore and discover that Euclid's algorithm is a more efficient means of finding the greatest common factor of larger numbers, and they determine that Euclid's algorithm is based on long division.

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Overview

Eureka Acceleration Tools include:

1. a diagnostic assessment to help teachers determine the misunderstandings or gaps in mathematical knowledge related to a specific Topic in the Eureka curriculum
2. guidance for teachers to analyze student work on the diagnostic assessment
3. suggested materials for targeted remedial instruction

Note: The use of this guidance is not intended to delay students' engagement with on-grade-level learning. On-grade-level learning should be the focus of instructional time and be treated as an opportunity for students to "finish" learning previous skills and deepen conceptual understanding.

Diagnostic Assessment

The diagnostic assessment is designed to be administered to targeted students prior to beginning instruction on the given Topic. It is broken into parts (Part A, Part B, and so on); each part addresses a different prerequisite standard and contains three problems. If a student correctly answers at least 2 out of the 3 problems, it can be assumed that he/she is ready to engage with the new content of the Topic with little to no support needed prior to engaging with the Topic. The diagnostic assessment is designed in this way so that teachers can determine the "entry point" to remedial instruction and/or opportunities for unfinished learning within the context of the new learning. The entry points and opportunities for unfinished learning will vary between students.

Guidance for Acceleration

The Acceleration Guidance is designed for teacher use. It is also broken into parts (Part A, Part B, and so on) and correlates to the parts on the diagnostic assessment. Each part contains the following:

1. **The focus standard**: The focus standards are strategically chosen to address prerequisite skills and are purposefully arranged in the order that students typically master the skills and knowledge.
2. **Why this is important for current grade level work**: This section describes how the work of the prerequisite standard relates to the standard(s) addressed in the Topic of instruction.
3. **Using the diagnostic assessment to identify gaps**: This section identifies common errors students make on the diagnostic assessment items.
4. **Acceleration Resources for Targeted Instruction**: The resources pinpoint specific Eureka lessons and parts of lessons for teachers to use to address gaps in mathematical knowledge. Using Eureka materials to address acceleration ensures alignment to the standards, consistency in approach to learning, and similarities in strategies for solving problem

Part A: 4.OA.B.4a

1. List all of the factors of 15.

2. List all of the factor pairs of 40.

3. List all of the factor pairs of 56.

Part B: 4.OA.B.4c

4. Is 28 a multiple of 2? Explain and/or justify your answer.

5. Is 85 a multiple of 10? Explain and/or justify your answer.

6. Is 35 a multiple of 5? Explain and/or justify your answer.

Part C: 4.OA.B.4d

7. Is 10 *prime*, *composite*, or *neither*. Explain and/or justify your choice.

8. Is 15 *prime*, *composite*, or *neither*. Explain and/or justify your choice.

9. Is 2 *prime*, *composite*, or *neither*. Explain and/or justify your choice.

Solutions:

1. 1, 15, 3, 5
2. 1 and 40, 2 and 20, 4 and10, 5 and 8
3. 1 and 56, 2 and 28, 4 and 14, 7 and 8
4. Yes, explanations will vary.
5. No, explanations will vary.
6. Yes, explanations will vary.
7. Composite, explanations will vary.
8. Composite, explanations will vary.
9. Prime, explanations will vary.

Part A Focus: 4.OA.B.4a Using whole numbers in the range 1 – 100. a. Find all factors pairs for a given whole number.

<table>
<tr><td colspan="2">Why this is important for current grade level work:
In Grade 6 students will extend their knowledge of factor pairs to finding the greatest common factor of two whole numbers. Students will apply this knowledge using the distributive property to decompose a number multiplicatively into equal groups and express it as a product of two factors; therefore, it is important for students to be fluent in finding factor pairs before starting Topic D.</td><td rowspan="2">Acceleration Resources for Targeted Instruction:
adapted from
4th Grade, Module 3, Topic F, Lesson 22

Teacher Companion
Student Activity Sheet</td></tr>
<tr><td colspan="2">Using the Diagnostic Assessment to identify gaps:</td></tr>
<tr><td>Problem 1:
Students may list the factors as factor pairs or as a list. If students list the factors, they are probably skip-counting to determine factors, rather than thinking of multiplication facts to come up with factor pairs, which is a more efficient method.</td><td>Problems 2 & 3:
Students may list these as multiplication facts rather than pairs. Students may also use a strategy such as a table or a factor tree.</td><td></td></tr>
</table>

Part B Focus: 4.OA.B.4c Using whole numbers in the range 1 – 100. c. Determine whether a given whole number is a multiple of a given one-digit number.

Why this is important for current grade level work:
In Grade 6 students will use their knowledge of factors and multiples and apply that to the distributive property. Students will not only need to be fluent in finding multiples of whole numbers, but it is also important that students attend to precision when distinguishing between a factor and a multiple. This understanding is critical when they begin factoring expressions using the distributive property, so they are able to understand how a common factor can be used as a multiple of a sum of two numbers, without getting confused.

Using the Diagnostic Assessment to identify gaps:

Problem 4:
Take note as to whether students list multiples of 2 to determine that 28 is a multiple of 2 or use the rule that 28 is a multiple of 2 because 28 is an even number.

Problem 5:
Check to see if students list multiples 10 to determine that 85 is not a multiple of 10 or use the rule that 80 is not a multiple of 10 because it does not end in a 0.

Problem 6:
Check to see if students list multiples 5 to determine that 35 is a multiple of or by using the rule that any number ending in a 5 is a multiple of 5.

Acceleration Resources for Targeted Instruction:
adapted from
4th Grade, Module 3, Topic F, Lesson 24

Teacher Companion
Student Activity Sheet

Part C. 4.OA.B.4d Using whole numbers in the range 1 – 100. d. Determine whether a given whole number is prime or composite.

Why this is important for current grade level work:
Having a conceptual understanding of prime and composite numbers will help students become fluent in finding the greatest common factor in Grade 6. For example, if students can recognize that a number is prime, they immediately know that the number only has two factors; therefore, if they are determining the GCF between a prime number and another number, students know that the prime number must be a factor of the second number in order for them to share a factor other 1.

Using the Diagnostic Assessment to identify gaps:

Problem 7:
Students may list factors or write multiplication facts to prove that 10 is a composite number.

Problem 8:
A common misconception is for students to mistake all odd numbers as prime numbers.

Problem 9:
A common misconception is for students to think that all even numbers are composite and therefore may mistake 2 as a composite number.

Acceleration Resources for Targeted Instruction:
adapted from
4th Grade, Module 3, Topic F, Lessons 22 - 23

Teacher Companion
Student Activity Sheet

Acceleration Guidance:
Grade 6 Eureka Module 2, Topic D

Appendix

Eureka Acceleration Tools

Prerequisite Support Lesson for Grade 6, Module 2, Topic D

Part A: Find Factor Pairs for a Given Whole Number

Adapted from Eureka Lesson: 4th Grade, Module 3, Topic F, Lesson 22

Use Prior to: 6th Grade, Module 2, Topic D, Lesson 16

Standard: 4.OA.B.4a

Using whole numbers in the range 1 – 100.

a. Find all factors pairs for a given whole number.

Purpose: In Grade 6 students will extend their knowledge of factor pairs to finding the **greatest** common factor of two whole numbers. Students will apply this knowledge using the distributive property to decompose a number multiplicatively into equal groups and express it as a product of two factors; therefore, it is important for students to be fluent in finding factor pairs before starting Topic D

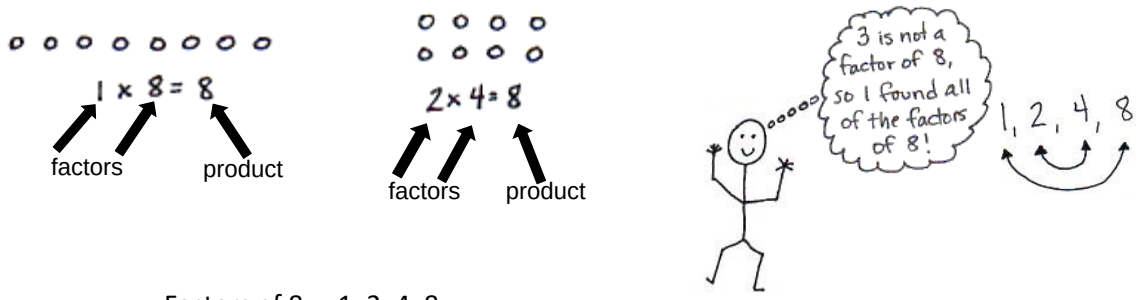
Note: Guide students through the Example Problems.

- Encourage use of mathematical language as you work through the problems, such as "factors" and "product."
- Students will first use arrays to think about the factors of a given number.
- Students will also use multiplication sentences to assist them in finding the factors of the number.
- Showing the students factor pairs, in order starting with 1 and the number, and drawing arrows connecting pairs will help students identify factors without skipping any factors.
- The Example Problems are designed to build students' capacity of identifying factors from a concrete approach using arrays and multiplication sentences to being able to use basic fact knowledge and writing out the factors as pairs and in order from least to greatest.

Example Problems

Identify the factors and product represented in an array by writing multiplication sentences represented by the arrays.

1)



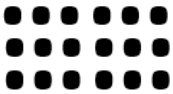
Factors of 8: 1, 2, 4, 8

- Draw the first array on the board and ask students to say the multiplication sentence for the array (1 x 8 = 8)
- Write 1 and 8 as factor pairs in line, leaving space in between for other factors, as show above. Connect the pairs with an arrow.
- Draw the second array and ask the students to say the multiplication sentence for the array (2 x 4 = 8)
- Write 2 and 4 as factor pairs in between the 1 and 8 (separated by commas). Connect with an arrow.
- Say, Yes both arrays show a product of 8. The factors 1 and 8 and the factors 2 and 4 are multiplied to give a product of 8. Can we arrange the array in any other way to show equal groups? (No)
- Remind students that if we think about factors in order, starting with 1 and the number, they can work their way up in order to find all the factors. Have students explain why 3 is not a factor (3 does not go into 8 evenly)
- 1, 2, 4, and 8 are all factors of 8.

2)

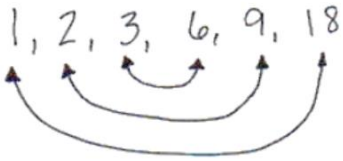
1 x 18 = 18

2 x 9 = 18



3 x 6 = 18

Factors: 1, 2, 3, 6, 9, 18



- What is the product in each of the arrays? (18)
- Record the multiplication sentence for each array. Circle the factors. Write out the factors in order from least to greatest (1, 2, 3, 6, 9, 18).
- How can we make sure we found all the factors of 18? (I can use my multiplication chart, I can think through my multiplication facts)
- Look at the factor pairs. Draw an arrow from 1 to 18. These are factor pairs because 1 x 18 = 18. Draw the rest of the arrows to connect the factor pairs.
- Notice that 3 and 6 are the middle pairs. We've checked everything up to 3, and the counting numbers between 3 and 6 are not factors of 18, so we found all of the pairs.

Identify the factors of the given numbers as multiplication sentences and as a list in order from least to greatest.

- For the next set of problems, move away from the pictorial representation and have students write out the multiplication sentences that represent the given product.
- Encourage students to start with 1 and the number itself, and then continue finding factors in an increasing order until all factors have been found. List factors in increasing order out to the side as they find the factors.
- Remind students of divisibility rules when determining factors:
  - All even numbers are multiples of 2
  - All even numbers that can be halved twice (with a whole number result) are multiples of 4
  - All numbers ending in 0 or 5 are multiples of 5

	Multiplication Sentences	Factors
3) 4	1 x 4 = 4; 2 x 2 = 4 Point out that they don't need to write "2" two times.	1, 2, 4
4) 7	1 x 7 = 7 Point out that 7 only has 2 factors, 1 and itself.	1, 7
5) 15	1 x 15 = 15, 3 x 5 = 15	1, 3, 5, 15
6) 24	1 x 24 = 24, 2 x 12 = 24, 3 x 8 = 24, 4 x 6 = 24 Emphasize that by starting with 1 and the number itself and then continue finding factor pairs in an increasing order can help students in finding all of the factors without skipping any. Once they meet in the middle, like 6 and 8, students consider 7, realize it is not a factor of 24, and then know that they have found all of the factors.	1, 2, 3, 4, 6, 8, 12, 24

Identify the factors pairs of the given numbers. List the factors below, from least to greatest.

- For this next set of problems, encourage students to move away from both using pictorial representations and writing multiplication sentences, and use basic fact knowledge in identifying factor pairs.
- Some students may still feel the need to draw an array, write multiplication sentences, or even use a multiplication chart. Students can use whatever strategy needed, so long as they are able to identify the factors.
- Remind students that for each problem all rows might not be filled in as each number may have a different total of factors/factor pairs.

7) Factor Pairs for 25	
1	25
5	5

Factors: 1, 5, 25

8) Factor Pairs for 28	
1	28
2	14
4	7

Factors: 1, 2, 4, 7, 14, 28

9) Factor Pairs for 40	
1	40
2	20
4	10
5	8

Factors: 1, 2, 4, 5, 8, 10, 20, 40

Problem Set

Identify the factors of the given numbers as multiplication sentences, and list in order from least to greatest.

Note: Students should complete the Problem Set independently. Students may need additional workspace in order to complete.

	Multiplication Sentences	Factors
1) 6	1 x 6 = 6; 2 x 3 = 6	1, 2, 3, 6
2) 9	1 x 9 = 9; 3 x 3 = 9	1, 3, 9
3) 12	1x 12 = 12; 2 x 6 = 12; 3 x 4 = 12	1, 2, 3, 4, 6, 12
4) 13	1 x 13 = 13	1, 13
5) 16	1 x 16 = 16; 2 x 8 = 16; 4 x 4 = 16	1, 2, 4, 8, 16
6) 19	1 x 19 = 19	1, 19
7) 21	1 x 21 = 21; 3 x 7 = 21	1, 3, 7, 21
8) 30	1 x 30 = 30, 2 x 15 = 30, 3 x 10 = 30, 5 x 6 = 30	1, 2, 3, 5, 6, 10, 15, 30
9) 36	1 x 36 = 36; 2 x 18 = 36, 3 x 12 = 36, 4 x 9 = 36; 6 x 6 = 36	1, 2, 3, 4, 6, 9, 12, 18, 36
10) 45	1 x 45; 3 x 15; 5 x 9;	1, 3, 5, 9, 15, 45

Identify the factors for the given numbers.

11) 54: 1, 2, 3, 6, 9, 18, 27, 54

12) 57: 1, 3, 19, 57

13) 48: 1, 2, 3, 4, 6, 8, 12, 16, 24, 48

14) 11: 1, 11

15) 23: 1, 23

16) 60: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

17) 42: 1, 2, 3, 6, 7, 14, 21, 42

18) 27: 1, 3, 9, 27

19) 35: 1, 5, 7, 35

20) 100: 1, 2, 4, 5, 10, 20, 25, 50, 100

Name: _____

Eureka Acceleration Tools

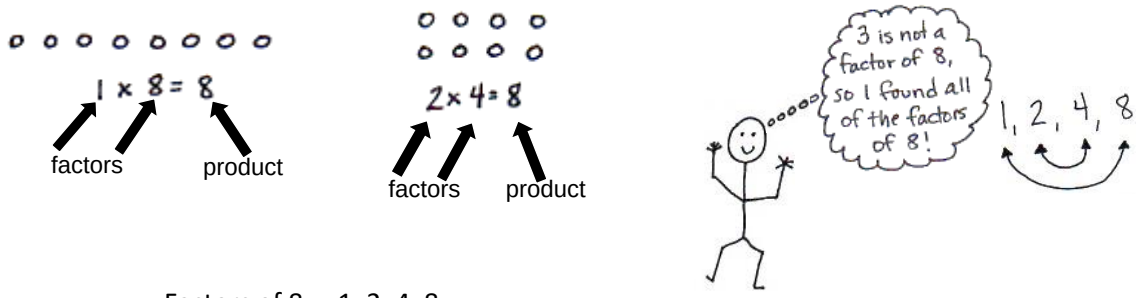
Prerequisite Support Lesson for Grade 6, Module 2, Topic D

Part A: Find Factor Pairs for a Given Whole Number

Example Problems

Identify the factors and product represented in an array by writing multiplication sentences represented by the arrays.

1)



Factors of 8: 1, 2, 4, 8

2)

Factors:

Identify the factors of the given numbers as multiplication sentences and as a list in order from least to greatest.

	Multiplication Sentences	Factors
3) 4		
4) 7		
5) 15		
6) 24		

Identify the factors pairs of the given numbers. List the factors below, from least to greatest.

7) Factor Pairs for 25	

Factors: _____

8) Factor Pairs for 28	

Factors: _____

9) Factor Pairs for 40	

Factors: _____

Problem Set

Identify the factors of the given numbers as multiplication sentences, and list in order from least to greatest.

	Multiplication Sentences	Factors
1) 6		
2) 9		
3) 12		
4) 13		
5) 16		
6) 19		
7) 21		
8) 30		
9) 36		
10) 45		

Identify the factors for the given numbers.

11) 54: _____

12) 57: _____

13) 48: _____

14) 11: _____

15) 23: _____

16) 60: _____

17) 42: _____

18) 27: _____

19) 35: _____

20) 100: _____

Eureka Acceleration Tools

Prerequisite Support Lesson for Grade 6, Module 2, Topic D

Part B: Multiples of One-digit Numbers

Adapted from Eureka Lesson: 4th Grade, Module 3, Topic F, Lesson 24

Use Prior to: 6th Grade, Module 2, Topic D, Lesson 17

Standard: 4.OA.B.4c

Using whole numbers in the range 1 – 100.

c. Determine whether a given whole number is a multiple of a given one-digit number.

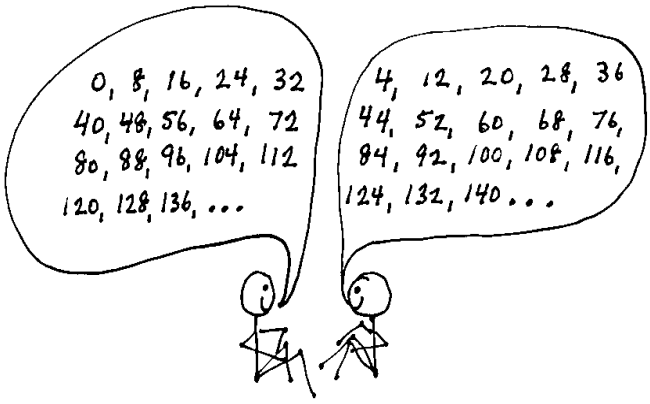
Purpose:

In Grade 6 students will use their knowledge of factors and multiples and apply that to the distributive property. Students will not only need to be fluent in finding multiples of whole numbers, but it is also important that students attend to precision when distinguishing between a factor and a multiple. This understanding is critical when they begin factoring expressions using the distributive property, so they are able to understand how a common factor can be used as a multiple of a sum of two numbers, without getting confused.

Note: This lesson focuses on the conceptual understanding of multiples and factors, and how they differ. Students often confuse the two, so frequent use of the correct terms (factor, multiple) and practice with these ideas together instead of in isolation will help students distinguish between the two. The Example Problems take a conceptual approach as students are guided to use what they know about numbers in order to determine multiples and multiples concepts. The Problem Set provides students the opportunity to apply these concepts independently.

Example Problems

1) Starting at 0, skip count by 4 with a partner.
   - Tell students to turn to a partner and count by 4s, taking turns with each new number. Give them one minute.
   - Ask the students if they noticed any patterns in the numbers (even numbers, pattern in how the numbers ended)
   - Tell students to continue counting by 4s for another minute.
   - Ask if the patterns continue (yes, the digits in the ones place of every number follow its own pattern, 0, 4, 8, 2, 6, over and over again).
   - Why does this pattern continue? (We keep adding 4 more each time)



2) These numbers are **multiples** of 4. How can we define the term multiples?

Possible responses: the numbers we get when we skip count; products of a number when multiplied with another number; a number that can be divided by another number without a remainder; what we get after multiplying a number by another number; the result of skip counting by each of the factors;

- Ask students if multiples of numbers ever stop. (No, we can keep adding 4 forever)
- Ask students if 0 is a multiple 4. (Yes, because when you multiply 4 by 0, the answer is 0)
- So is 0 a multiple of every number? (Yes, because any number you multiply by 0 results in 0).

3) List multiples of 5: 0, 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95,100…

4) List multiples of 9: 0, 9, 18, 27, 36, 45, 54, 63, 72, 81, 90, 99, 108…
   - Remind students that 0 is a multiple of every number since 0 times any number is 0.

5) List the multiples and factors of each of the numbers:

	Factors	Multiples
8	1, 2, 4, 8	0, 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96
10	1, 2, 5, 10	0, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 110, 120
12	1, 2, 3, 4, 6, 12	0, 12, 24, 36, 48, 60, 72, 84, 96, 108, 120, 132

- Discuss with students the difference between factors and multiples.
  - Have students define each in their own words. Let students share out.
  - Ask students what they notice about the set of factors or the multiples of the numbers. Do they notice any patterns? (Factors always have 1 and itself, There is a certain set of factors, but multiples go on forever, Factors include numbers equal to or less than the number; Besides 0, multiples of a number increase in size by that number and go on forever)
  - Consider taking a quick check for understanding at this point. Ask students to write factors for a number and then multiples of a number on personal boards.

6) Multiples Workspace
   - Instruct students to use the workspace as necessary as you ask questions about multiples.
   - How are factors and multiples different? (Factors are the numbers that go into a number. Multiples are the numbers we get when we multiply numbers. When listing factors, there is a certain number of factors, but when we find multiples, the list could go on forever.
   - How can you determine if one number (a) is a multiple of another number (b)? (You have to figure out if b is divisble by a or if b can divided by a evenly/without any remainders.
   - Use your workspace; explain why 24 is a multiple of 4. (Students could either draw an array to show 4 groups of 6 or write a multiplication sentence and explain that 4 times 6 is 24. 4 is a factor of 24)

- Is 24 a multiple of 5? (No, because we cannot skip count by 5 to 24, 24 divided by 5 has a remainder, 5 is not a factor of 24)
- Is 96 a multiple of 4? It is sort of a big number, what are some ways we can figure it out? (We could divide 96 by 4 and see if we get answer with no remainder. We could also try to half 96 two times).
- Yes, let's try something else first. What are some other factors of 96? (12 x 8 = 96). Okay, is 4 a factor of 8? (Yes)

- Let's use the associative property to help us determine if 4 is a factor of 96. Remember, the associative property states that when multiplying numbers, changing how they are grouped will not change the product. *Show students how to decompose 8 to 4 x 2 and then use the associative property to group 12 and 2, resulting in 24. This helps students realize that 24 x 4 = 96. Encourage students to use their workspace to do the math.*
    - 96 = 12 x 8
      96 = 12 x (4 x 2)
      96 = (12 x 2) x 4
      96 = 24 x 4
      96 = 96

- We can follow this approach when we think about other numbers. Think about multiples of 6, such as 12, 24, 30, 60, and 600. Is any multiple of 6 also a multiple of 2 and 3? Students can also divide to see if a number is a multiple of another number. *Some students may have grasped that using the associative property can help answer this question. If not, you may need to break it down again for them.*
    - 60 = 10 x 6
      60 = 10 x (2 x 3)
      60 = (10 x 2) x 3
      60 = 20 x 3
      60 = 60

      60 = 10 x 6
      60 = 10 x (2 x 3)
      60 = (10 x 3) x 2
      60 = 30 x 2
      60 = 60
    - Would this work for every multiple of 6? (Yes, because 6 is a multiple of 2 and 3)
    - How can this help us when determining if a number is a multiple of 2 or 3? (If a number is a multiple of 6, then we know that it is a multiple of both 2 and 3)
    - Try a few more examples with multiples of 6 if necessary.

- Let's try that with 8. Think of multiples of 8, such as 16, 24, 32, 40, 48, 80, and 800. What other factors could go into all of these multiples? (Since 8 is a multiple of 2 and 4, maybe 2 and 4 are also factors of each of these numbers.)
    - 80 = 10 x 8
      80 = 10 x (4 x 2)
      80 = (10 x 4) x 2
      80 = 40 x 2
      80 = 80

      80 = 10 x 8
      80 = 10 x (4 x 2)
      80 = (10 x 2) x 4
      80 = 20 x 4
      80 = 80
    - So what can be said about multiples of 8? (They are also multiples of 4 and 2)

7) Follow the directions below.

a. Circle in red the multiples of 2.

- Prompt students to look at the ones digits: 0, 2, 4, 6, 8

b. Shade in green the multiples of 3.

- Prompt students to find the sum of the digits, which is also a multiple of 3.

c. Circle in blue the multiples of 5.

- Prompt students to look at the ones digits: 0 or 5.

d. Draw an X over the multiples of 10.

- Prompt students find the commonality in all multiples of ten; end in a 0

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

- Use this activity to help students identify patterns among multiples. Use information gained in this lesson to complete the chart below. Answers may vary depending on the patterns discussed.

8) Use properties of multiplication to find patterns.

A number is a multiple of….	
2	if the number is even (ends in 0, 2, 4, 6, 8)
3	if the sum of the digits is a multiple of 3.
5	if the last digit is 0 or 5
10	if the number ends in a 0 or 5

Problem Set

1) Write the first 10 multiples of 8 starting from 0. 0, 8, 16, 24, 32, 40, 48, 56, 64, 72

2) Write the first 10 multiples of 3 starting from 0. 0, 3, 6, 9, 12, 15, 18, 21, 24, 27

3) Write the first 10 multiples of 7 starting from 0. 0, 7, 14, 21, 28, 35, 42, 49, 56, 63

4) Write the first 10 multiples of 5 starting from 100. 100, 105, 110, 115, 120, 125, 130, 135, 140, 140, 145, 150

5) Write the first 10 multiples of 4 starting from 20. 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, 60

6) Write the 1first 10 multiples of 6 starting from 36. 36, 42, 48, 54, 60, 66, 72, 78, 84, 90, 96

7) List the numbers that have 24 as a multiple. 1, 2, 3, 4, 6, 8, 12, 24

8) Use mental math, division, or the associative property to solve. (Use scratch paper if you like.)

   a. Is 12 a multiple of 4? Yes Is 4 a factor of 12? Yes

   b. Is 42 a multiple of 8? No Is 8 a factor of 42? No

   c. Is 84 a multiple of 6? Yes Is 6 a factor of 84? Yes

   d. Is 12 a multiple of 3? Yes Is 3 a factor of 12? Yes

   e. Is 48 a multiple of 8? Yes Is 48 a factor of 8? Yes

   f. Is 56 a multiple of 6? No Is 6 a factor of 56? No

9) Complete the pattern of multiples by skip-counting.

   0, 7, 14, 21, 28, 35, 42, 49, 56, 63, 70

10) Follow the instructions for each part.

   a. List the numbers that have 18 as a multiple.

      1, 2, 3, 6, 9, 18

   b. What are the factors of 18?

      1, 2, 3, 6, 9, 18

   c. Are the two lists the same? Why or why not?

      Yes they are the same. The numbers that have 18 as a multiple are the factors of 18.

Name: _____

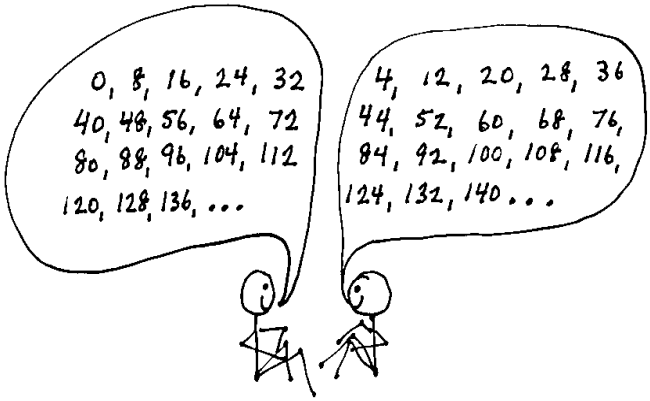
Eureka Acceleration Tools

Prerequisite Support Lesson for Grade 6, Module 2, Topic D

Part B: Multiples of One-digit Numbers

Example Problems

1) Starting at 0, skip count by 4 with a partner.



2) These numbers are **multiples** of 4. How can we define the term multiples?

_____

3) List multiples of 5: _____

4) List multiples of 9: _____

5) List the multiples and factors of each of the numbers:

	Factors	Multiples
8		
10		
12		

6) Multiples Workspace:

7) Follow the directions below.

a. Circle in red the multiples of 2.

b. Shade in green the multiples of 4.

c. Circle in blue the multiples of 5.

d. Draw an X over the multiples of 10.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

8) Use properties of multiplication to find patterns.

A number is a multiple of….	
2	If
3	If
5	If
10	If

Problem Set

1) Write the first 10 multiples of 8 starting from 0.

2) Write the first 10 multiples of 3 starting from 0.

3) Write the first 10 multiples of 7 starting from 0.

4) Write the first 10 multiples of 5 starting from 100.

5) Write the first 10 multiples of 4 starting from 20.

6) Write the first 10 multiples of 6 starting from 36.

7) List the numbers that have 24 as a multiple.

8) Use mental math, division, or the associative property to solve. (Use scratch paper if you like.)

   a. Is 12 a multiple of 4? _____ Is 4 a factor of 12? _____

   b. Is 42 a multiple of 8? _____ Is 8 a factor of 42? _____

   c. Is 84 a multiple of 6? _____ Is 6 a factor of 84? _____

   d. Is 12 a multiple of 3? _____ Is 3 a factor of 12? _____

   e. Is 48 a multiple of 8? _____ Is 48 a factor of 8? _____

   f. Is 56 a multiple of 6? _____ Is 6 a factor of 56? _____

9) Complete the pattern of multiples by skip-counting.

   0, 7, 14, _____, 28, _____, _____, _____, _____, _____, _____

10) Follow the instructions for each part.

   a. List the numbers that have 18 as a multiple.

   b. What are the factors of 18?

   c. Are the two lists the same? Why or why not?

Eureka Acceleration Tools

Prerequisite Support Lesson for Grade 6, Module 2, Topic D

Part C: Prime and Composite Numbers & Factors

Adapted from Eureka Lesson: 4th Grade, Module 3, Topic F, Lesson 22 - 23

Use Prior to: 6th Grade, Module 2, Topic D, Lesson 18

Standard: 4.OA.B.4a

Using whole numbers in the range 1 – 100.

d. Determine whether a given whole number is prime or composite.

Purpose:

Having a conceptual understanding of prime and composite numbers will help students become fluent in finding the greatest common factor in Grade 6. For example, if students can recognize that a number is prime they immediately know that the number only has two factors; therefore, if they are determining the GCF between a prime number and another number, students know that the prime number must be a factor of the second number in order for them to share a factor other 1.

Note: Before starting this lesson, students should have a solid understanding of factors and multiples. They will list factors to determine if numbers are prime or composite. Students will then use basic fact knowledge, divisibility rules, and the associative property to find factors of numbers, including larger numbers. Work through the first set of Example Problems together, and then allow the students to complete the first Problem Set on their own. Follow the same procedure for the next set of Example Problems and the Problem Set. Prompt students to be precise in their math language and encourage the use of "factors" and "multiples" as they explain their answers.

Example Problems

1. Identify the factors of the following numbers:
   a. 7: 1, 7
   b. 5: 1, 5
   c. 13: 1, 13
2. What do you notice about the number of factors for each of these numbers? There are only 2 factors or they only have one pair of factors, 1 and the number itself; Nothing else goes into the numbers evenly; The factors are odd.

3. The numbers in Problem 1 are prime numbers, which means that they only have one set of factors or exactly 2 factors, 1 and the number itself.
   - Encourage students to define prime numbers on their own based off the examples.
   - Ask students to give some more examples of prime numbers.

4. Identify the factors of the following numbers:
   a. 4: 1, 2, 4
   b. 12: 1, 2, 3, 4, 6, 12
   c. 15: 1, 3, 5, 15

5. What do you notice about the number of factors for each of these numbers? They each have more than two factors; They have at least one factor other than one and the number itself.

6. The numbers in Problem 4 are composite numbers, which means that they have more than two factors.

   - Encourage students to define composite numbers on their own based off the examples.

7. Identify the following statements as true or false. Explain why you chose that answer.
   - After students identify each statement after true and false, prompt them to explain why with examples. Consider letting them complete this part with a partner, and then have a class discussion.

   a. All odd numbers are prime numbers. False. Prime numbers are all odd, but not all odd numbers are prime. For example, 23 is a prime number because the only factors for 23 are 1 and 23, but 27 is composite because the factors for 27 are 1, 3, 9, 27.
   b. 2 is a composite number. False. 2 is not composite. It is prime because it only has two factors, 1 and itself.
   c. All even numbers greater than two are composite numbers. True. Besides 2, every even number is divisible by 2, so it will at least have 2 sets of factors, 1, itself, 2, and the number that can be doubled to equal the multiple.

Practice Problems

	Number	Factors	Prime (P) or Composite (C)
a.	9	The factors of 9 are: 1, 3, 9	C
b.	12	The factors of 12 are: 1, 2, 3, 4, 6, 12	C
c.	19	The factors of 19 are: 1, 19	P
d.	27	The factors of 27 are: 1, 3, 9, 27	C
e.	29	The factors of 29 are: 1, 29	P
f.	37	The factors of 37 are: 1, 37	P
g.	45	The factors of 45 are: 1, 3, 5, 9, 15, 45	C

Example Problems

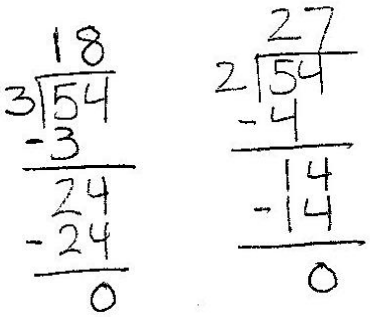
1. Use division to find factors of larger numbers.

   a. Is 3 a factor of 54?

   - Divide 54 and 3. Is there a remainder? (No)
   - Is 3 a factor of 54? (yes) Why? (No remainder)

   b. Is 2 a factor of 54?

   - Can you tell if 2 is a factor of 54 without dividing it? (Yes)



- How do you know? (It is an even number, 2 will always be a factor of that number).
- Divide to find out how many times 2 goes into 54. (27 times).

c. Is 3 a factor of 78?

- Is there a way to find out if 3 is factor of 78 without dividing? (Yes, if we add the digits of 78 the sum is 15, 15 is divisible by 3, or 15 is a multiple of 3, so we know that 78 is a multiple of 3, or that 3 is a factor of 78).
- Divide to find out how many times 3 will go into 78. (26)

d. Is 4 a factor of 94?

- Is there a way to find out if 4 is a factor of 94? (Yes, if the number can be halved twice, then we know the number is a multiple of 4 or that 4 is a factor of that number. 94 cannot be halved twice since 47 is half of 94 and 47 cannot be halved without a decimal or remainder.)
- Divide to find out if 4 is a factor of 94. (23 times with 2 left over)

e. Is 3 a factor of 87?

- Is there a way to find out if 3 is factor of 87 without dividing? (Yes, if we add the digits of 87 the sum is 15, 15 is divisible by 3, or is a multiple of 3, so we know that 87 is a multiple of 3, or that 3 is a factor of 87).
- Divide to find out how many times 3 will go into 87. (29)

2. Use the associative property to find additional factors of larger numbers.

a. Is 5 a factor of 54?

- Do we need to divide to find this answer? (No, because all multiple of 5 end in either a zero or 5, so 54 is not a multiple of 54, or 5 is not a factor of 54).

b. Is 2 or 3 a factor of 54?

- We found out that 3 and 2 are factors of 54 by dividing, but let's use the associative property to prove that 2 and 3 are factors of 54. Use 54 = 6 x 9 to prove this.
- Break apart 6 into 2 x 3 and then use the associative property to find the factor pairs of 2 and 3 that result in the product of 54.

54 = 6 x 9	54 = 6 x 9
54 = (2 x 3) x 9	54 = (2 x 3) x 9
54 = 2 x (3 x 9)	54 = 3 x (2 x 9)
54 = 2 x 27	54 = 3 x 18
54 = 54	54 = 54

c. Is 2 or 3 a factor of 42?

- Let's try that same procedure with 42. Is 6 a factor of 42? (Yes, 6 x 7 = 42).

42 = 6 x 7	42 = 6 x 7
42 = (2 x 3) x 7	42 = (2 x 3) x 7
42 = 2 x (3 x 7)	42 = 3 x (2 x 7)
42 = 2 x 21	42 = 3 x 14
42 = 42	42 = 42

3. Use division or the associative property to find factors of larger numbers.

a. Prove that 2 and 3 are factors of 72 since 6 is a factor of 72.

72 = 6 x 12	72 = 6 x 12
72 = (2 x 3) x 12	72 = (2 x 3) x 12
72 = 2 x (3 x 12)	72 = 3 x (2 x 12)
72 = 2 x 36	72 = 3 x 24
72 = 72	72 = 42

b. Prove that 2 and 4 are factors of 56, since 8 is a factor of 56.

56 = 8 x 7	56 = 8 x 7
56 = (2 x 4) x 7	56 = (2 x 4) x 7
56 = 2 x (4 x 7)	56 = 4 x (2 x 7)
56 = 2 x 28	56 = 4 x 14
56 = 56	56 = 42

Problem Set

1. Explain your thinking or use division to answer the following.

a. Is 2 a factor of 84?

Yes. 84 is an even number. 2 is a factor of every even number.

	4	2
2	8	4
−	8	
	0	4
−		4
		0

b. Is 2 a factor of 83?

No. 83 is an odd number. 2 is not a factor of odd numbers.

	4	1
2	8	3
−	8	
	0	3
−		2
		1

c. Is 3 a factor of 84?

Yes. The sum of the digits of the product 84 is 12. 12 is divisible by 3, so 84 is divisible by 3.

	2	8
3	8	4
−	6	
	2	4
−	2	4
		0

d. Is 2 a factor of 92?

Yes. 92 is an even number. 2 is a factor of every even number.

	4	6
2	9	2
−	8	
	1	2
−	1	2
		0

e. Is 6 a factor of 84?

Yes. 6 times 12 is 72. If I add 6 two more times, the sum will be 84.

	1	4
6	8	4
−	6	
	2	4
−	2	4
		0

f. Is 4 a factor of 92?

Yes. 92 can be halved twice. Half of 92 is 46. Half of 46 is 23.

	2	3
4	9	2
−	8	
	1	2
−	1	2
		0

g. Is 5 a factor of 84?

No. Multiples of 5 end in either 0 or 5. 84 does not, so 5 is not a factor of 84.

	1	6
5	8	4
−	5	
	3	4
−	3	0
		4

h. Is 8 a factor of 92?

No. 8 times 11 is 88, and 8 times 12 is 96.

	1	1
8	9	2
−	8	
	1	2
−		8
		4

Graphic Solutions Sources: Calculator Soup https://www.calculatorsoup.com/calculators/math/longdivision.php

2. Use the associative property to explain why any number that has 9 as a factor also has 3 as a factor. Provide an explanation and three examples.

3 is a factor of 9. 3 times 3 equals 9. So whatever you multiply 9 by, you can also multiply the same number by 3 two times, and you will get the same answer. For example:

9 is a factor of 63.	9 is a factor of 36.	9 is a factor of 81.
63 = 9 x 7	36 = 9 x 4	81 = 9 x 9
63 = (3 x 3) x 7	36 = (3 x 3) x 4	81 = (3 x 3) x 9
63 = 3 x (3 x 7)	36 = 3 x (3 x 4)	81 = 3 x (3 x 9)
63 = 3 x 21	36 = 3 x 12	81 = 3 x 27
63 = 63	36 = 36	81 = 81

Name: _____

Eureka Acceleration Tools

Prerequisite Support Lesson for Grade 6, Module 2, Topic D

Part C: Prime and Composite Numbers & Factors

Example Problems

1. Identify the factors of the following numbers:
   a. 7:
   b. 5:
   c. 13:
2. What do you notice about the number of factors for each of these numbers?
   _____
3. The numbers in Problem 1 are _____ numbers, which means that they
   _____.

4. Identify the factors of the following numbers:
   a. 4:
   b. 12:
   c. 15:
5. What do you notice about the number of factors for each of these numbers?
   _____
6. The numbers in Problem 4 are _____ numbers, which means that they
   _____.

7. Identify the following statements as true or false. Explain why you chose that answer.
   a. All odd numbers are prime numbers. _____
   b. 2 is a composite number. _____
   c. All even numbers greater than two are composite numbers. _____

Practice Problems

	Number	Factors	Prime (P) or Composite (C)
a.	9	The factors of 9 are:	
b.	12	The factors of 12 are:	
c.	19	The factors of 19 are:	
d.	27	The factors of 27 are:	
e.	29	The factors of 29 are:	
f.	37	The factors of 37 are:	

g.	45	The factors of 45 are:	

Example Problems

1. Use division to find factors of larger numbers.

    a. Is 3 a factor of 54?

    b. Is 2 a factor of 54?

    c. Is 3 a factor of 78?

    d. Is 4 a factor of 94?

    e. Is 3 a factor of 87?

2. Use the associative property to find additional factors of larger numbers.

    a. Is 5 a factor of 54?

    b. Is 2 or 3 a factor of 54?

    c. Is 2 or 3 a factor of 42?

3. Use division or the associative property to find factors of larger numbers.

    a. Prove that 2 and 3 are factors of 72 since 6 is a factor of 72.

    b. Prove that 2 and 4 are factors of 56, since 8 is a factor of 56.

Problem Set

1. Explain your thinking or use division to answer the following.

a. Is 2 a factor of 84?	b. Is 2 a factor of 83?
c. Is 3 a factor of 84?	d. Is 2 a factor of 92?
e. Is 6 a factor of 84?	f. Is 4 a factor of 92?
g. Is 5 a factor of 84?	h. Is 8 a factor of 92?

2. Use the associative property to explain why any number that has 9 as a factor also has 3 as a factor. Provide an explanation and three examples.